CSCI-GA.3033-004

Graphics Processing Units (GPUs): Architecture and Programming

Lecture 4:

CUDA Threads & Memories

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Software <-> Hardware

• From a programmer’s perspective:
  – Blocks
  – Kernel
  – Threads
  – Grid

• Hardware Implementation:
  – SMs
  – SPs (per SM)
  – Warps
Some Restrictions First

- All threads in a grid execute the same kernel function.
- A grid is organized as a 2D or 3D array of blocks (gridDim.x, gridDim.y, and gridDim.z).
- Each block is organized as a 3D array of threads (blockDim.x, blockDim.y, and blockDim.z).
- Once a kernel is launched, its dimensions cannot change.
- All blocks in a grid have the same dimension.
- The total size of a block has an upper bound.
- Once assigned to an SM, the block must execute in its entirety by the SM.
Compute Capability

• A standard way to expose hardware resources to applications.
• CUDA compute capability starts with 1.0 and latest one is 7.x (as of today)
• API: cudaGetDeviceProperties()
cudaError_t cudaGetDeviceProperties(
    struct cudaDeviceProp * prop, int device)

cudaError_t
    cudaGetDeviceCount(
        int * count )

struct cudaDeviceProp {
    char name[256];
    size_t totalGlobalMem; /* in bytes */
    size_t sharedMemPerBlock; /* in bytes */
    int regsPerBlock;
    int warpSize;
    int maxThreadsPerBlock;
    int maxThreadsDim[3];
    int maxGridSize[3];
    int clockRate; /* in KHz */
    size_t totalConstMem;
    int major;    int minor;
    int multiProcessorCount;
    int concurrentKernels;
    int unifiedAddressing;
    int memoryClockRate;
    int memoryBusWidth;
    int l2CacheSize;
    int maxThreadsPerMultiProcessor;
    ... and a lot of other stuff}
## Compute Capability Example

Table 1. A Comparison of Maxwell GM107 to Kepler GK107

<table>
<thead>
<tr>
<th></th>
<th>GK107 (Kepler)</th>
<th>GM107 (Maxwell)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA Cores</td>
<td>384</td>
<td>640</td>
</tr>
<tr>
<td>Base Clock</td>
<td>1058 MHz</td>
<td>1020 MHz</td>
</tr>
<tr>
<td>GPU Boost Clock</td>
<td>N/A</td>
<td>1085 MHz</td>
</tr>
<tr>
<td>GFLOP/s</td>
<td>812.5</td>
<td>1305.6</td>
</tr>
<tr>
<td>Compute Capability</td>
<td>3.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Shared Memory / SM</td>
<td>16KB / 48 KB</td>
<td>64 KB</td>
</tr>
<tr>
<td>Register File Size / SM</td>
<td>256 KB</td>
<td>256 KB</td>
</tr>
<tr>
<td>Active Blocks / SM</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>Memory Clock</td>
<td>5000 MHz</td>
<td>5400 MHz</td>
</tr>
<tr>
<td>Memory Bandwidth</td>
<td>80 GB/s</td>
<td>86.4 GB/s</td>
</tr>
<tr>
<td>L2 Cache Size</td>
<td>256 KB</td>
<td>2048 KB</td>
</tr>
<tr>
<td>TDP</td>
<td>64W</td>
<td>60W</td>
</tr>
<tr>
<td>Transistors</td>
<td>1.3 Billion</td>
<td>1.87 Billion</td>
</tr>
<tr>
<td>Die Size</td>
<td>118 mm(^2)</td>
<td>148 mm(^2)</td>
</tr>
<tr>
<td>Manufacturing Process</td>
<td>28 nm</td>
<td>28 nm</td>
</tr>
</tbody>
</table>
Figure 3.2. An Example of CUDA Thread Organization.
Revisiting Matrix Multiplication

```c
// Matrix multiplication kernel - thread specification
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
    // 2D Thread ID
    int tx = threadIdx.x;
    int ty = threadIdx.y;

    // Pvalue stores the Pd element that is computed by the thread
    float Pvalue = 0;

    for (int k = 0; k < Width; ++k)
    {
        float Mdelement = Md[ty * Width + k];
        float Ndelement = Nd[k * Width + tx];
        Pvalue += Mdelement * Ndelement;
    }

    // Write the matrix to device memory each thread writes one element
    Pd[ty * Width + tx] = Pvalue;
}
```

This is what we did before... What is the main shortcoming??
Revisiting Matrix Multiplication

// Matrix multiplication kernel - thread specification
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
    // 2D Thread ID
    int tx = threadIdx.x;
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    // Pvalue stores the Pd element that is computed by the thread
    float Pvalue = 0;

    for (int k = 0; k < Width; ++k)
    {
        float Mdelement = Md[ty * Width + k];
        float Ndelement = Nd[k * Width + tx];
        Pvalue += Mdelement * Ndelement;
    }

    // Write the matrix to device memory each thread writes one element
    Pd[ty * Width + tx] = Pvalue;
}

Can only handle limited elements in each dimension!

