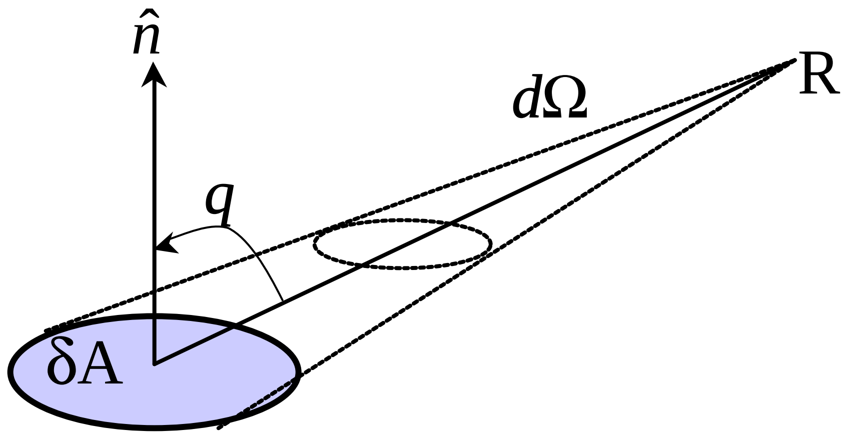
Image Formation


Light can change the image and appearances (images from D. Jacobs)
What is the relation between pixel brightness and scene radiance?
Later: what is the relation between pixel brightness and scene reflectance ?

Image Formation

$$d\Omega = \frac{dA\cos q}{R^2}$$ solid angle subtended by a small patch of area A.

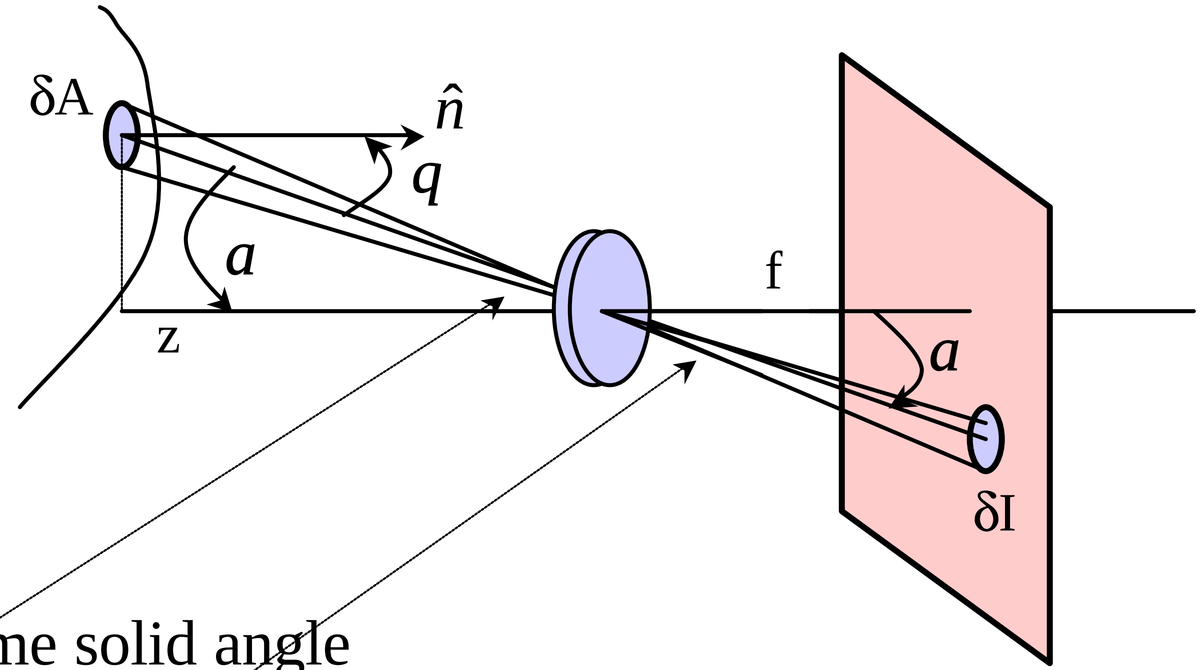


L - radiance is the amount of light radiated from a surface per solid angle
(power per unit area per unit solid angle emitted from a surface. $W\,m^{-2} sr^{-1}$)

E - irradiance is the amount of light falling in a surface
(power per unit area incident in a surface. $W\,m^{-2}$)

