Learning Hierarchies of Visual Features

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Unsupervised Feature Learning with Sparse Coding

The learning algorithm minimizes the loss function:

$$L(W_d) = \sum_i F(Y^i; W_d) = \sum_i (\min_Z E(Y^i, Z; W_d))$$

The columns of $W_d$ are normalized

$$E(Y^i, Z; W_d) = \|Y^i - W_d Z\|^2 + \lambda \sum_j |z_j|$$

Free Energy: 

$$F(Y^i; W_d) = F(Z^i) = \min_z E(Y^i, z; W_d)$$
Problem with Sparse Coding: Inference is slow

Inference: find Z that minimizes the energy for a given Y

\[ E(Y^i, Z^i ; W_d) = \|Y^i - W_d Z^i\|^2 + \lambda \sum_j |z^i_j| \]

\[ Z^i = \text{argmin}_z E(Y^i, z ; W_d) \]

- For each new Y, an optimization algorithm must be run to find the corresponding optimal Z
- This would be very slow for large scale vision tasks
- Also, the optimal Z are very unstable:
  - A small change in Y can cause a large change in the optimal Z
Solution: Predictive Sparse Decomposition (PSD)

Prediction the optimal code with a trained encoder

Energy = reconstruction_error + code_prediction_error + code_sparsity

\[ E(Y^i, Z) = \|Y^i - W_d Z\|^2 + \|Z - g_e(W_e, Y^i)\|^2 + \lambda \sum_j |z_j| \]

\[ g_e(W_e, Y^i) = D \tanh(W_e Y) \]
Inference by gradient descent starting from the encoder output

\[ E(Y^i, Z) = \|Y^i - W_d Z\|^2 + \|Z - g_e(W_e, Y^i)\|^2 + \lambda \sum_j |z_j| \]

\[ Z^i = \text{argmin}_z E(Y^i, z; W) \]
Learning by minimizing the average energy of the training data with respect to \( W_d \) and \( W_e \).

**Loss function:**

\[
L(W_d, W_e) = \sum_i F(Y^i; W_d, W_e)
\]

\[
F(Y^i; W_d, W_e) = \min_z E(Y^i, z; W_d, W_e)
\]
1. Initialize $Z = \text{Encoder}(Y)$

2. Find $Z$ that minimizes the energy function

3. Update the Decoder basis functions to reduce reconstruction error

4. Update Encoder parameters to reduce prediction error

Repeat with next training sample
Simple: $D \cdot \tanh(We.Y)$

Sophisticated $Z(t) = \text{shrink}(We.Y - S.Z(t-1))$

For a discussion of best encoders, see [Gregor & LeCun, ICML 2010]
PSD trained on handwritten digits: decoder filters are “parts” (strokes).

- Any digit can be reconstructed as a linear combination of a small number of these “parts”.
Basis functions are like Gabor filters (like receptive fields in V1 neurons)

- 256 filters of size 12x12
- Trained on natural image patches from the Berkeley dataset
- Encoder is linear-tanh-diagonal
Learned Features on natural patches: V1-like receptive fields
Learned Features: V1-like receptive fields

- 12x12 filters
- 1024 filters
Classification Error Rate on MNIST

- Supervised Linear Classifier trained on 200 trained sparse features
  - Red: linear-tanh-diagonal encoder; Blue: linear encoder

![Graphs showing error rate vs. RMSE for different sample sizes: 10, 100, and 1000 samples.](image-url)
Phase 1: train first layer using PSD

\[ \|Y^i - \hat{Y}\|^2 \]

\[ WDZ \]

\[ g_e(W_e, Y^i) \]

\[ \|Z - \hat{Z}\|^2 \]

\[ |z_j| \]

\[ \lambda \sum_j \]

\[ FEATURES \]
Using PSD to Train a Hierarchy of Features

- Phase 1: train first layer using PSD
- Phase 2: use encoder + absolute value as feature extractor
Using PSD to Train a Hierarchy of Features

- Phase 1: train first layer using PSD
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- Phase 3: train the second layer using PSD
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- Phase 4: use encoder + absolute value as 2nd feature extractor
Using PSD to Train a Hierarchy of Features

- Phase 1: train first layer using PSD
- Phase 2: use encoder + absolute value as feature extractor
- Phase 3: train the second layer using PSD
- Phase 4: use encoder + absolute value as 2nd feature extractor
- Phase 5: train a supervised classifier on top
- Phase 6 (optional): train the entire system with supervised back-propagation
The “deep learning” method was popularized by Hinton for training “deep belief networks”.

