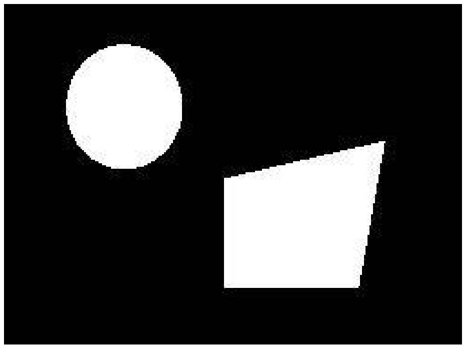
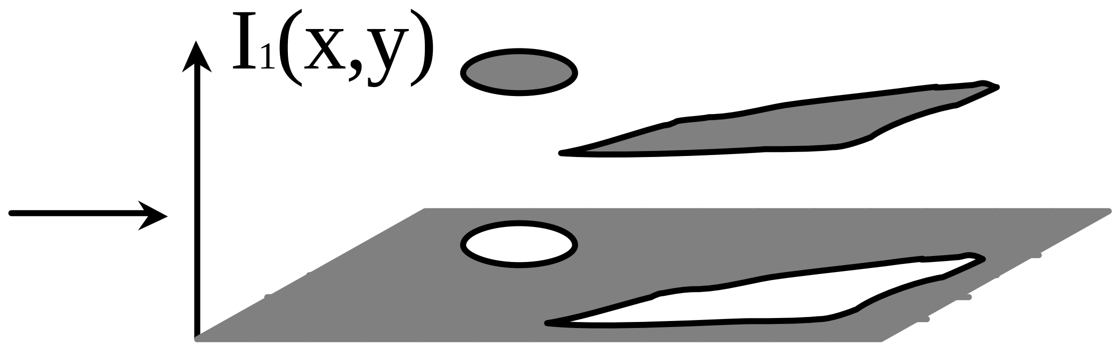
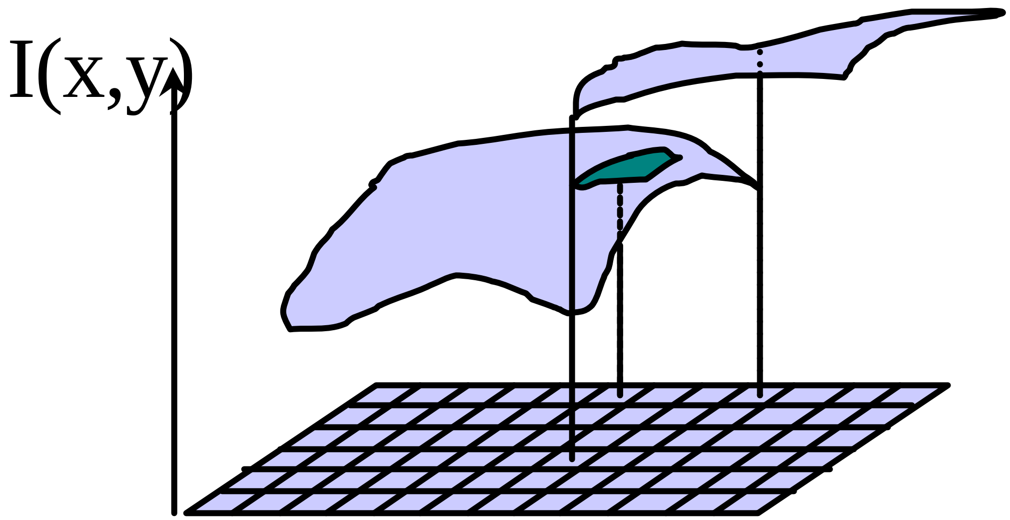
Image Measurements