Reason:
We used 1 block, and a block is limited to X threads (X depends on GPU type)
Revisiting Matrix Multiplication

- Break-up $P_d$ into tiles
- Each block calculates one tile
  - Each thread calculates one element
  - Block size equals tile size
Revisiting Matrix Multiplication

TILE_WIDTH = 2

Block(0,0)  Block(1,0)

\[
\begin{array}{cccc}
P_{0,0} & P_{1,0} & P_{2,0} & P_{3,0} \\
P_{0,1} & P_{1,1} & P_{2,1} & P_{3,1} \\
P_{0,2} & P_{1,2} & P_{2,2} & P_{3,2} \\
P_{0,3} & P_{1,3} & P_{2,3} & P_{3,3} \\
\end{array}
\]
Revisiting Matrix Multiplication

// Setup the execution configuration
    dim3 dimGrid(Width/TILE_WIDTH, Width/TILE_WIDTH);
    dim3 dimBlock(TILE_WIDTH, TILE_WIDTH);

// Launch the device computation threads!
MatrixMulKernel<<<dimGrid, dimBlock>>>(Md, Nd, Pd, Width);

__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
    // Calculate the row index of the Pd element and M
    int Row = blockIdx.y*TILE_WIDTH + threadIdx.y;
    // Calculate the column index of Pd and N
    int Col = blockIdx.x*TILE_WIDTH + threadIdx.x;

    float Pvalue = 0;
    // each thread computes one element of the block sub-matrix
    for (int k = 0; k < Width; ++k)
        Pvalue += Md[Row*Width+k] * Nd[k*Width+Col];

    Pd[Row*Width+Col] = Pvalue;
}
Synchronization

\texttt{__syncthreads()}

- called by a kernel function
- The thread that makes the call will be held at the calling location until every thread in the block reaches the location
- Beware of if-then-else
- Threads in different blocks cannot synchronize -> CUDA runtime system can execute blocks in any order
Each block can execute in any order relative to other blocks.

The ability to execute the same application code on hardware with different number of execution resources is called transparent scalability.
Thread Assignment

• Threads assigned to execution resources on a block-by-block basis.

• CUDA runtime automatically reduces number of blocks assigned to each SM until resource usage is under limit.

• Runtime system:
  – maintains a list of blocks that need to execute
  – assigns new blocks to SM as they compute previously assigned blocks

• Example of SM resources
  – computational units
  – number of threads that can be simultaneously tracked and scheduled.
  – registers
  – shared memory
GT200 can accommodate 8 blocks/SM and up to 1024 threads can be assigned to an SM.

What are our choices for number of blocks and number of threads/block?

Thread scheduling is an implementation concept.
Warps

- Once a block is assigned to an SM, it is divided into units called warps.
  - Thread IDs within a warp are consecutive and increasing
  - Warp 0 starts with Thread ID 0
  - Partitioning is always the same

- Warp size is implementation specific.
- Warp is unit of thread scheduling in SMs
Warps

- **DO NOT** rely on any execution order among warps
  - That is, we cannot tell which warp will finish first.

- Each warp is executed in a SIMD fashion (i.e. all threads within a warp must execute the same instruction at any given time).
  - Problem: **branch divergence**
Branch Divergence in Warps

- occurs when threads inside warps branches to different execution paths.

