CSCI-GA.2420-001/MATH-GA.2010-001
Numerical Methods I Fall 2013
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Homework set 2: Due September 27 at 12 noon.

Homework should be given to me in class or put under my office door.
Do not put it in my mail box.

In all cases, discuss your findings.

1. What is the smallest positive integer which is not exactly represented as a single precision IEEE floating point number? What is the largest finite integer which is part of the double precision IEEE floating point system?

2. 3 Find the IEEE single and double precision floating point representation of the numbers $4, 100, 1/100, 2^{-100}, 2^{-200}$, and 2^{-1050}.

3. Find three double precision IEEE floating point numbers a, b, and c for which the relative error of $a + b + c$ is very large. Try to make it as bad as possible and explain your reasoning. Combine paper and pencil work with MATHLAB experiments. Note that MATLAB uses IEEE double precision.

4. Consider the following MATLAB program:

```
f(1)=1;
f(2)=1;

for i=2:(n-1)
f(i+1) = f(i) + f(i-1);
end
g(n)=f(n);
g(n-1)=f(n-1);
for i=(n-1):-1:2
g(i-1)=g(i+1) - g(i);
end
```

For any input n, a positive integer, the program first computes the n first Fibonacci numbers in the normal order and it then computes the same sequence of numbers in reverse order.

For small enough n there is no round-off errors at all. Explain why.

For very large values of n, the numbers will overflow. Find out how large n must be for this to happen and relate your finding to what you can learn from the tables of IEEE floating point numbers which is part of the handout on September 12.

Finally, for values of n in between, the $f(i)$ will differ very much from the $g(i)$. Get some experience with this and provide as full an explanation as you can for what is happening. In particular, explain why breakdown first happens at that a particular value of n.

Does this failure indicates a serious problem with the arithmetic or can it be explained by saying that the problem is very sensitive to relatively small perturbations?