- DBN use a special kind of encoder-decoder architecture called Restricted Boltzmann Machines (RBM)

1. Train each layer in an unsupervised fashion, layer by layer

2. Stick a supervised classifier on top, and refine the entire system with gradient descent (back-prop) on a supervised criterion.
**Unsupervised Learning: Capturing Dependencies Between Variables**

- **Energy function:** viewed as a negative log probability density

- **Probabilistic View:**
  - Produce a probability density function that:
  - has high value in regions of high sample density
  - has low value everywhere else (integral = 1).

- **Energy-Based View:**
  - produce an energy function $E(Y,W)$ that:
  - has low value in regions of high sample density
  - has high(er) value everywhere else
Unsupervised Learning: Capturing Dependencies Between Variables

- Energy function viewed as a negative log density

  - Example: \( y = x^2 \)
\[ P(Y|W) = \frac{e^{-\beta E(Y,W)}}{\int_Y e^{-\beta E(y,W)}} \]

\[ E(Y, W) \propto -\log P(Y|W) \]
Training an Energy-Based Model

- Make the energy around training samples low
- Make the energy everywhere else higher

\[ P(Y, W) = \frac{e^{-\beta E(Y, W)}}{\int_y e^{-\beta E(y, W)}} \]
Training an Energy-Based Model to Approximate a Density

Maximizing $P(Y|W)$ on training samples

$$P(Y|W) = \frac{e^{-\beta E(Y,W)}}{\int_y e^{-\beta E(y,W)}}$$

Minimizing $-\log P(Y,W)$ on training samples

$$L(Y,W) = E(Y, W) + \frac{1}{\beta} \log \int_y e^{-\beta E(y,W)}$$
Training an Energy-Based Model with Gradient Descent

Gradient of the negative log-likelihood loss for one sample $Y$:

$$\frac{\partial L(Y, W)}{\partial W} = \frac{\partial E(Y, W)}{\partial W} - \int_y P(y|W) \frac{\partial E(y, W)}{\partial W}$$

Gradient descent:

$$W \leftarrow W - \eta \frac{\partial L(Y, W)}{\partial W}$$

- Pushes down on the energy of the samples
- Pulls up on the energy of low-energy $Y$'s

$$W \leftarrow W - \eta \frac{\partial E(Y, W)}{\partial W} + \eta \int_y P(y|W) \frac{\partial E(y, W)}{\partial W}$$
How do we push up on the energy of everything else?

**Solution 1: contrastive divergence [Hinton 2000]**
- Move away from a training sample a bit
- Push up on that

**Solution 2: score matching**
- On the training samples: minimize the gradient of the energy, and maximize the trace of its Hessian.

**Solution 3: denoising auto-encoder (not really energy-based)**
- Train the inference dynamics to map noisy samples to clean samples

**Solution 4: MAIN INSIGHT! [Ranzato, ..., LeCun AI-Stat 2007]**
- Restrict the information content of the code (features) Z
- If the code Z can only take a few different configurations, only a correspondingly small number of Ys can be perfectly reconstructed
- Idea: impose a sparsity prior on Z
- This is reminiscent of sparse coding [Olshausen & Field 1997]
Restricted Boltzmann Machines

[Hinton & Salakhutdinov 2005]