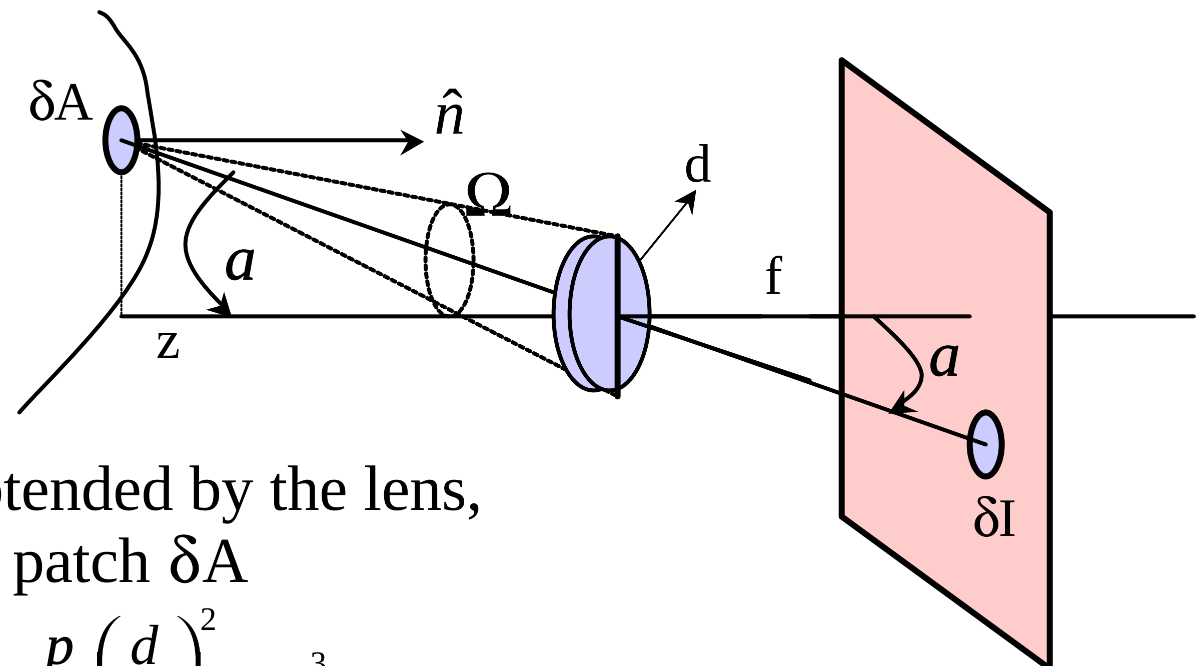
Surface Radiance and Image Irradiance

Pinhole
Camera
Model



$$\frac{dA\cos q}{(z/\cos a)^2} = \frac{dI\cos a}{(f/\cos a)^2} \rightarrow \frac{dA}{dI} = \frac{\cos a}{\cos q}\left(\frac{z}{f}\right)^2$$

Surface Radiance and Image Irradiance



Solid angle subtended by the lens,
as seen by the patch δA

$$\Omega = \frac{p}{4}\frac{d^2 \cos a}{(z/\cos a)^2} = \frac{p}{4}\left(\frac{d}{z}\right)^2 \cos^3 a$$

Power from patch δA through the lens

$$dP = L\,\Omega\; dA\; \cos q = L\,dA\,\frac{p}{4}\left(\frac{d}{z}\right)^2 \cos^3 a\, \cos q$$

Thus, we conclude

$$E = \frac{dP}{dI} = L\,\frac{dA}{dI}\frac{p}{4}\left(\frac{d}{z}\right)^2 \cos^3 a\, \cos q = L\,\frac{p}{4}\left(\frac{d}{f}\right)^2 \cos^4 a$$

Conclusions

$$E = L\frac{p}{4}\left(\frac{d}{f}\right)^2 \cos^4 a$$

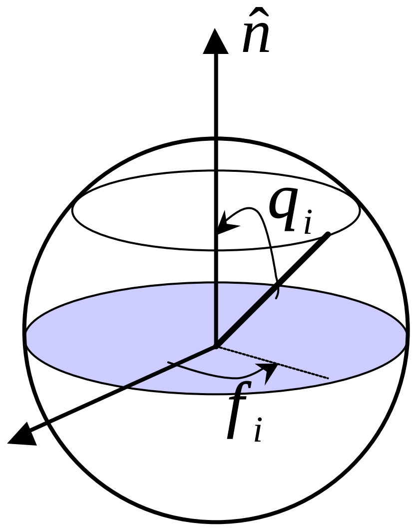
- The irradiance at the image pixel is converted into the brightness of the pixel

