Problem Set 5
(due Monday, November 23)

Problem 1 Given a digraph $G$, say that a vertex $v$ is a sink in $G$ if every other vertex is an in-neighbor of $v$, but no vertex is an out-neighbor of $v$. Design an algorithm that, given the adjacency matrix of a digraph, determines whether the digraph contains a sink. Your algorithm should run in time $O(n)$, where $n$ is the number of vertices.

Problem 2 Design an algorithm that, given a dag, finds the longest path in the dag. Your algorithm should run in time $O(m + n)$, where $m$ and $n$ are the number of edges and vertices, respectively.

Problem 3 Design an efficient algorithm that, given a weighted graph with at least 2 spanning trees, finds the second-cheapest spanning tree.

**Hint:** See Problem 24.1 in the textbook for some help.

Problem 4 Suppose that you have a weighted digraph $G$ whose weights are integers in the interval $1..5$. Design an algorithm that, given $G$ and two vertices $s$ and $t$, finds the shortest path from $s$ to $t$. Your algorithm should run in time $O(m + n)$. 