Honors Theory, Spring 2002, Yap
Homework 1
Due: Wed Feb 6 in class

In the following questions, when you describe a dfa or nfa, please use state transition diagrams only. Do not explicitly describe the δ function. This makes grading much easier. Be sure to use our convention for indicating start states and final states.

1. Construct dfa's to accept the following languages:
   (i) $\{w \in \{0,1\}^* : w$ has 0101 as substring$\}$.
   (ii) $\{w \in \{0,1\}^* : w$ has neither 00 nor 11 as substring$\}$.

2. Write regular expressions for (i) and (ii) in the previous question.

3. Construct nfa's to accept the languages indicated by the following expressions:
   (i) $(01)^*(10)^* + 00^*$.
   (ii) $((01 + 001)^*0^*)^*$.
   Be sure to parse these regular expressions correctly when we paretheses are omitted.

4. Prove or disprove that each of the following languages is regular:
   (i) $\{1^n : n$ is divisible by $7\}$.
   (ii) $\{w \in \{0,1\}^* : w$ is a binary number divisible by $7\}$.
   (iii) $\{0^n 1 0^m 1 0^{m+n} : n, m \geq 1\}$.
   (iv) $\{ww' \in \{0,1\}^* : w'$ is obtained from w by interchanging 0 and 1$\}$.

5. A **letter homomorphism** is a function $h : \Sigma \to \Gamma^*$ where Σ, Γ are alphabets. We can extend h to the function

$$h : \Sigma^* \to \Gamma^*$$

   in the natural way, so that $h(vw) = h(v)h(w)$ for all $v, w \in \Sigma^*$. In particular, $h(\epsilon) = \epsilon$. If $A \subseteq \Sigma^*$, then define $h[A] = \{h(w) : w \in A\} \subseteq \Gamma^*$. Show the following.
   (i) If $A \subseteq \Sigma^*$ is regular, so is $h[A]$.
   (ii) If $B \subseteq \Gamma^*$ is regular, so is that $h^{-1}[B] = \{w \in \Sigma^* : h(w) \in B\}$. HINT: start from a dfa M for B and construct one that, when it reads an input symbol a, tries to simulate M on $h(a)$.