Special Topics in Complexity Theory, Spring 2006, Yap
Homework 2
Due: Wed Feb 22

INSTRUCTIONS

- REMEMBER OUR HOMEWORK RULES: you prepare a solution in writing, and then we will have appointments in which you need to give an oral defence of your solution. I will grade BOTH the oral defence AND your written solution.

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1. (15 Points)
   Suppose $f = \omega(g^2)$ and both f, g are "nice" complexity functions. A function f is **nice** means that there exists a Turing machine that, on inputs of size n, will run for $f(n)$ steps and halt. Prove that there is a $DTIME(f) \setminus DTIME(g)$ is non-empty. HINT: Use a UTM that diagonalizes over every machine that runs in time $g(n)$. The UTM need to use $O(g(n)^2)$ time to simulate such machines. NOTE: This is the kind of result that is needed in the proof of Theorem 2.8 [p. 47].

2. (30 Points)
   Prove that the set of computable reals forms a field. That means that if x, y are computable reals, then so is $x \pm y$ and xy and x/y provided $y \neq 0$.

3. (15 Points)
   We have discussed two ways to formulate the computation of a real function $f : \mathbb{R} \to \mathbb{R}$. The text uses oracle machines. The other model assumes the input tape contains the Cauchy representation of a real number x, which is an infinite sequence of rapidly converging dyadic numbers. The Turing machine will output an infinite sequence that is a Cauchy representation of $f(x)$.

   Show that both models leads to the same concept of computable real functions.