Image Measurements

Images are Intensity Surfaces and Edges are Surface Discontinuities

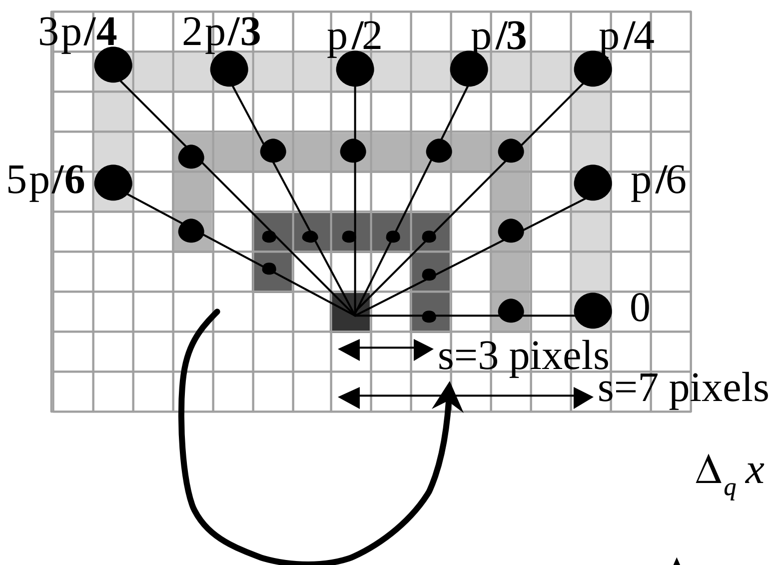






Measurements:Intensity Accumulation, $\tilde{I}\,(x, y, q\,, s)$

For each pixel *(x,y)* we evaluate the accumulation of the intensity along sixteen (16) directions, θ=0, π/6, π/4, π/3, π/2, 2π/3, 3π/4, 5π/6, π, 7π/6, 5π/4, 4π/3, 3π/2, 5π/3, 7π/4, 13π/6, and four (4) different scales, *s= 3, 5, 7, 9*.



Locations of $\tilde{I}\,(x, y, q\,, s)$

$$\tilde{x}_{\tilde{I}}(x,q) = \frac{1}{s}\sum_{i=0}^{s-1}(x + i\,\Delta_q x\,\cos q),$$

$$\tilde{y}_{\tilde{I}}(y,q) = \frac{1}{s}\sum_{i=0}^{s-1}(y + i\,\Delta_q y\,\sin q)$$

$$\tilde{I}\,(x, y, q\,, s) = \frac{1}{s}\int_0^s I(a\,\cos q\,, a\,\sin q)\; da$$

Discrete approximation

$$\tilde{I}\,(x, y, q\,, s) \approx \frac{1}{s}\sum_{i=0}^{s-1} I(x + i\,\Delta_q\,x\,\cos q\,, y + i\,\Delta_q\,y\,\sin q)$$

where,

$\Delta_q x = 1$ and $\Delta_q y = 1$ for $\theta = 0, \pi/2, \pi, 3\pi/2$

$\Delta_q x = \sqrt{2}$ and $\Delta_q y = \sqrt{2}$ for $\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$\Delta_q x = \frac{2}{\sqrt{3}}$ and $\Delta_q y = 1$ for $\theta = \pi/6, 5\pi/6, 7\pi/6, 13\pi/6$

$\Delta_q x = 1$ and $\Delta_q y = \frac{2}{\sqrt{3}}$ for $\theta = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3.$

Measurements: Intensity Accumulation II, $\hat{I}(x, y, q, s)$

In order to obtain more robust measures, for an angle θ, $\tilde{I}(x, y, q, s)$
we can average the value of along $\pi/2 + \theta$ to obtain $\hat{I}(x, y, q, s)$

$$\hat{I}(x, y, q, s) = \frac{1}{4}(\tilde{I}(x, y, q, s) + \tilde{I}(x, y+1, q, s) + \tilde{I}(x, y-1, q, s) + \tilde{I}(x+1, y, q, s))$$