50% performance loss
Example of underutilization

Computational Resource Utilization

32 warps, 32 threads per warp, round-robin scheduling
Dealing With Branch Divergence

• A common case: avoid divergence when branch condition is a function of thread ID
  – Example with divergence:
    • If (threadIdx.x > 2) { }
    • This creates two different control paths for threads in a block
  – Example without divergence:
    • If (threadIdx.x / WARP_SIZE > 2) { }
    • Also creates two different control paths for threads in a block
    • Branch granularity is a whole multiple of warp size; all threads in any given warp follow the same path

• There is a big body of research for dealing with branch divergence
Latency Tolerance

• When an instruction executed by the threads in a warp must wait for the result of a previously initiated long-latency operation, the warp is not selected for execution -> latency hiding
• Priority mechanism used to schedule ready warps
• Scheduling does not introduce idle time -> zero-overhead thread scheduling
• Scheduling is used for tolerating long-latency operations, such as:
  – pipelined floating-point arithmetic
  – branch instructions
This ability of tolerating long-latency operation is the main reason why GPUs do not dedicate as much chip area to cache memory and branch prediction mechanisms as traditional CPUs.
**Exercise:** Suppose 4 clock cycles are needed to dispatch the same instruction for all threads in a Warp in G80. If there is one global memory access every 4 instructions, how many warps are needed to fully tolerate 200-cycle memory latency?
Exercise

The GT200 has the following specs (maximum numbers):

- 512 threads/block
- 1024 threads/SM
- 8 blocks/SM
- 32 threads/warp

What is the best configuration for thread blocks to implement matrix multiplications 8x8, 16x16, or 32x32?
Exercise

If a CUDA device’s SM can take up to 1,536 threads and up to 4 blocks, which of the following block configs would result in the most number of threads in the SM?

- 128 threads/blk
- 256 threads/blk
- 512 threads/blk
- 1,024 threads/blk
Exercise

• For a vector addition, assume that the vector length is 2,000, each thread calculates one output element, and the thread block size 512 threads. How many threads will be in the grid?

• Given the above, how many warps do you expect to have divergence due to the boundary check on the vector length?
Exercise

A CUDA programmer says that if they launch a kernel with only 32 threads in each block, they can leave out the __syncthreads() instruction wherever barrier synchronization is needed. Do you think this is a good idea? Explain.
Motivational Example

• G80 supports 86.4 GB/s of global memory access
• Single precision floating point = 4 bytes
• Then we cannot load more than $86.4/4 = 21.6$ giga single precision data per second
• Theoretical peak performance of G80 is 367 gigaflops!
Let’s talk about memory ...
Computation vs Memory Access

• Compute to global memory access (CGMA) ratio

• Definition: The number of FP calculations performed for each access to the global memory within a region in a CUDA program.
Computation vs Memory Access

```c
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
    // Calculate the row index of the Pd element and M
    int Row = blockIdx.y*TILE_WIDTH + threadIdx.y;
    // Calculate the column index of Pd and N
    int Col = blockIdx.x*TILE_WIDTH + threadIdx.x;

    float Pvalue = 0;
    // each thread computes one element of the block sub-matrix
    for (int k = 0; k < Width; k++)
        Pvalue += Md[Row*Width+k] * Nd[k*Width+Col];

    Pd[Row*Width+Col] = Pvalue;
}
```

2 memory accesses
1 FP multiplication
1 FP addition
so CGMA = 1
Main Goals

- How to make the best use of the GPU memory system?
- How to deal with hardware limitation?
Registers
- Fastest.
- Do not consume off-chip bandwidth.
- Only accessible by a thread.
- Lifetime of a thread.

- Registers
- Shared Memory
- Thread (0, 0)
- Thread (1, 0)
- Block (0, 0)
- Block (1, 0)
- Global Memory
- Constant Memory
- Host
**Shared Memory**
- Extremely fast
- Highly parallel
- Restricted to a block
- Example: Fermi’s shared/L1 is 1+TB/s aggregate
Global Memory

- Typically implemented in DRAM
- High access latency: 400-800 cycles
- Finite access bandwidth
- Potential of traffic congestion
- Throughput of about 320GB/s (as of 2017)

Traffic congestion prevents all but a few threads from making progress.
Grid

Block (0, 0)
  - Shared Memory
  - Registers
  - Thread (0, 0)
  - Thread (1, 0)

Block (1, 0)
  - Shared Memory
  - Registers
  - Thread (0, 0)
  - Thread (1, 0)

Host

Global Memory

Constant Memory

Constant Memory
  - Read only
  - Short latency and high bandwidth when all threads access the same location
Important!

• Each access to registers involves fewer instructions than global memory.

• Aggregate register files bandwidth = ~two orders of magnitude that of the global memory!

• Energy consumed for accessing a value from the register file =~ at least an order of magnitude lower than accessing global memory!

