No Need for Liveness Model Checking Algorithms?

April, 2004

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Beyond Safety

International Workshop

April 25–28, 2004, Ringberg Castle, Germany

Simple Temporal Properties

- simple **safety** properties:
  - LTL: $\mathbf{G}p$
  - CTL: $\mathbf{AG}p$

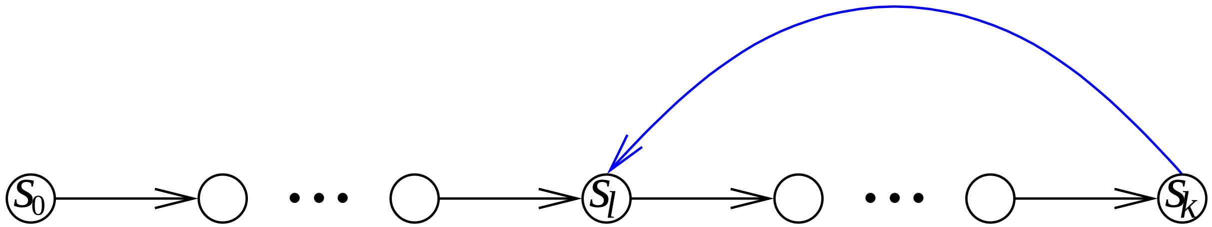
- simple **liveness** properties:
  - LTL: $\mathbf{F}p$
  - CTL: $\mathbf{AF}p$
  - plus fairness constraints (generalized Büchi Automata)

- full LTL can be translated to simple liveness + fairness

Completeness Thresholds in BMC

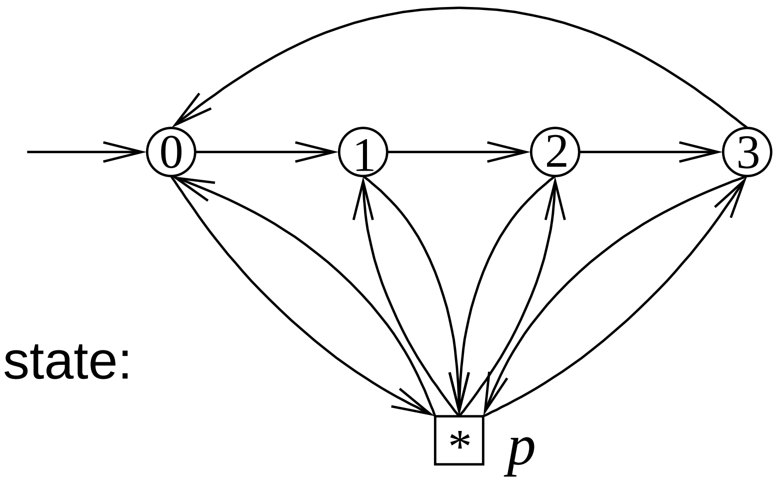
- counter examples to a **safety** property are finite traces
  - **radius** is the length of shortest initialized path to an arbitrary state
  - radius is a **completeness threshold** for (simple) safety properties
  - no longer potential counter example traces have to be checked

- every counter example trace to a liveness property is *lasso* shaped:



  - **diameter** is the length of the longest shortest path between two states
  - wrong: completeness threshold for liveness properties is $\boxed{\text{radius + diameter}}$

Completeness Threshold for Liveness Revised



- modulo n (here $n = 4$) counter with an explicit set state:

- radius and diameter both constant, but shortest counter example is of length n

- solution: use $\neg p$ predicated $\boxed{\text{radius + diameter}}$:
  - restrict Kripke structure to states in which $\neg p$ holds
  - calculate radius and diameter in restricted Kripke structure

Translating Liveness to Safety for Finite State Systems

- liveness is actually bounded liveness: $\mathbf{F}p \quad \equiv \quad \mathbf{F}_{\leq |S|}\, p$