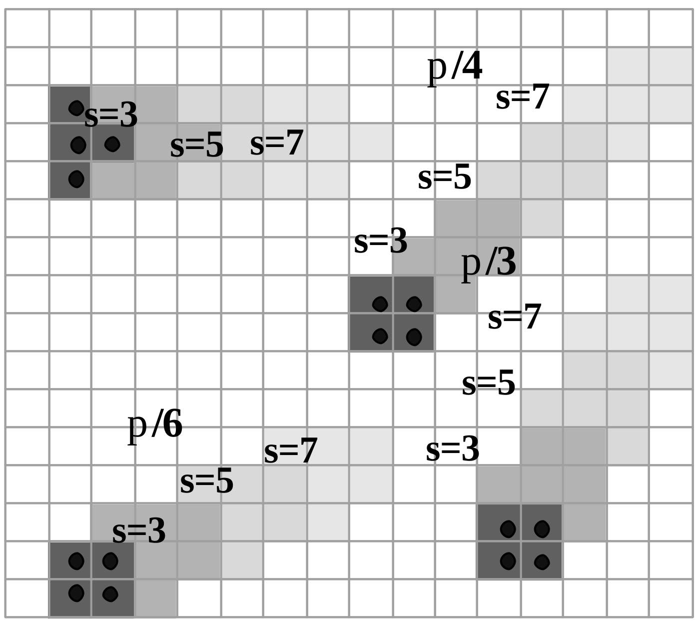
for $\theta = 0, \pi$,

$$\frac{1}{4}(\tilde{I}(x, y, q, s) + \tilde{I}(x+1, y, q, s) + \tilde{I}(x-1, y, q, s) + \tilde{I}(x, y+1, q, s))$$

for $\theta = \pi/2, 3\pi/2$,

$$\frac{1}{4}(\tilde{I}(x, y, q, s) + \tilde{I}(x-1, y, q, s) + \tilde{I}(x, y+1, q, s) + \tilde{I}(x-1, y+1, q, s))$$

for $\theta = \pi/6, \pi/4, \pi/3$.



Measurements: Homogenity

The average intensity should not vary across scales if there no discontinuities along θ.

$$\tilde{R}(x, y, q, s) = |\, \tilde{I}_c(x, y, q, s) - \tilde{I}_c(x, y, q, s - \Delta s)\,| \,\big/\, |\, \tilde{I}_c(x, y, q, s)\,|$$

where $\tilde{I}_c(x, y, q, s) = \frac{s}{s-1}\left(\tilde{I}(x, y, q, s) - \frac{1}{s} I(x, y)\right)$ does not include the center pixel.

Finally

$$\tilde{H}(x, y, s) = \sum_{q=0}^{11p/6} \tilde{R}(x, y, q, s) \;,$$

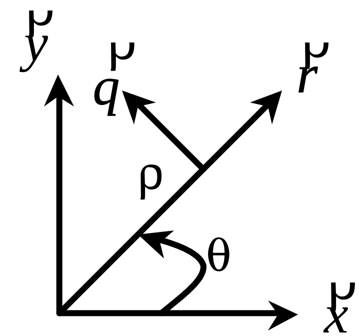
is a measure of homogenity, the lower it is the more homogeneous is the region centered on (x,y). Analogously one can compute $\hat{H}(x, y, s)$.

Edges and Junctions are expected to be localized at homogeneous pixels.