- Y and Z are binary
- Enc and Dec are linear
- Distance is negative dot product

\[ E(Y, Z) = \text{Dist}[Y, \text{Dec}(Z)] + \text{Dist}[Z, \text{Enc}(Y)] \]

\[ \text{Enc}(Y) = -W.Y \quad \text{Dist}(Z, W.Y) = -\frac{1}{2}Z^T.W.Y \]

\[ \text{Dec}(Y) = -W^T.Z \quad \text{Dist}(Y, E^T.Z) = -\frac{1}{2}Y^T.W^T.Z \]

\[ E(Y, Z) = -Z^T.W.Y \quad F(Y) = -\log \sum_{Z} e^{Z^T.W.Y} \]
Non-Linear Dimensionality Reduction with Stacked RBMs

[Hinton and Salakhutdinov, Science 2006]

Fig. 1. Pretraining consists of learning a stack of restricted Boltzmann machines (RBMs), each having only one layer of feature detectors. The learned feature activations of one RBM are used as the “data” for training the next RBM in the stack. After the pretraining, the RBMs are “unrolled” to create a deep autoencoder, which is then fine-tuned using backpropagation of error derivatives.
Non-Linear Dimensionality Reduction with Deep Learning

[Hinton and Salakhutdinov, Science 2006]

Fig. 2. (A) Top to bottom: Random samples of curves from the test data set; reconstructions produced by the six-dimensional deep autoencoder; reconstructions by “logistic PCA” (b) using six components; reconstructions by logistic PCA and standard PCA using 18 components. The average squared error per image for the last four rows is 1.44, 7.64, 2.45, 5.90. (B) Top to bottom: A random test image from each class; reconstructions by the 30-dimensional autoencoder; reconstructions by 30-dimensional logistic PCA and standard PCA. The average squared errors for the last three rows are 3.00, 8.01, and 13.87. (C) Top to bottom: Random samples from the test data set; reconstructions by the 30-dimensional autoencoder; reconstructions by 30-dimensional PCA. The average squared errors are 126 and 135.
Non-Linear Dimensionality Reduction: MNIST

[Hinton and Salakhutdinov, Science 2006]

Fig. 3. (A) The two-dimensional codes for 500 digits of each class produced by taking the first two principal components of all 60,000 training images. (B) The two-dimensional codes found by a 784-1000-500-250-2 autoencoder. For an alternative visualization, see (B).
Non-Linear Dimensionality Reduction: Text Retrieval

[Hinton and Salakhutdinov, Science 2006]

Fig. 4. (A) The fraction of retrieved documents in the same class as the query when a query document from the test set is used to retrieve other test set documents, averaged over all 402,207 possible queries. (B) The codes produced by two-dimensional LSA. (C) The codes produced by a 2000-500-250-125-2 autoencoder.
Examples of LabelMe retrieval using RBMs

- [Torralba, Fergus, Weiss, CVPR 2008]
- 12 closest neighbors under different distance metrics
LabelMe Retrieval Comparison of methods

% of 50 true neighbors in retrieval set

Size of retrieval set
Prediction the optimal code with a trained encoder

Energy = reconstruction_error + code_prediction_error + code_sparsity

\[ E(Y^i, Z) = \|Y^i - W_d Z\|^2 + \|Z - g_e(W_e, Y^i)\|^2 + \lambda \sum_j |z_j| \]

\[ g_e(W_e, Y^i) = D \tanh(W_e Y) \]
If the information content of the feature vector is limited (e.g. by imposing sparsity constraints), the energy MUST be large in most of the space.

Pulling down on the energy of the training samples will necessarily make a groove.

The volume of the space over which the energy is low is limited by the entropy of the feature vector.

Input vectors are reconstructed from feature vectors.

If few feature configurations are possible, few input vectors can be reconstructed properly.
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

Training based on minimizing the reconstruction error over the training set
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

BAD: machine does not learn structure from training data!!

It just copies the data.
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

IDEA: reduce number of available codes.
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
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**IDEA:** reduce number of available codes.
Why Limit the Information Content of the Code?

- Training sample
- Input vector which is NOT a training sample
- Feature vector

**IDEA:** reduce number of available codes.
We are going to impose a sparsity penalty on the code to restrict its information content.

We will allow the code to have higher dimension than the input.

Categories are more easily separable in high-dim sparse feature spaces.
  - This is a trick that SVM use: they have one dimension per sample.