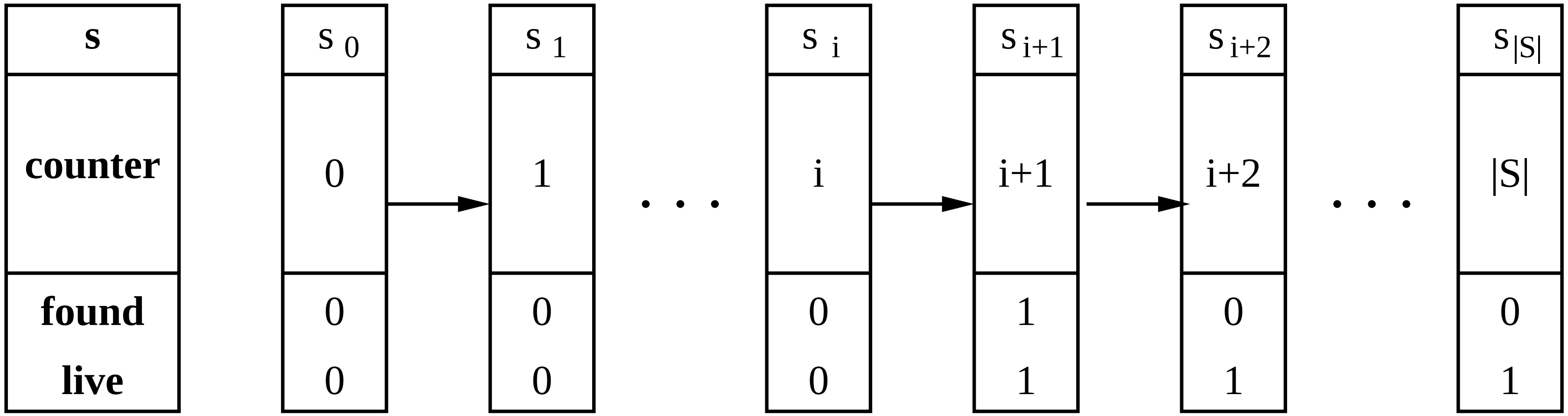
$$\mathbf{F}_{\leq |S|}\, p \quad \equiv \quad p \lor \mathbf{X}p \lor \ldots \lor \underbrace{\mathbf{X} \cdots \mathbf{X}}_{|S|} p$$

- brute force expansion needs exponential space for symbolic model checking

  (via the standard Büchi-Automata translation)

- counting translation requires twice the number of state bits

Counting Translation of Liveness to Safety



s = original state component

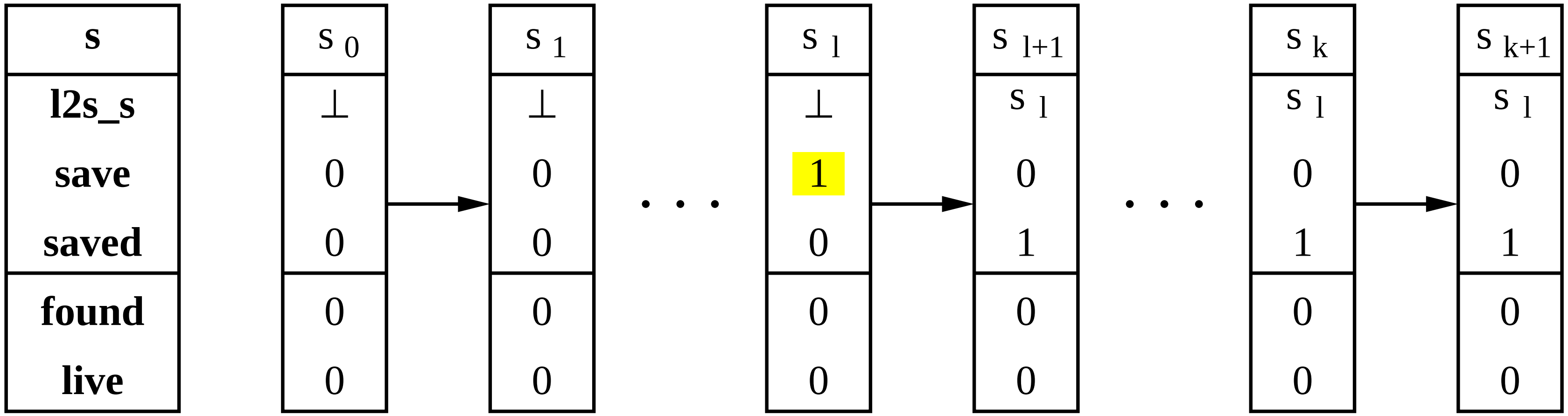
counter = $\lceil \log_2 |S| \rceil$-bit counter ($|S|$ = number of original states)