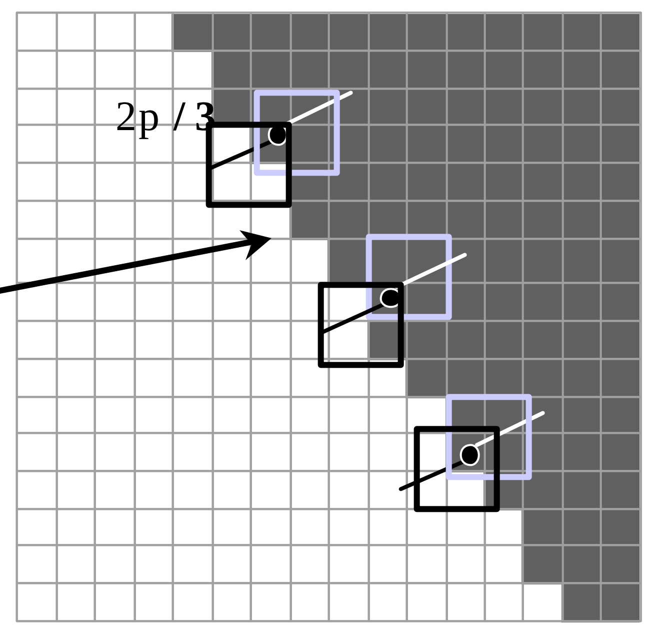
Measurements: Derivatives $D_r \hat{I}(x, y, \theta, \sigma)$

Taking the derivative along the angle θ, i.e., in a direction $\hat{r} = (\cos\theta, \sin\theta)$

$$
\begin{aligned}
D_r \hat{I}(x, y, \theta, \sigma) &= \nabla \hat{I}(x, y, \theta, \sigma) \bullet (\cos\theta, \sin\theta) \\
&= \frac{\partial \hat{I}(x, y, \theta, \sigma)}{\partial x} \cos\theta + \frac{\partial \hat{I}(x, y, \theta, \sigma)}{\partial y} \sin\theta \\
&\approx \hat{I}(x, y, \theta, \sigma) - \hat{I}(x, y, \pi + \theta, \sigma) \ .
\end{aligned}
$$



Analogously we obtain $D_r \tilde{I}(x, y, \theta, \sigma)$. We have now a bank of eight (8) distinct oriented filters at different scales, namely $\{ D_r \hat{I}(x, y, \theta, \sigma)$; θ = 0, π/6, ..., 5π/6 , and s= 1, 3, 5, 7, ...}, with the property that $D_r \hat{I}(x, y, \theta, \sigma) = -D_r \hat{I}(x, y, \theta + \pi, \sigma)$

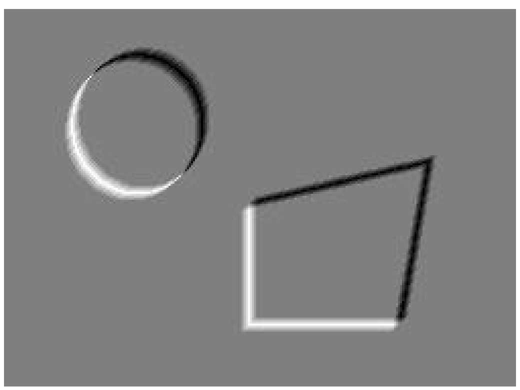


$| D_r \hat{I}(x, y, \theta = \pi/6, \sigma) |$ will have maximum response (highest value) among the possible orientations, while $| D_r \hat{I}(x, y, \theta = \frac{2\pi}{3}, \sigma) |$, will have the minimum response.