• Shared memory is part of the address space → accessing it requires load/store instructions.
**Scope**: the range of threads that can access a variable

**Lifetime**: the portion of the program’s execution when the variable is available for use.

__device__ is optional when used with __shared__, or __constant__.  

Automatic variables reside in a register.
<table>
<thead>
<tr>
<th>Variable declaration</th>
<th>Memory</th>
<th>Scope</th>
<th>Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>int LocalVar;</td>
<td>register</td>
<td>thread</td>
<td>thread</td>
</tr>
<tr>
<td><strong>device</strong> <strong>shared</strong> int SharedVar;</td>
<td>shared</td>
<td>block</td>
<td>block</td>
</tr>
<tr>
<td><strong>device</strong> int GlobalVar;</td>
<td>global</td>
<td>grid</td>
<td>application</td>
</tr>
<tr>
<td><strong>device</strong> <strong>constant</strong> int ConstantVar;</td>
<td>constant</td>
<td>grid</td>
<td>application</td>
</tr>
</tbody>
</table>

Automatic array variables local to a thread reside in **local memory**.

Does not physically exist. It is an abstraction to the local scope of a thread. Actually put in global memory by the compiler.
The variable must be declared within the kernel function body; and will be available only within the kernel code.

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<td>register</td>
<td>thread</td>
<td>thread</td>
</tr>
<tr>
<td><strong>device</strong> <strong>shared</strong> int SharedVar;</td>
<td>shared</td>
<td>block</td>
<td>block</td>
</tr>
<tr>
<td><strong>device</strong> int GlobalVar;</td>
<td>global</td>
<td>grid</td>
<td>application</td>
</tr>
<tr>
<td><strong>device</strong> <strong>constant</strong> int ConstantVar;</td>
<td>constant</td>
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</tr>
<tr>
<td>------------------------------------------</td>
<td>--------</td>
<td>-------</td>
<td>----------------</td>
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<td>thread</td>
<td>thread</td>
</tr>
<tr>
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<td>shared</td>
<td>block</td>
<td>block</td>
</tr>
<tr>
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<td>global</td>
<td>grid</td>
<td>application</td>
</tr>
<tr>
<td><strong>device</strong> <strong>constant</strong> int ConstantVar;</td>
<td>constant</td>
<td>grid</td>
<td>application</td>
</tr>
</tbody>
</table>

The variable must be declared outside of any function.
• Declaration of constant variables must be outside any function body.
• Currently total size of constant variables in an application is limited to 64KB.
By declaring a CUDA variable in one of the CUDA memory types, a CUDA programmer dictates the visibility and access speed of the variable.
A Note About Local Memory

• Local for each thread
• Check local memory usage: `nvcc -Xptxas -v`
  – will print number of lmem bytes for each kernel, if the kernel uses local memory.
• Used for:
  – Arrays not accessed by constant indices
  – Large structures and arrays that do not fit into registers
  – If thread uses a lot of registers the runtime uses register spilling into local memory to increase concurrency in the SM.
• Internally, the hardware has a limit in the number of registers per thread.
• As programmer you can specify the maximum registers a thread can use
  o `nvcc -maxrregcount num` ...
  o If `num >` hardware limit, then register spilling happens.
Reducing Global Memory Traffic

• Global memory access is performance bottleneck.
• The lower CGMA the lower the performance
• Reducing global memory access enhances performance.
• A common strategy is **tiling**: partition the data into subsets called tiles, such that each tile fits into the shared memory. We saw this earlier.
Outline of Tiling

- Identify a tile of global memory contents that are accessed by multiple threads
- Load the tile from global memory into on-chip memory (e.g. shared memory)
- Use barrier synchronization to make sure that all threads are ready to start the phase
- Have the multiple threads to access their data from the on-chip memory
- Use barrier synchronization to make sure that all threads have completed the current phase
- Move on to the next tile
Back to Matrix Multiplication

