Special Topics in Complexity Theory, Spring 2006, Yap
Homework 1
Due: Wed Feb 8

INSTRUCTIONS

- Please download the latest copy of Chapter 0 (Intro. to Computability). It should be dated after Feb 1, 2006. These exercises are based on material there. The sections you are responsible for are:
- REMEMBER OUR HOMEWORK RULES: you prepare a solution in writing, and then we will have appointments in which you need to give an oral defence of your solution. I will grade BOTH the oral defence AND your written solution.
- In the following questions, when you describe a dfa or nfa, please use state transition diagrams only. In particular, do not explicitly describe the δ function. Use our convention for indicating start states and final states.

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1. (10+10 Points)
   Construct dfa's to accept the following languages:
   (i) $\{w \in \{0,1\}^* : w \text{ has } 0101 \text{ as substring}\}$.
   (ii) $\{w \in \{0,1\}^* : w \text{ has neither } 00 \text{ nor } 11 \text{ as substring}\}$.

2. (10+10 Points)
   Write regular expressions for (i) and (ii) in the previous question.

3. (10x4 Points)
   Prove or disprove that each of the following languages is regular:
   (i) $\{1^n : n \text{ is divisible by } 7\}$.
   (ii) $\{w \in \{0,1\}^* : w \text{ is a binary number divisible by } 7\}$.
   (iii) $\{0^n 1 0^m 1 0^{m+n} : n, m \geq 1\}$.
   (iv) $\{ww' \in \{0,1\}^* : w' \text{ is obtained from } w \text{ by interchanging } 0 \text{ and } 1\}$.

4. (10 Points)
   Verify that the STM in Figure 6 of Chapter 0 accepts the marked duplicate language $L^{\#}_{\mathsf{dup}}$.

5. (15 Points)
   (i) Let $K_0 = \{L \in RE : \epsilon \in L\}$. Show that $L(K_0)$ is r.e..
   (ii) Let $K_1 = \{A_1\}$ where A_1 is any r.e. set. Show that $L(K_1)$ is not r.e..

6. (10 Points)
   Students often assume that if A is r.e., then any enumeration $\alpha : \mathbb{N} \to A$ is recursive. To dispel this idea, prove the following: if A is infinite and r.e., then there exists a non-recursive denumeration

$$f : \mathbb{N} \to A.$$

REMARK: If A is non-recursive, there is a simple argument. So it may be enough to assume A is recursive.

HINT: Let $\alpha : \mathbb{N} \to A$. be a recursive denumeration of A. Now split A into $A_0 \uplus A_1$, and define $f : \mathbb{N} \to A$ such that $f(i) \in A_0$ iff $i \in \mathrm{HALT}$.

7. (15 Points)
   Use the Rice-Shapiro theorem to show the following sets to be non-r.e.:
   (i) $\mathrm{TOT} = \{i \in \mathbb{N} : \phi_i \text{ is total}\}$
   (ii) co-TOT.