Sparse features are optimal when an active feature costs more than an inactive one (zero).
  - e.g. neurons that spike consume more energy
  - The brain is about 2% active on average.
2 dimensional toy dataset
- Mixture of 3 Cauchy distrib.
- Visualizing energy surface
  (black = low, white = high)

[ Ranzato 's PhD thesis 2009 ]
- 2 dimensional toy dataset
- spiral

Visualizing energy surface (black = low, white = high)

<table>
<thead>
<tr>
<th>Method</th>
<th>Encoder</th>
<th>Decoder</th>
<th>Energy</th>
<th>Loss</th>
<th>Pull-up</th>
<th>Dimens.</th>
<th>Code Units</th>
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<tbody>
<tr>
<td>PCA</td>
<td>$W$</td>
<td>$WZ$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>dimens.</td>
<td></td>
<td>(1 code unit)</td>
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<td>$\sigma(W_e Y)$</td>
<td>$W_d Z$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>dimens.</td>
<td></td>
<td>(1 code unit)</td>
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<tr>
<td>Sparse coding</td>
<td>$\sigma(W_e Z)$</td>
<td>$W_d Z$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>sparsity</td>
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<td>(20 code units)</td>
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<tr>
<td>K-Means</td>
<td>$WZ$</td>
<td>$|Y - WZ|^2$</td>
<td>$F(Y)$</td>
<td>1-of-N code</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- PCA
- Autoencoder
- Sparse coding
- K-Means
Using PSD to learn the features of an object recognition system

1. Train filters on images patches with PSD
2. Plug the filters into a ConvNet architecture
3. Train a supervised classifier on top
“Modern” Object Recognition Architecture in Computer Vision

Example:
- Edges + Rectification + Histograms + SVM [Dalal & Triggs 2005]
- SIFT + classification

Fixed Features + “shallow” classifier
“State of the Art” architecture for object recognition

Example:
- SIFT features with Spatial Pyramid Match Kernel SVM [Lazebnik et al. 2006]

Fixed Features + unsupervised features + “shallow” classifier
Can't we get the same results with (deep) learning?

Stacking multiple stages of feature extraction/pooling.

Creates a hierarchy of features

ConvNets and SIFT+PMK-SVM architectures are conceptually similar

Can deep learning make a ConvNet match the performance of SIFT+PNK-SVM?
How well do PSD features work on Caltech-101?

Recognition Architecture

- Filter Bank
- Non-Linearity
- Spatial Pooling
- Classifier

- SVM
Procedure for a single-stage system

1. Pre-process images
   - remove mean, high-pass filter, normalize contrast

2. Train encoder-decoder on 9x9 image patches

3. use the filters in a recognition architecture
   - Apply the filters to the whole image
   - Apply the tanh and D scaling
   - Add more non-linearities (rectification, normalization)
   - Add a spatial pooling layer

4. Train a supervised classifier on top
   - Multinomial Logistic Regression or Pyramid Match Kernel SVM
64 filters on 9x9 patches trained with PSD

with Linear-Sigmoid-Diagonal Encoder

weights \( \pm 0.2828 \) - \( 0.3043 \)
Feature Extraction

Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
Feature Extraction

Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- C  Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- Abs Rectification layer: needed?

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- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
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Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
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\[
\frac{x - \mu}{\max(t, \sigma)}
\]

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
- **N** Normalization layer: needed?

![Diagram showing feature extraction process]

- Convolution/sigmoid layer
- Rectification layer
- Normalization layer

Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
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Pinto, Cox and DiCarlo, PloS 08
Feature Extraction

- C  Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- Abs Rectification layer: needed?
- N  Normalization layer: needed?

Pooling Down-Sampling Layer
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
- **N** Normalization layer: needed?
- **P** Pooling down-sampling layer: average or max?
Feature Extraction

- **C** Convolution/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
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Feature Extraction

- **C** Convolutions/sigmoid layer: filter bank? Learning, fixed Gabors?
- **Abs** Rectification layer: needed?
- **N** Normalization layer: needed?
- **P** Pooling down-sampling layer: average or max?