Experiments: $D_r \hat{I}(x, y, q, s)$

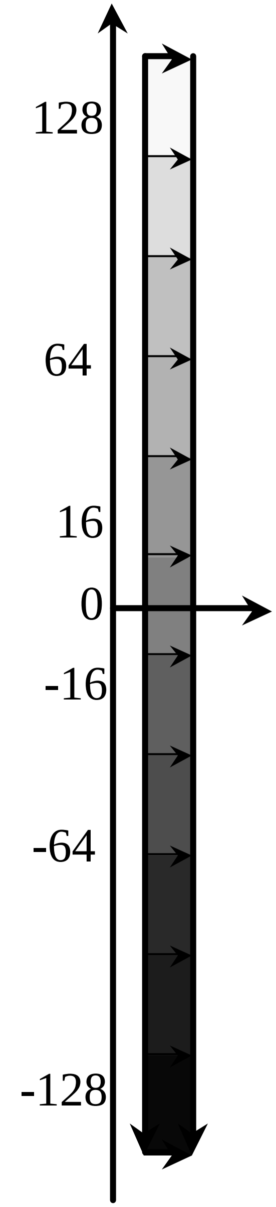
$D_r \hat{I}(x, y, 0, 3)$

$D_r \hat{I}(x, y, \frac{p}{6}, 3)$

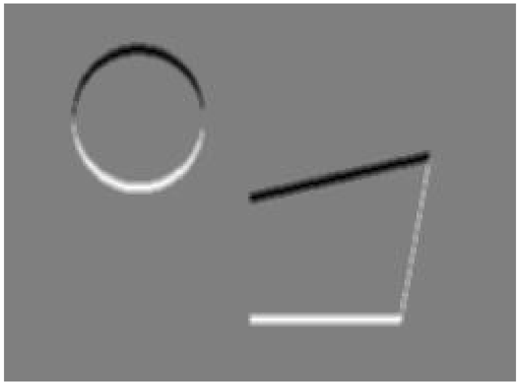


$D_r \hat{I}(x, y, \frac{p}{4}, 3)$

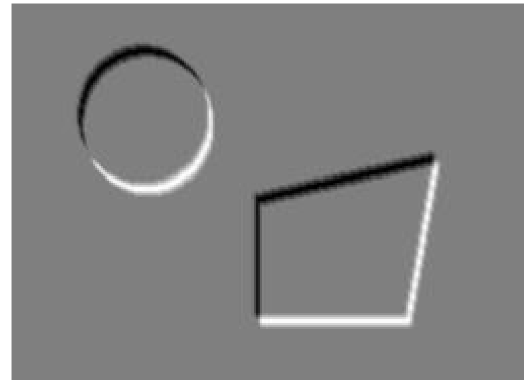
$D_r \hat{I}(x, y, \frac{p}{3}, 3)$



Exeriments (cont.): $D\hat{I}(x, y, q, s)$



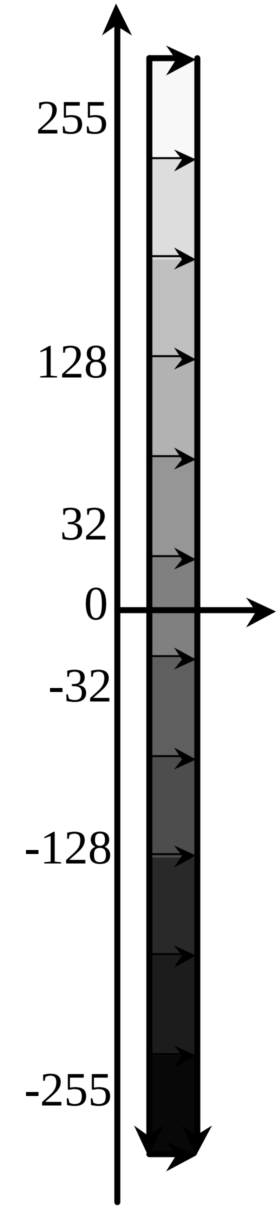
$D_r \hat{I}(x, y, \frac{p}{2}, 3)$



$D_r \hat{I}(x, y, \frac{2p}{3}, 3)$

$D_r \hat{I}(x, y, \frac{3p}{4}, 3)$

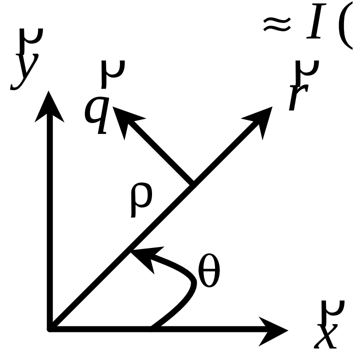
$D_r \hat{I}(x, y, \frac{5p}{6}, 3)$



Measurements: Derivatives $D_\theta \tilde{I}(x, y, \theta, s)$

$$D_\theta \tilde{I}(x,y,\theta,s) = \nabla \tilde{I}(x,y,\theta,s) \bullet (-\sin\theta, \cos\theta) = -\frac{\partial \tilde{I}(x,y,\theta,s)}{\partial x}\sin\theta + \frac{\partial \tilde{I}(x,y,\theta,s)}{\partial y}\cos\theta$$

