Correspondence with intuitionistic propositional logic

- This interpretation correponds exactly to that of intuitionistic logic, reading negative events e^{co} as $\neg e$, transition slashes / as logical implication, and the composition of events in triggers and actions, and parallel composition // of configurations, as conjunction

- Interaction steps M are then linear Kripke structures

- This leads to the following def of logical satisfaction $\models$:
An interaction step $M = (M_0,\ldots,M_n)$ satisfies configuration C, $M \models C$, if $M,i \models C$ for all $0 \le i \le n$, where

  - $M,i \models 0$ always (i.e.,configuration 0 is identified with true)

  - $M,i \models P,N^{co}/A$ if $P \subseteq M_i$ and $N \cap M_n = \emptyset$ implies $A \subseteq M_i$

  - $M,i \models C_1//C_2$ if $M,i \models C_1$ and $M,i \models C_2$

- Now, $M \models C$ iff C is valid in the linear Kripke structure M

Main Result

- Note that for interaction steps of lenght 1, the notions of interaction model and classical model coincide, and we simply write M_1 for (M_1)
- Step responses of a config C in the sense of Pnueli and Shalev are now exactly those interaction models of lenght 1, called response models, that are not suffixes of interaction models $N=(N_0,\dots,N_m,M)$ of C with lenght $m\geq 0$.
  For, if such a singleton interaction model was suffix of a longer interaction model, the reaction would be separable and hence not causal. Thus we have
- **Theorem 3 (Correctness and Completeness). If C is a configuration and $M \subseteq \Pi$, then M is a Pnueli-Shalev step response of C iff M is a response model of C**

- Firstly, consider the configuration $\overline{a}/b$ which exhibits the Pnueli-Shalev step response $\{b\}$ for the empty environment. Indeed, $\{b\}$ is a response model, i.e., a model and not a suffix of a longer interaction model. The only possibility would be the interaction step $(\emptyset, \{b\})$, but this is not an interaction model since $(\emptyset, \{b\}), 0 \not\models \overline{a}/b$: by definition, we have to consider $\emptyset \subseteq \emptyset$ and $\{a\} \cap \{b\} = \emptyset$ implies $\{b\} \subseteq \emptyset$, and this implication is false because $b \notin \emptyset$.

- Secondly, configuration $C_2 =_{\mathrm{df}} \overline{a}/b \,\|\, b/a$ has no response model. Although $\{a, b\}$ is a classical model of C_2, it may be left-extended to the interaction model $(\emptyset, \{a, b\})$. Note in particular that $(\emptyset, \{a, b\}), 0 \models \overline{a}/b$: by definition, we have to consider $\emptyset \subseteq \emptyset$ and $\{a\} \cap \{a, b\} = \emptyset$ implies $\{b\} \subseteq \emptyset$, and this implication trivially holds. In other words, event a is absent at position 0 of the interaction step $(\emptyset, \{a, b\})$ since it is added later in the step, namely at position 1, and thus is *not* absent.

- Thirdly, consider configuration $C_3 =_{\mathrm{df}} a/b \,\|\, b/a$ with its Pnueli-Shalev step response $\emptyset$. It is easy to see that $\emptyset$ is trivially a response model. In contrast, the set $\{a, b\}$ — while being a classical model of C_3 — is not a response model since the suffix extension $(\emptyset, \{a, b\})$ is an interaction model of C_3.

- Fourthly, configuration $\overline{a}/b \,\|\, \overline{b}/a$ offers two response models, namely $\{a\}$ and $\{b\}$, which are exactly the configuration's Pnueli-Shalev step responses. As in the example regarding configuration C_2 above, neither response model can be left-extended to an interaction model of length greater than one.

Full abstraction. The interaction models of a configuration C encode all possible interactions of C with all its environments and nothing more. Firstly, any differences between the interaction models of C are differences in the interactions of C with its environments and thus can be observed. Secondly, any observable difference in the interaction of C with its environments should imply a difference in the interaction models, and this holds by the very construction of interaction models. Therefore, the above interaction step semantics provides the desired compositional and fully abstract semantics for Pnueli-Shalev steps:

Theorem 4 (Compositionality & Full Abstraction). *Let C_1, C_2 be configurations. Then, C_1 and C_2 have the same interaction models if and only if, for all configurations C_3, the parallel configurations $C_1 \| C_3$ and $C_2 \| C_3$ have the same Pnueli-Shalev step responses.*

3.4 Algebraic Perspective

We now turn to characterising the Pnueli-Shalev step semantics, or more precisely the largest congruence contained in equality on step responses, in terms of axioms. These are derived from general axioms of propositional intuitionistic formulas over linear Kripke models. Thus, the algebraic characterisation presented here is closely related to the above denotational characterisation.

Table 1. Axiom system for the Pnueli-Shalev step semantics

(A1)	$C_1 \,\|\, C_2 = C_2 \,\|\, C_1$	
(A2)	$(C_1 \,\|\, C_2) \,\|\, C_3 = C_1 \,\|\, (C_2 \,\|\, C_3)$	
(A3)	$C \,\|\, C = C$	
(A4)	$C \,\|\, 0 = C$	
(B1)	$P, I/P = 0$	
(B2)	$I/A \,\|\, I/B = I/(A \cup B)$	
(B3)	$I/A = I/A \,\|\, I, J/A$	
(B4)	$I/A \,\|\, A, J/B = I/A \,\|\, A, J/B \,\|\, I, J/B$	
(B5)	$P, \overline{N}/A = 0$	*if* $P \cap N \neq \emptyset$
(C1)	$P, \overline{N}/A = P, \overline{N}/A, B$	*if* $N \cap A \neq \emptyset$
(C2)	$P, \overline{N}/A = P, e, \overline{N}/A \,\|\, P, \overline{N}, \overline{e}/A$	*if* $N \cap A \neq \emptyset$
(C3)	$I, \overline{N}/B \,\|\, P, \overline{N}/A = \{I, \overline{N}, \overline{e}/B : e \in P\} \,\|\, P, \overline{N}/A,$	*if* $N \cap A \neq \emptyset$ *and* $P \neq \emptyset$

