Homework set 5: Due April 17.

1. Paper and pencil problems:

2. (a) Write and test a program for the adaptive Simpson quadrature rule. Recall, that there was a handout about this method.
   The integrand is defined by a subroutine which provides values of the integrand $f(x)$ for any given input value $x$. This algorithm is recursive and if your program is properly designed the value of $f(x)$ should never be computed more than once for any particular value of $x$. You can check if you do it right by counting the number of quadrature nodes and the number of function calls.
   When the program is running, there is an active interval the contribution of which to the final approximate value of the integral, we are trying to compute accurately enough. The overall tolerance, a positive number, $\epsilon$, is provided as input. The contribution of each subinterval, to the overall error should not exceed $\epsilon \times$ the length of the subinterval measured as a fraction of the entire, given interval.
   The first active interval is the entire interval. If we decide that the quadrature rule is not accurate enough, the left half of the active interval becomes the active interval. Once the contribution from this interval has been computed accurately enough, we compute the contribution of the right half of the interval. This is a recursive procedure.
You can either use recursive calls if it is available in the pro-
gramming language that you are using, or construct stacks to
accomplish the savings of the function values, already computed,
that you need later. A stack can easily be implemented using a
vector and an index that serves as a pointer. Note that MATLAB
provides for the use of recursion.
Test your program by finding the approximate value, for several
values of the tolerance, of
\[
\int_0^1 x^{1/2} dx,
\]
\[
\int_0^1 (1 - x^2)^{3/2} dx
\]
and
\[
\int_0^1 \frac{\sin(x)}{x^{3/2}} dx
\]
and compare the results with the exact values of the integrals if
they are available.
Note that the integrand can take on infinite values at some points
and that a small neighborhood of such a point has to be treated
in a different way. Please discuss what you do in such a case.
Also, make graphs of the functions which should also display the
location of the quadrature nodes.
(b) Find the numerical quadrature methods provided by Matlab and
use them to solve the same problems. Compare the results with
those obtained by your own program.
(c) Finally, use whatever program you wish to solve problems 8.4 and
8.6 on pp. 265–266 of the text book.