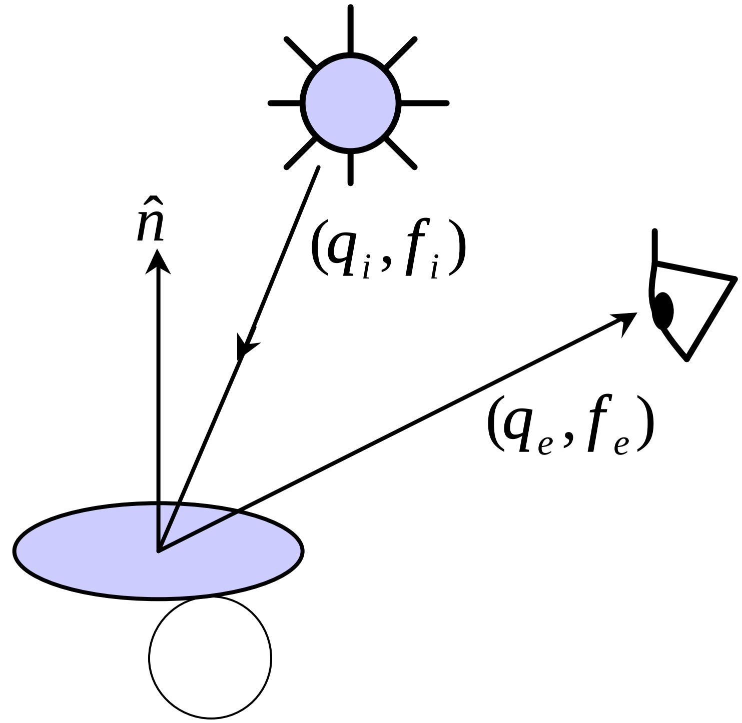
- Image Irradiance is proportional to Scene Radiance

- Scene distance, z, does not affect/reduce image brightness (the model is too simplified, since in practice it does.)

- The angle of the scene patch with respect to the view (α) reduces the brightness by the $\cos^4 a$. In practice the effect is even stronger.

The Bidirectional Reflectance Distribution Function (BRDF)



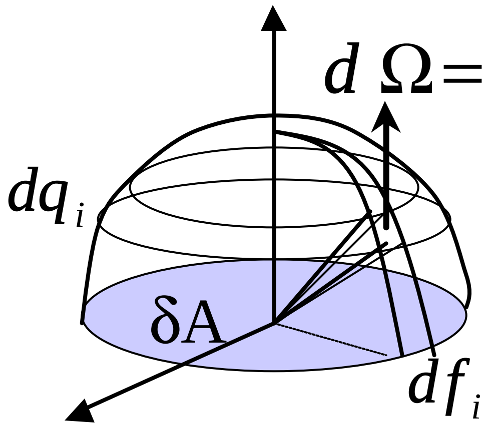


$$f(q_i, f_i, q_e, f_e) = \frac{d\,L(q_e, f_e)}{d\,E(q_i, f_i)}$$

BRDF - How bright a surface appears when viewed from one direction while light falls on it from another.

Usually *f* depends only on $f_e - f_i$, q_i, q_e : true for matte surfaces and specularly reflecting surfaces.

Extended Light Sources and BRDF



$$d\,\Omega = \sin q_i \; dq_i \; df_i$$

Light source radiance arriving through solid angle δΩ

$$dE\,(q_i, f_i) = E(q_i, f_i)\; \sin q_i \; dq_i \; df_i$$

Power arriving at patch δA from δΩ

$$dP = dA \cos q_i \, dE\,(q_i, f_i) = dA \; E(q_i, f_i)\; \cos q_i \; \sin q_i \; dq_i \; df_i$$

Foreshortening

thus the irradiance arriving at patch δA is

$$E_0 = \frac{\int_\Omega dP}{dA} = \int_{-p}^{p} \int_0^{p/2} E(q_i, f_i)\; \cos q_i \; \sin q_i \; dq_i \; df_i$$

The radiance of a patch δA at direction (q_e, f_e) is thus, given by

$$L(q_e, f_e) = \int_{-p}^{p} \int_0^{p/2} f\,(q_i, f_i, q_e, f_e)\; E(q_i, f_i)\; \cos q_i \; \sin q_i \; dq_i \; df_i$$

Special Cases of BRDF

1. **Lambertian Surfaces (matte)-** appears equally bright from all viewing directions and reflects all incident light, absorbing none, i.e. the BRDF is constant and $L = E_0$. What constant f?