$$\approx \tilde{I}(x,y,\theta,s) - \tilde{I}(x,y,\theta - \Delta_\theta \theta, s)$$



where

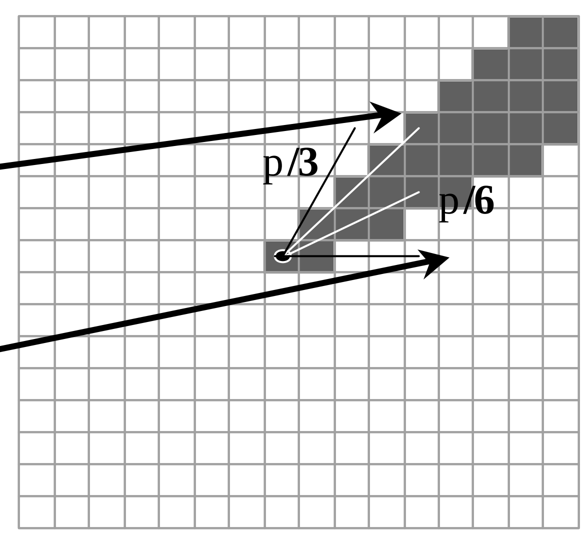
$\Delta_\theta \theta = \frac{\pi}{6}$ for θ=0, π/6, π/2, 2π/3, , π, 7π/6, 3π/2, 5π/3,

$\Delta_\theta \theta = \frac{\pi}{12}$ for θ= π/4, π/3,3π/4, 5π/6, 5π/4, 4π/3, 7π/4, 11π/6.

Corners and Junctions will respond to high values of $D_\theta \hat{I}(x,y,\theta,s)$

The maximum responses here, among different θ are for

$D_\theta \hat{I}(x,y,\frac{\pi}{3},s)$ and $D_\theta \hat{I}(x,y,\frac{\pi}{6},s)$



Measurements: 2nd Derivatives

$$H(x,y) = \begin{pmatrix} \partial^2 \hat{I} / \partial_2 x & \partial^2 \hat{I} / \partial x \partial y \\ \partial^2 \hat{I} / \partial y \partial x & \partial^2 \hat{I} / \partial_2 y \end{pmatrix}$$

Hessian allows for computing the second derivative

in order to compute the (second) derivative along a direction $\hat{v}$ of a (first) derivative along a direction $\hat{u}$, we compute the projections

$$D_{vu}\hat{I}(x,y,q,f,s) = \hat{v}\, H(x,y)\ \hat{u}^T$$

For example for $\hat{u} = (1,0) = \hat{x};\ \hat{v} = (1,0) = \hat{x}$ we obtain $D_{vu}\hat{I}(x,y,q,f,s) = \partial^2 \hat{I} / \partial x^2$

and for $\hat{u} = (1,0) = \hat{x};\ \hat{v} = (0,1) = \hat{y}$ we obtain $D_{vu}\hat{I}(x,y,q,f,s) = \partial^2 \hat{I} / \partial y \partial x$

Measurements: 2nd Derivatives (cont.)

in order to compute the (second) derivative along a direction φ of a (first) derivative along a direction θ, we obtain a continuous and a lattice approximation as

$$D_{\phi\theta}\hat{I}(x,y,\theta,\phi,\sigma) = (\cos\phi,\sin\phi)\ H(x,y)\ (\cos\theta,\sin\theta)^T$$
$$\approx D\hat{I}(x,y,\theta,\sigma) - D\hat{I}(x-\Delta_\phi x\ \cos\phi,\ y-\Delta_\phi y\ \sin\phi,\theta,\sigma)\big/\sigma\ \Delta$$