This is one stage of feature extraction.
Training Protocol

Training

- Logistic Regression on Random Features: $R$
- Logistic Regression on PSD features: $U$
- Refinement of whole net from random with backprop: $R^+$
- Refinement of whole net starting from PSD filters: $U^+$

Classifier

- Multinomial Logistic Regression or Pyramid Match Kernel SVM
## Using PSD Features for Recognition

\[
[64.F_{CSG}^{9\times9} - R/N/P_{5\times5}] - \text{log\_reg}
\]

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<td>32.1%</td>
<td>15.3%</td>
<td>12.1(±2.2)%</td>
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\[
[64.F_{CSG}^{9\times9} - R/N/P_{5\times5}] - \text{PMK}
\]

| U | 65.0\% |

\[
[96.F_{CSG}^{9\times9} - R/N/P_{5\times5}] - \text{PCA - lin\_svm}
\]

| U | 58.0\% |

96. Gabors - PCA - lin\_svm (Pinto and DiCarlo 2006)

| Gabors | 59.0\% |

SIFT - PMK (Lazebnik et al. CVPR 2006)

| Gabors | 64.6\% |
Using PSD Features for Recognition

- **Rectification makes a huge difference:**
  - 14.5% -> 50.0%, without normalization
  - 44.3% -> 54.2% with normalization

- **Normalization makes a difference:**
  - 50.0 → 54.2

- **Unsupervised pretraining makes small difference**

- **PSD works just as well as SIFT**

- **Random filters work as well as anything!**
  - If rectification/normalization is present

- **PMK_SVM classifier works a lot better than multinomial log_reg on low-level features**
  - 52.2% → 65.0%
Comparing Optimal Codes Predicted Codes on Caltech 101

Approximated Sparse Features Predicted by PSD give better recognition results than Optimal Sparse Features computed with Feature Sign!

- PSD features are more stable.

Feature Sign (FS) is an optimization methods for computing sparse codes [Lee...Ng 2006]
Approximated Sparse Features Predicted by PSD give better recognition results than Optimal Sparse Features computed with Feature Sign!

Because PSD features are more stable. Feature obtained through sparse optimization can change a lot with small changes of the input.

How many features change sign in patches from successive video frames (a,b), versus patches from random frame pairs (c)
PSD features are much cheaper to compute

Computing PSD features is hundreds of times cheaper than Feature Sign.
How Many 9x9 PSD features do we need?

Accuracy increases slowly past 64 filters.
Yann LeCun

Training a Multi-Stage Hubel-Wiesel Architecture with PSD

1. Train stage-1 filters with PSD on patches from natural images
2. Compute stage-1 features on training set
3. Train state-2 filters with PSD on stage-1 feature patches
4. Compute stage-2 features on training set
5. Train linear classifier on stage-2 features
6. Refine entire network with supervised gradient descent

What are the effects of the non-linearities and unsupervised pretraining?
Multistage Hubel-Wiesel Architecture on Caltech-101

Y (luminance)

U

V

CONVOLUTIONS (9x9)

MAX/SUBSAMPLING (4x4)

CONVOLUTIONS (9x9)

MAX/SUBSAMPLING (5x5)

INPUT 3@140x140

32@132x132

32@33x33

64@25x25

64@5x5
Multistage Hubel-Wiesel Architecture

**Image Preprocessing:**
- High-pass filter, local contrast normalization (divisive)

**First Stage:**
- Filters: 64 9x9 kernels producing 64 feature maps
- Pooling: 10x10 averaging with 5x5 subsampling

**Second Stage:**
- Filters: 4096 9x9 kernels producing 256 feature maps
- Pooling: 6x6 averaging with 3x3 subsampling
- Features: 256 feature maps of size 4x4 (4096 features)

**Classifier Stage:**
- Multinomial logistic regression

**Number of parameters:**
- Roughly 750,000
### Multistage Hubel-Wiesel Architecture on Caltech-101

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**Two Stage System:** \([64.F^{9 \times 9}_{\text{CSG}} - R/N/P^{5 \times 5}] - [256.F^{9 \times 9}_{\text{CSG}} - R/N/P^{4 \times 4}] - \text{log_reg}\)

