**Trapezoid Rule**

Let $\text{Trap}(f, a, b, N) = \Delta \cdot \left[ \frac{f(a) + f(b)}{2} + \sum_{i=1}^{N-1} f(a + i \cdot \Delta) \right]$ where $\Delta = (b - a)/N$

Then $\int_a^b f(x)dx \approx \text{Trap}(f, a, b, N)$

Error estimate:

$$|\int_a^b f(x)dx - \text{Trap}(f, a, b, N)| \leq \frac{M \cdot (b - a)^3}{12N^2}$$

where $M$ is the maximum value of $f''(x)$ between $a$ and $b$.

Using the order-of-magnitude notation we will talk about in a couple of weeks, the error

$$|\int_a^b f(x)dx - \text{Trap}(f, a, b, N)| \approx O\left(\frac{1}{N^2}\right)$$

Estimates the area under the curve as a sum of trapezoids.

The trapezoid from $a + i \cdot \Delta$ to $a + (i + 1) \cdot \Delta$ has area $\Delta \cdot (f(a + i \cdot \Delta) + f(a + (i + 1) \cdot \Delta))/2$, so adding these up gives the formula.