found = boolean flag: body of liveness property is satisfied

live = boolean state bit: **found** is or was true

$$\mathbf{G}\ (\mathbf{counter} = |S| \quad \rightarrow \quad \mathbf{live})$$

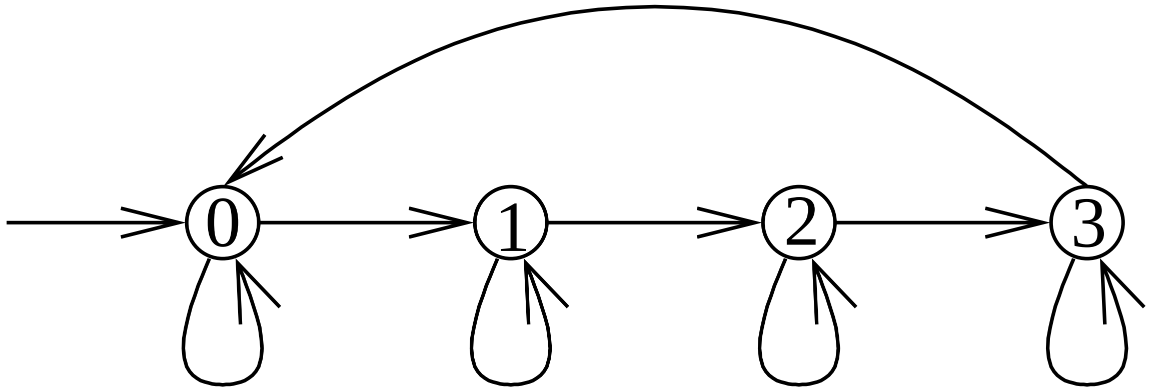
State Recording Translation of Liveness to Safety



s	=	original state component
l2s_s	=	copy of original state component to save a state
save	=	oracle (new primary input) to control when a state is saved
saved	=	boolean flag set to true when state has been saved
found	=	boolean flag: body of liveness property is satisfied
live	=	boolean state bit: **found** is or was true

$$\mathbf{G}\,(\mathbf{s} = \mathbf{l2s\_s} \quad \rightarrow \quad \mathbf{live})$$

Example: 2-Bit Counter with Self-Loops



$$\not\models \mathbf{F}(\mathtt{s} = 3)$$

```
DULE main
R
s: {0, 1, 2, 3};
SIGN
init(s) := 0;
next(s) := case
  s = 0: {1, s};
  s = 1: {2, s};
  s = 2: {3, s};
  s = 3: {0, s};
esac;

EC AF s = 3
```

```
MODULE main
VAR
  s: {0, 1, 2, 3};
ASSIGN
  init(s) := 0;
  next(s) := case
    s = 0: {1, s};
    s = 1: {2, s};
    s = 2: {3, s};
    s = 3: {0, s};
  esac;

-- loop detection part
VAR
  counter: 0..4;
ASSIGN
  init(counter) := 0;
  next(counter) := case
    counter < 4: counter + 1;
    1: counter;
  esac;

DEFINE
  looped := counter = 4;

-- property observing part
VAR live: boolean;
DEFINE found := s = 3;
ASSIGN
  init(live) := 0;
  next(live) := live | found;
SPEC AG (looped -> live)
```

```
MODULE main
VAR
  s: {0, 1, 2, 3};
ASSIGN
  init(s) := 0;
  next(s) := case
    s = 0: {1, s};
    s = 1: {2, s};
    s = 2: {3, s};
    s = 3: {0, s};
  esac;

-- loop detection part
VAR
  save: boolean;
  saved: boolean;
  l2s_s: {0, 1, 2, 3};
ASSIGN
  init(saved) := 0;
  next(saved) := on_loop;
  init(l2s_s) := s;
  next(l2s_s) := case
    save & !saved: s;
    1: l2s_s;
  esac;
DEFINE
  looped := saved & (s = l2s_
  on_loop := save | saved;

-- property observing part
VAR live: boolean;
DEFINE found := s = 3;
ASSIGN
  init(live) := 0;
  next(live) := live | found;
SPEC AG (looped -> live)
```

Comparison

- both translations are *complete*

- both translations double the number of state bits (in symbolic model checking)

- both translations may double the number of reachable states
  (really bad for explicit model checking)

- radius in counting translation may increase exponentially:
  (in symbolic model checking)

$$r^{\text{counting}} \quad \geq \quad |S|$$

- radius in state recording translation (optimizations possible):

$$r^{\text{recording}} \quad \leq \quad max\{r+2d+2, r_{\neg p}+d_{\neg p}+1\} \quad = \quad O(max\{d, d_{\neg p}\})$$

- counter examples found are indeed counter examples (correctness)

- conditions for completeness (modulo reachability):
  - if there is a counter example, then there is also a lasso shaped one
  - each trace visits only finite many states

- examples where it works (state variables $\in \mathbb{N}$):

$$
\begin{aligned}
I(s) &\equiv s \in \mathbb{N} \\
T(s,s') &\equiv s \geq s' \wedge \mathit{details}(s,s')
\end{aligned}
$$

$$
\begin{aligned}
I(s,b) &\equiv s = 0 \wedge b \in \mathbb{N} \\
T((s,b),(s',b')) &\equiv I(s',b') \vee \\
& s \leq s' \wedge b = b' \wedge \mathit{details}(s,s')
\end{aligned}
$$

Conclusion

- completeness threshold is different for liveness and safety
  - predicated diameter instead of ordinary diameter as bound
- finite states: efficient translation of liveness to safety
  - through state-recording translation
  - works in practice for symbolic model checking (e.g. with interpolation)
- infinite states: state recording workds for some examples
  - combination of state recording with fairness?
  - can we always (efficiently) translate liveness to safety?