Avoiding Coordination
Announcements
Office Hours

- No office hours on 11/15: I am not here.
- No office hours on 11/22: Everyone should be out having 🍰 and 🦃.
- No office hours on 11/29: I am not here.
- E-mail me if you want to meet/need help.
  - Probably going to be over Skype or some such medium.
- Or in person on 11/26 or 11/27.
Final Project

• Hopefully everyone has started...
  
  • ... at least thinking about the project.

• Remember: no extensions or late days -- must receive final report on Dec 12.
Onto Science...
So Far...

- Looked at mechanisms to achieve consensus, get machines to work together.
  - A few different algorithms, a few different failure conditions.
- Powerful primitive: can replicate any deterministic state machine.
  - Don't even need to implement or understand consensus to do this.
- Powerful abstraction: replicate most programs, get fault tolerance.
So Far...

• But everything is not rosy:
  • Configure and initialize system for correctness.
  • Need to communicate and wait before responding to any request.
  • Can we do better?
Yes?
Self Stabilizing Algorithms
Setting

• Treat program as transition machine.
Setting

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- Where some states are good.
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- Self-stabilization:
  - From any state arrive at good state.
  - In bounded steps.
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  • Where some states are good.
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Djikstra's Example

• Mutual exclusion: only one process gets to write (or compute) at a time.
Djikstra's Example

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At process 0
do {
    if (x_4 == x_0) {
        x_0 = (x_0 + 1) % 5;
    }
} while (true);

At process n
do {
    if (x_n != x_{n-1}) {
        x_n = x_{n-1};
    }
} while (true);
Dijkstra's Example

- When and why can this provide mutual exclusion?

At process 0
    do {
        if (x₄ == x₀) {
            x₀ = (x₀ + 1) % 5;
        }
    } while (true);

At process n
    do {
        if (xₙ != xₙ₋₁) {
            xₙ = xₙ₋₁;
        }
    } while (true);
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Dijkstra's Example

What happens with random initial values?

At process 0
    do {
        if (x_4 == x_0) {
            x_0 = (x_0 + 1) % 5;
        }
    } while (true);

At process n
    do {
        if (x_n != x_{n-1}) {
            x_n = x_{n-1};
        }
    } while (true);
Dijkstra's Example

- Assuming a fair execution.
- **Lemma 1**: At least every $n$ rounds $x_0$ changes its value.
  - Round here is every processor got a chance to run.
  - Why?

```java
At process 0
do {
  if (x_4 == x_0) {
    x_0 = (x_0 + 1) % 5;
  }
} while (true);

At process n
do {
  if (x_n != x_{n-1}) {
    x_n = x_{n-1};
  }
} while (true);
```
Djikstra's Example

- Assuming a fair execution.
- **Lemma 1**: At least every $n$ rounds $x_0$ changes its value.
- **Lemma 2**: There is some value $c$ in $0..(n+1)$ s.t. $x_i \neq c$ $\forall i$
- Why?
Dijkstra's Example

- Assuming a fair execution.
- **Lemma 1**: At least every $n$ rounds $x_0$ changes its value.
- **Lemma 2**: There is some value $c$ in $0..(n+1)$ s.t. $x_i \neq c \ \forall i$
- **Theorem**: Get to all $x_i$s being equal in $O(n^2)$ rounds.
- Why?
Other Self Stabilizing Algorithms?
Finding a Minimal Spanning Tree
Thoughts on how?
Finding a Minimal Spanning Tree

At root
do {
    for n in nbr {
        send(n, <d=0, parent=false>)
    }
} while (true);

At process n
do {
    n[i], parent = recv(nbr i)
    d = min(n)
    parent = find(i s.t. n[i] = d-1)
    send(parent, <d=d, parent=true>)
    for n in nbr {
        if n!= parent {
            send(n, <d = d, parent=false>)
        }
    }
} while (true);
Finding a Minimal Spanning Tree

Why does this work?
Finding a Minimal Spanning Tree

Why does this work?
Maximal Matching
How?
partner = null
do {
    broadcast(id, partner)
    collect partners for id
    if partner = null and exists j s.t. partners[j]=id {
        partner = j
    }
    if partner = null and exists j s.t. partners[j] = null {
        partner = j
    }
    if partner = j and partners[j] != id {
        partner = null
    }
} while (true);
Why?
CRDTs
Revisiting RSMs

Application
Ordering

Application
Ordering

Application
Ordering

Application
Ordering

Client

Client

Client

Client
What if we didn't care about ordering

Diagram showing a sequence of Application-Gossip-Client connections.
Challenge: Ensuring correctness despite reordering
## Challenge

<table>
<thead>
<tr>
<th>Peer 0</th>
<th>Peer 1</th>
<th>Peer 2</th>
<th>Peer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>m(A, m(Y, m(Z, X)))</td>
<td>m(Z, m(A, m(X, Y)))</td>
<td>m(A, m(X, m(Y, Z)))</td>
<td>m(Z, m(Y, m(X, A)))</td>
</tr>
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<td>m(Y, m(X, A))</td>
</tr>
<tr>
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<td>A</td>
</tr>
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</table>
Challenge

\[
\begin{align*}
\text{Peer 0} & : m(A, m(Y, m(Z, X))) & \text{Peer 1} & : m(Z, m(A, m(X, Y))) & \text{Peer 2} & : m(A, m(X, m(Y, Z))) & \text{Peer 3} & : m(Z, m(Y, m(X, A))) \\
\text{Peer 0} & : m(Y, m(Z, X)) & \text{Peer 1} & : m(A, m(X, Y)) & \text{Peer 2} & : m(X, m(Y, Z)) & \text{Peer 3} & : m(Y, m(X, A)) \\
\text{Peer 0} & : \text{merge}(Z, X) & \text{Peer 1} & : \text{merge}(A, m(X, Y)) & \text{Peer 2} & : \text{merge}(X, m(Y, Z)) & \text{Peer 3} & : \text{merge}(Y, m(X, A)) \\
\text{Peer 0} & : X & \text{Peer 1} & : Y & \text{Peer 2} & : Z & \text{Peer 3} & : A
\end{align*}
\]

Need all of these to be equal.
Modelling a Merge Function

• Treat updates as a set.

• For previous example \{A, X, Y, Z\}

• Define merge function to be the **least upper bound** (similar to supremum).

  • Commutative, associative and idempotent.

  • Thus \(\text{LUB}(A, \text{LUB}(X, \text{LUB}(Y, Z)))) = \text{LUB}(X, \text{LUB}(A, \text{LUB}(Y, Z)))) = \ldots\)

• In abstract algebra posets with LUBs are called semilattices.
Is this enough?
Modelling a Merge Function

- Sufficient for consistency.
- Not sufficient to make sure all semantics are preserved.
- In particular picking the least upper bound might lose operations.
Counters

\[ x = [\emptyset, 0, 0] \quad x = [0, 0, 0] \quad x = [0, 0, \emptyset] \]

Suppose we use element wise max as LUB.

Is element wise max a LUB?
Suppose we use element wise max as LUB.

Is element wise max a LUB?
Modelling a Merge Function

- Need it to be a least upper bound for the state which is a semilattice.
- Need LUB to be monotonic.
  - Each application preserves more information.
Build a Collaborative Editor using CRDTs
It was the best of times, it was the poohest of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of credulity, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness
it was the poorest
It was the poorest very very.
Sometimes it fails.
What consistency guarantees do both of today's mechanisms provide?
When are these good?