Lectures on Computational Geometry and Modeling
Spring 2005 (Yap)

January 19, 2005

1 Course Description

Computational Geometry has traditionally emphasized combinatorial analysis and efficient algorithmic techniques to solve a variety of geometric problems. Most of these problems are linear problems, involving points and piecewise linear objects (e.g., polytopes).

In recent years, the push to extend such techniques to nonlinear geometric problems such as curves and surfaces have exposed new algorithmic issues: non-robustness issues, computing with curves and surfaces, interaction between algebraic and numerical computations, etc.

This course will give you some of the necessary background and try to bring you to the frontier of some issues.

Here are the main references:


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[M+Y] Robust Geometric Computation,
by Mehlhorn and Yap.
Preliminary draft from http://cs.nyu.edu/~yap/book/egc/.

[Edel] Geometry and Topology for Mesh Generation,
by H. Edelsbrunner.
Cambridge Press, 2001.

[Walk] Algebraic Curves,
by R.J.Walker.
        Springer-Verlag, Berlin-New York 1978.

[B+G] Curves and Singularities,
        by J.W.Bruce and P.J.Giblin.
        Cambridge Press (2nd Edition), 1992.
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The range of material is quite vast, but we shall emphasize two important topics: meshes and algebraic curves.

The mesh book [Edel] is available in the NYU book store.

For curves, we emphasize the algebraic properties, using [Walk]. We will not have for analytic properties of curves, so [B+G] is mainly for side reading.

2 Outline of Lectures

I will try to adhere as closely as possible to the following schedule. For each lecture, I give the references which I hope you will read up in advance.

1. Introduction to Nonrobustness and EGC:

   Jan 19. REF: [M+Y], Chapter 1 and 2.

   Topics: nonrobustness phenomenon, various approaches, extended example (Convex Hull), nature of geometric computation, the EGC approach.

2. Numerical Computation

   Jan 26. REF: [M+Y, Chap. 2]

   Topics: Newton iteration, Brent's algorithms, composition of elementary function, theory of numerical approximation. .

3. Algebraic Computation

   Feb 2. REF: [M+Y, Chap.6]

   Topics: Resultants, Bezout's Theorem,

4. Tetrahedralization:

   Feb 9. REF: [Edel, Chap.5 and 6].

5. Delaunay Triangulations:

   Feb 16. REF: [Edel, Chap. 1 and 2]

6. Combinatorial Topology and Simplification:

   Feb 23. REF: [Edel, Chap.3 and 4]

7. The Zero Problem

   Mar 2. REF: [M+Y]

   Topics: Mahler's measure, BFMSS Bound, Richardson's work.

8. Algebraic Curves 1

   Mar 9. REF: [Walk]

   Mar 16. SPRING BREAK

9. Guest Lecture by Kaltofen.

   Mar 23. Numerical algebra.

10. Algebraic Curves 2

    Mar 30. REF: [Walk]

11. Algebraic Curves 3

    Apr 6. REF: [Walk]

12. Parametric Curves and Surfaces

    Apr 13. REF: [M+Y, Chap.30], [Farin]

    Topics: intersection of quadric surfaces, Bezier curves, NURBs, Parametric Surfaces.

13. Non-algebraic Computation

    Apr 20. REF: Decidability of Disc Shortest Path, Hypergeometric Functions

    Topics: transcendental number theory, hypergeometric functions, decidability of disc shortest path problem.

14. Guest Lecture by Pion/Yvinec.

    Apr 27. Filters, Geometric Predicates.