$$L(q_e, f_e) = f \int_{-p}^{p} \int_{0}^{p/2} E(q_i, f_i)\, \cos q_i \,\sin q_i \, dq_i \, df_i = f\, E_0$$

Thus, the total "reflected power" from patch δA becomes

$$dp = \int_{-p}^{p} \int_{0}^{p/2} L(q_e, f_e)\, dA \cos q_e \,\sin q_e \, dq_e \, df_e = f\, E_0 \, dA\, p$$

Foreshortening

since $\int_{-p}^{p} \int_{0}^{p/2} \cos q_e \,\sin q_e \, dq_e \, df_e = p$

Using that $L = \frac{dp}{dA} = f\, E_0\, p$ and for Lambertian surfaces $L = E_0$,

we finally obtain $f = \frac{1}{p}$

Special Cases of BRDF

2. Specular Surfaces (mirrors) – reflects all light arriving from the direction (q_i, f_i) into the direction $(q_i, f_i + p)$. The BRDF is in this case proportional to the product of two impulses, $d(q_e - q_i)$ and $d(f_e - f_i - p)$. What is the factor of proportionality $k(q_i, f_i)$?

$$L(q_e, f_e) = \int_{-p}^{p}\int_{0}^{p/2} k(q_i, f_i)\, d(q_e - q_i)\, d(f_e - f_i - p)\, E(q_i, f_i)\, \cos q_i\, \sin q_i\, dq_i\, df_i$$

$$= k(q_e, f_e)\, E(q_e, f_e)\, \cos q_e\, \sin q_e$$

$$E_0 = \int_{-p}^{p}\int_{0}^{p/2} E(q_i, f_i)\, \cos q_i\, \sin q_i\, dq_i\, df_i$$

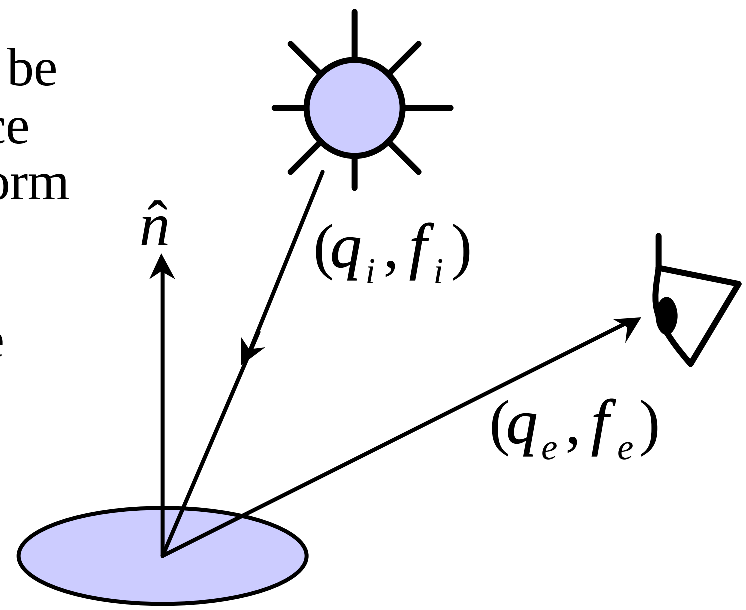
$$L = \int_{-p}^{p}\int_{0}^{p/2} L(q_e, f_e)\, \cos q_e\, \sin q_e\, dq_e\, df_e$$

and for specular surfaces $L = E_0 \Rightarrow k(q_i, f_i) = \dfrac{1}{\cos q_i\, \sin q_i}$

we finally obtain $f(q_i, f_i, q_e, f_e) = \dfrac{d(q_e - q_i)\, d(f_e - f_i - p)}{\cos q_i\, \sin q_i}$

Lambertian Surface Brightness

How bright will a Lambertian surface be when it is illuminated by a point source of radiance *E*? and by a “sky” of uniform radiance *E*?



For a point source the irradiance at the surface is $E_0 = E \cos q_i$ and the radiance must then be

$$L(q_e, f_e) = f \;\; E_0 = \frac{1}{p} E \cos q_i$$

Familiar cosine or “Lambert’s law” of reflection from matte surfaces (surfaces covered with finely powdered transparent materials such as barium sulfate or magnesium carbonate), and can approximate paper, snow and matte paint.

Finally, for a “sky” of uniform radiance E we obtain

$$L(q_e, f_e) = \int_{-p}^{p} \int_{0}^{p/2} \frac{1}{p} \; E \; \cos q_i \; \sin q_i \; dq_i \, df_i = E!$$