TILE_WIDTH = 2
## Back to Matrix Multiplication

<table>
<thead>
<tr>
<th></th>
<th>$P_{0,0}$ thread$_{0,0}$</th>
<th>$P_{1,0}$ thread$_{1,0}$</th>
<th>$P_{0,1}$ thread$_{0,1}$</th>
<th>$P_{1,1}$ thread$_{1,1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{0,0} \cdot N_{0,0}$</td>
<td>$M_{0,0} \cdot N_{1,0}$</td>
<td>$M_{0,1} \cdot N_{0,0}$</td>
<td>$M_{0,1} \cdot N_{1,0}$</td>
<td></td>
</tr>
<tr>
<td>$M_{1,0} \cdot N_{0,1}$</td>
<td>$M_{1,0} \cdot N_{1,1}$</td>
<td>$M_{1,1} \cdot N_{0,1}$</td>
<td>$M_{1,1} \cdot N_{1,1}$</td>
<td></td>
</tr>
<tr>
<td>$M_{2,0} \cdot N_{0,2}$</td>
<td>$M_{2,0} \cdot N_{1,2}$</td>
<td>$M_{2,1} \cdot N_{0,2}$</td>
<td>$M_{2,1} \cdot N_{1,2}$</td>
<td></td>
</tr>
<tr>
<td>$M_{3,0} \cdot N_{0,3}$</td>
<td>$M_{3,0} \cdot N_{1,3}$</td>
<td>$M_{3,1} \cdot N_{0,3}$</td>
<td>$M_{3,1} \cdot N_{1,3}$</td>
<td></td>
</tr>
</tbody>
</table>

**Access order**

- $P_{0,0}$ thread$_{0,0}$
- $P_{1,0}$ thread$_{1,0}$
- $P_{0,1}$ thread$_{0,1}$
- $P_{1,1}$ thread$_{1,1}$

The first access is $M_{0,0} \cdot N_{1,0}$ and the last access is $M_{3,1} \cdot N_{1,3}$. The order of access is indicated by the circled terms.
Back to Matrix Multiplication

- The basic idea is to make threads that use common elements collaborate.
- Each thread can load different elements into the shared memory before calculations.
- These elements will be used by the thread that loaded them and other threads that share them.
## Back to Matrix Multiplication

<table>
<thead>
<tr>
<th>Time</th>
<th>Phase 1</th>
<th>Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{0,0}$</td>
<td>$\text{Md}_{0,0}$</td>
<td>$\text{Md}_{2,0}$</td>
</tr>
<tr>
<td></td>
<td>$\downarrow$ $\text{Mds}_{0,0}$</td>
<td>$\downarrow$ $\text{Mds}_{0,0}$</td>
</tr>
<tr>
<td></td>
<td>$\text{Nd}_{0,0}$</td>
<td>$\text{Nd}_{0,2}$</td>
</tr>
<tr>
<td></td>
<td>$\downarrow$ $\text{Nds}_{0,0}$</td>
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</tr>
<tr>
<td></td>
<td>$\text{PValue}<em>{0,0} = \text{Mds}</em>{0,0} \cdot \text{Nds}<em>{0,0} + \text{Mds}</em>{1,0} \cdot \text{Nds}_{0,1}$</td>
<td>$\text{PValue}<em>{0,0} = \text{Mds}</em>{0,0} \cdot \text{Nds}<em>{0,0} + \text{Mds}</em>{1,0} \cdot \text{Nds}_{0,1}$</td>
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<tr>
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<td>$\text{PValue}<em>{1,0} = \text{Mds}</em>{0,0} \cdot \text{Nds}<em>{1,0} + \text{Mds}</em>{1,0} \cdot \text{Nds}_{1,1}$</td>
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**Phase 1** and **Phase 2** represent different stages in the process. Time progresses from left to right.
Back to Matrix Multiplication

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<thead>
<tr>
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</table>

Shared Memory

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<tr>
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Shared Memory

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Back to Matrix Multiplication

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</table>

Shared Memory

Shared Memory
Back to Matrix Multiplication

• Potential reduction in global memory traffic in matrix multiplication example is proportional to the dimension of the blocks used.
  – With $N \times N$ blocks the potential reduction would be $N$

• If an input matrix is of dimension $M$ and the tile size is $TILE\_WIDTH$, the dot product will be performed in $M/TILE\_WIDTH$ phases.
Back to Matrix Multiplication

```
<table>
<thead>
<tr>
<th>Md</th>
<th>by</th>
<th>Pd</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>bx</td>
<td>k</td>
</tr>
<tr>
<td></td>
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<td>Pd_sub</td>
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</tbody>
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<th>012 TILE_WIDTH-1</th>
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<tbody>
<tr>
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</tbody>
</table>
```
Back to Matrix Multiplication

```c
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
    __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
    __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

    int bx = blockIdx.x; int by = blockIdx.y;
    int tx = threadIdx.x; int ty = threadIdx.y;

    // Identify the row and column of the Pd element to work on
    int Row = by * TILE_WIDTH + ty;
    int Col = bx * TILE_WIDTH + tx;

    float Pvalue = 0;
    // Loop over the Md and Nd tiles required to compute the Pd element
    for (int m = 0; m < Width/TILE_WIDTH; ++m) {

        // Collaborative loading of Md and Nd tiles into shared memory
        Mds[ty][tx] = Md[Row*Width + (m*TILE_WIDTH + tx)];
        Nds[ty][tx] = Nd[(m*TILE_WIDTH + ty)*Width + Col];
        __syncthreads();

        for (int k = 0; k < TILE_WIDTH; ++k)
            Pvalue += Mds[ty][k] * Nds[k][tx];
        __syncthreads();
    }
    Pd[Row*Width + Col] = Pvalue;
}
```