Theorem 5 (Correctness & Completeness). *$C_1 = C_2$ can be derived from the axioms of Table 1 via standard equational reasoning if and only if, for all interaction steps M, $M \models C_1$ iff $M \models C_2$.*

Theorem 6 (Correctness & Completeness). *Let C be a configuration and M_C be the maze associated with C. Then, $A \subseteq \Pi$ is a Pnueli-Shalev step response of C if and only if there exists a lazy front line $(R_A, S \setminus R_A)$ in M_C such that $A = R_A \cap \Pi$.*

The proof of this theorem can be found in [1]. Note how the game model accommodates both the failure and nondeterminism of step responses. Depending on M_C, it may happen that there is no strategy to avoid a (visible) room m being visited by both players infinitely often. This corresponds to Pnueli and Shalev's step-construction procedure returning a failure. Also, a room m may occur in two different lazy front lines, which yields nondeterministic behaviour.

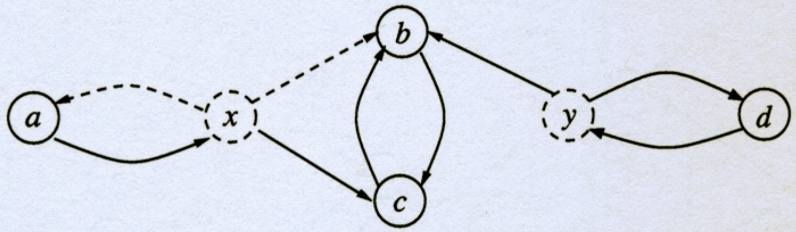

Fig. 4. The maze M_C for component $C = \overline{c}/b \parallel \overline{b}/c \parallel c, \overline{a}, \overline{b}/a \parallel b, d/d$ with maximal lazy front lines $(\{b, x, y\}, \{a, c, d\})$ and $(\{c, y\}, \{b, d\})$.

Game-Theoretic Perspective

the results. It has been observed in [1] that Pnueli and Shalev's interpretation of steps coincides exactly with the so-called *stable models* introduced by Gelfond and Lifschitz [21]. Consider configuration C as a propositional logic program. Given a set of events $E \subseteq \Pi$, let C_E be the program in which (i) all transitions with negative triggers in E are removed, i.e., we drop from C all $P, \overline{N}/A$ with $N \cap E \neq \emptyset$; and (ii) all remaining transitions are relieved from any negative events, i.e., every $P, \overline{N}/A$ with $N \cap E = \emptyset$ is simplified to P/A. The pruned program C_E has no negations, and thus it has a unique minimal classical model M. A classical model of C_E is a set $M \subseteq \Pi$ making all transitions/clauses of C_E true, i.e., for all P/A from C_E for which $P \subseteq M$ we have $A \subseteq M$. A set $M \subseteq \Pi$ is called a *stable model* of C if M is the minimal classical model of C_M. It has been shown in [21, 49] that stable models yield a more general semantics which consistently interprets a wider class of NLP programs than SLDNF.

Theorem 8 (Correctness & Completeness). *$M \subseteq \Pi$ is a stable model of configuration C if and only if M is a Pnueli-Shalev step response of C.*

Relation to Logic Programming

It is interesting to note that, while Pnueli and Shalev's notion of synchronous steps has not had much impact on synchronous programming tools, stable models have gained practical importance for NLP as the semantical underpinning of *answer set programming* [48]. From a wider perspective, therefore, it is fair to say that Pnueli-Shalev steps have indeed been implemented successfully in software engineering, albeit in a different domain. In addition, the theoretical results obtained around the Pnueli-Shalev semantics have ramifications in NLP. For instance, Thm. 4 of Sec. 3.3 implies that the standard intuitionistic semantics of logic provides a compositional and fully-abstract semantics for ground NLP programs under the stable interpretation.

Pnueli-Shalev semantics has been implemented in answer-set programming!

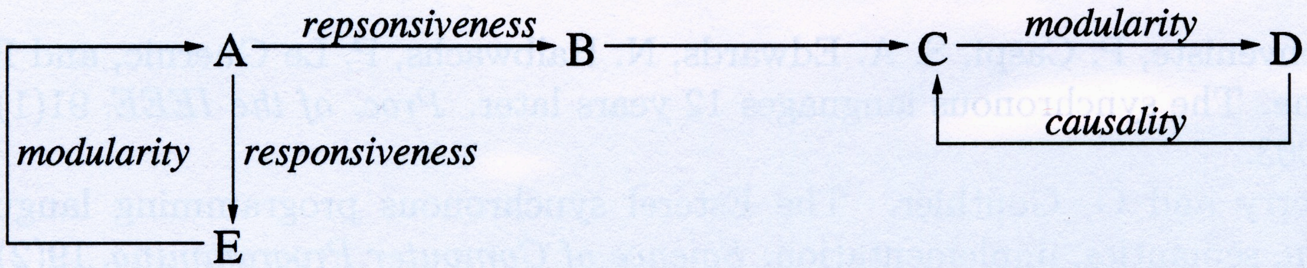

Fig. 5. Overview of the relationships among semantics A–E [32].

Amir’s view summer 1986