1 FDS’s and their Feasibility

Task 1: Prove Claim (7) in the lecture notes, which claims that every FDS which is derived from an SPL program is viable.

An infinite sequence \( \sigma : s_0, s_1, \ldots \) is said to be \textit{ultimately periodic} if there exist a \( k \geq 0 \) and an \( m > 0 \) such that \( s_{j+m} = s_j \) for every \( j \geq k \). An equivalent definition is that \( \sigma \) can be presented as \( \sigma = \sigma_1(\sigma_2)^\omega \), where \( \sigma_1 \) and \( \sigma_2 \) are finite sequences, and \( (\sigma_2)^\omega \) denotes an infinite repetition of the sequence \( \sigma_2 \).

Task 2: Let \( D \) be a finite-state FDS with no more than \( n \) states. Prove that if \( D \) is feasible then it has an eventually periodic computation. Can you give bounds on the sizes of \( \sigma_1 \) and \( \sigma_2 \)?

Task 3: Let \( \varphi \) be a propositional temporal formula. One way to check if \( \varphi \) is satisfiable is to construct the temporal tester \( T_\varphi \) and check whether \( T_\varphi \) is feasible. Based on the (explicit-state) algorithms given in class and the above considerations, give upper bounds on the time and space complexity of the problem “is \( \varphi \) satisfiable?”

Due Date: 11.29.01
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2 Temporal Logic and the Expression of Properties

Task 4: Following is a list of properties of sequences of states. Attempt to write temporal-logic formulas that specify these properties. Note that not every property mentioned can be described in LTL. Please identify explicitly the properties that are not LTL-expressible (in your opinion).

The properties refer to propositions $p$, $q$, $r$, and $t$, which can also be thought of as boolean variables.

1. Propositions $p$, $q$, and $r$ are pairwise exclusive. That is, when one of them holds, the other two cannot. For the rest of the property list, you may assume this to hold.

2. Every $p$ is followed by a $q$.

3. Every $q$ is preceded by a $p$.

4. Every two $p$'s are separated by at least one $q$.

5. Every $p$ is followed by a $q$ with no intermediate $r$.

6. Proposition $t$ changes its truth value (from true to false and vice versa) in every state which is the immediate successor of a $p$-state.

7. We say that an occurrence of $p$ is even if it is the $n$'th occurrence, where $n$ is an even natural number (i.e., evenly divisible by 2).

The property: $q$ may occur only in the interval between an odd and an even occurrences of $p$.

8. The combination of properties (6) and (7).

9. There are only finitely many occurrences of $q$ preceded by an occurrence of $p$.

10. It is not the case that every $p$ is followed by a $q$.

11. Let $\#p$ denote the number of $p$'s that occurred from the beginning of the sequence.

The property: At all points $\#p \geq \#q$. 

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12. At all points $0 \leq \#p - \#q \leq 1$.

13. At all points $0 \leq \#p - \#q \leq 2$.

**Task 5:** Following is a list of pairs of temporal formulas. Identify the pairs which are congruent, namely, a pair $(p, q)$ such that for every state sequence $\sigma$ and every position $j \geq 0$,

$$(\sigma, j) \models p \iff (\sigma, j) \models q.$$ 

For the cases that you claim congruence, give an informal argument, while for the other cases provide a counter example. For the cases that the two formulas are not congruent, also state whether they are equivalent, that is,

$$(\sigma, 0) \models p \iff (\sigma, 0) \models q.$$ 

1. $\Diamond p \land \Box q$ and $\Diamond (p \land \Box q)$
2. $\Diamond p \land \Box q$ and $\Box (\Diamond p \land q)$
3. $\Diamond \Box p \land \Diamond \Box q$ and $\Diamond (\Box p \land \Box q)$
4. $(pUq)Uq$ and $pUq$
5. $pU(pUq)$ and $pUq$
6. $\Diamond \Diamond p$ and $\Diamond \Diamond p$
7. $\Diamond p \land \Diamond q$ and $\Diamond (\Diamond p \land \Diamond q)$
8. $q \implies \Diamond p$ and $(-q)Wp$
9. $\neg(pUq)$ and $(\neg q)W(\neg p \land \neg q)$