Assignment Set 2, G22.2421/G63.2020
Spring 2003
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This assignment is due in two weeks.
Consider five different linear multistep schemes, using three levels, for solving
\[ y'(t) = f(t, y(t)), \quad y(t_0) = y_0, \]
namely, the Adams-Bashforth, the Adams-Moulton, the backward differentiation, the explicit mid-point rule, and the Milne’s scheme (based on Simpson’s rule.) Note that the fourth and fifth schemes were discussed in class two weeks ago.

Write down the five schemes, show that they all satisfy the root condition and determine their order of accuracy.

Implement them, e.g., in matlab, for fixed step size \( h \); do not try to use variable steps or step size control. For the implicit schemes, use Newton’s method to solve the nonlinear equation. Also assume that we only have one unknown \( y(t) \) rather than a system of differential equations.

Test the different methods for the problem given by
\[ f(t, y) = -100(y(t) - \sin(t)), \quad y(0) = 1, \]
and for a variety of mesh sizes. Note that after a short initial time interval, in which the solution decays rapidly, the solution is close to \( \sin(t) \).

Also construct other examples which show off different methods to their advantage. Recall that we showed in class that there should be cases where the Milne scheme is accurate and nonproblematic, while in other cases it fails badly. Note that given any function \( y(t) \), it is always possible to construct ordinary differential equations with \( y(t) \) as a solution; in fact all that has to be done is to construct a function \( y(t) \) with the desirable properties and then compute its derivative. More elaborate constructions are also possible.