The Phases
Back to Matrix Multiplication

```c
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x; int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  // Identify the row and column of the Pd element to work on
  int Row = by * TILE_WIDTH + ty;
  int Col = bx * TILE_WIDTH + tx;

  float Pvalue = 0;
  // Loop over the Md and Nd tiles required to compute the Pd element
  for (int m = 0; m < Width/TILE_WIDTH; ++m)
  {
    // Collaborative loading of Md and Nd tiles into shared memory
    Mds[ty][tx] = Md[Row*Width + (m*TILE_WIDTH + tx)];
    Nds[ty][tx] = Nd[(m*TILE_WIDTH + ty)*Width + Col];
    __syncthreads();

    for (int k = 0; k < TILE_WIDTH; ++k)
    {
      Pvalue += Mds[ty][k] * Nds[k][tx];
    }
    __syncthreads();
  }
  Pd[Row*Width + Col] = Pvalue;
}
```

to be sure needed elements are loaded
to be sure calculations are completed
Exercise

Can we use shared memory to reduce global memory bandwidth for matrix addition?
Do you Remember the G80 example?

- 86.4 GB/s global memory bandwidth
- In matrix multiplication if we use 16x16 blocks -> reduction in memory traffic by a factor of 16
- Global memory can now support \([(86.4/4) \times 16]\) = 345.6 gigaflops -> very close to the peak (367 gigaglops).
Memory As Limiting Factor to Parallelism

• Limited shared memory limits the number of threads that can execute simultaneously in SM for a given application
  – The more memory locations each thread requires, the fewer the number of threads per SM
  – Same applies to registers
Memory As Limiting Factor to Parallelism

- **Example: Shared memory**
  - G80 has 16KB of shared memory per SM
  - SM accommodates up to 8 blocks
  - To reach this maximum each block must not exceed 16KB/8 = 2KB of memory.
  - e.g. if each block uses 5KB -> no more than 3 blocks can be assigned to each SM
Registers As Limiting Factor to Parallelism

- **Example: Registers**
  - G80 has 8K registers per SM -> 128K registers for entire processor.
  - G80 can accommodate up to 768 threads per SM
  - To fill this capacity each thread can use only 8K/768 = 10 registers.
  - If each thread uses 11 registers -> threads per SM are reduced -> per block granularity
  - e.g. if block contains 256 threads the number of threads will be reduced by 256 -> lowering the number of threads/SM from 768 to 512 (i.e. 1/3 reduction of threads!)
Utilization Analysis Example

- Lets assume that:
  - We have a kernel that requires 48 registers per thread
  - Assume GTX 580 GPU (CC 2.0, 16 SMs, 32k registers/SM)
  - Execution configuration is a grid of 4x5x3 blocks, each 100 threads

- Each block requires 100*48=4800 registers

- The grid is made of 4*5*3 = 60 blocks that need to be distributed to the 16 SMs of the card. There will be 12 SMs that will receive 4 blocks and 4 SMs that will receive 3 blocks → inefficient

- Additionally, each of the 100-thread blocks would be split into ceil[100/warp_size] = ceil[100/32]
  - warps. The first three warps would have 32 threads and the last would have 4 threads! So during the execution of the last warp of each block most of the SPs will be idle → inefficient

Source: *Multicore and GPU Programming: An Integrated Approach* by G. Barlas
Myths About CUDA

- GPUs have very wide (1000s) SIMD machines
  - No, a CUDA Warp is only 32 threads
- Branching is not possible on GPUs
  - Incorrect.
- GPUs are power-inefficient
  - Nope, performance per watt is quite good
- CUDA is only for C or C++ programmers
  - Not true, there are third party wrappers for Java, Python, and more
Conclusions

• Using memory effectively will likely require the redesign of the algorithm.

• The only safe way to synchronize threads in different blocks is to terminate the kernel and start a new kernel for the activities after the synchronization point.

• The ability to reason about hardware limitations when developing an application is a key concept of computational thinking.