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Like HMux model

**Single Stage:** \([64.F^{9 \times 9}_{\text{CSG}} - R/N/P^{5 \times 5}] - \text{PMK-SVM}\)

| \( U \)  | 64.0%                         |

**Two Stages:** \([64.F^{9 \times 9}_{\text{CSG}} - R/N/P^{5 \times 5}] - [256.F^{9 \times 9}_{\text{CSG}} - R/N] - \text{PMK-SVM}\)

| \( UU \)  | 52.8%                         |
Two-Stage Result Analysis

Latest result: **69.7% correct**

Second Stage + logistic regression = PMK_SVM

Unsupervised pre-training doesn't help much :-(

Random filters work amazingly well with normalization

Supervised global refinement helps a bit

The best system is really cheap

Either use rectification and average pooling or no rectification and max pooling.
Multistage Hubel-Wiesel Architecture: Filters

Stage 1

After PSD

Stage 2

After supervised refinement

weights : -0.2232 - 0.2075

weights : -0.2828 - 0.3043

weights : -0.0778 - 0.064

weights : -0.0923 - 0.0784
Demo: real-time learning of visual categories

Input high-pass filtered contrast-normalized 83x83 (raw: 91x91)

STAGE 1
- Filter Bank + Tanh + Gain
- 64 features 75x75
- 64 filters
- 9x9 kernels
- Abs + Contrast Norm + Pooling + Downsampling
- 64 features 14x14
- 5x5 subsampling
- 10x10 pooling

STAGE 2
- Filter Bank + Tanh + Gain
- 256 features 6x6
- 4096 filters
- 9x9 kernels
- Abs + Contrast Norm + Pooling + Downsampling
- 256 features 1x1
- 4x4 subsampling
- 6x6 pooling

Parzen Windows Classifier
MNIST dataset

- 10 classes and up to 60,000 training samples per class
MNIST dataset

Architecture

U⁺U⁺: 0.53% error (this is a record on the undistorted MNIST!)

Comparison: $RR$ versus $UU$ and $R⁺R⁺$

Classification error on the MNIST dataset

- Supervised training of the whole network
- Unsupervised training of the feature extractors
- Random feature extractors
Why Random Filters Work?
## Replacing K-means with Sparse Coding

[Yang 2008] [Boureau, Bach, Ponce, LeCun 2010]

<table>
<thead>
<tr>
<th>Method</th>
<th>Caltech 15</th>
<th>Caltech 30</th>
<th>Scenes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiman et al. [1]</td>
<td>Nearest neighbor + spatial correspondence</td>
<td>65.00 ± 1.14</td>
<td>70.40</td>
</tr>
<tr>
<td>Jain et al. [8]</td>
<td>Fast image search for learned metrics</td>
<td>61.00</td>
<td>69.60</td>
</tr>
<tr>
<td>Lazebnik et al. [12]</td>
<td>Spatial Pyramid + hard quantization + kernel SVM</td>
<td>56.40</td>
<td>64.40 ± 0.80</td>
</tr>
<tr>
<td>van Gemert et al. [24]</td>
<td>Spatial Pyramid + soft quantization + kernel SVM</td>
<td>-</td>
<td>64.14 ± 1.18</td>
</tr>
<tr>
<td>Yang et al. [26]</td>
<td>SP + sparse codes + max pooling + linear</td>
<td>67.00±0.45</td>
<td>73.2±0.54</td>
</tr>
<tr>
<td>Zhang et al. [27]</td>
<td>kNN-SVM</td>
<td>59.10 ± 0.60</td>
<td>66.20 ± 0.50</td>
</tr>
<tr>
<td>Zhou et al. [29]</td>
<td>SP + Gaussian mixture</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Baseline:
- SP + hard quantization + avg pool + kernel SVM: 56.74 ± 1.31, 64.19 ± 0.94, 80.89 ± 0.21
- SP + soft quantization + avg pool + kernel SVM: 59.12 ± 1.51, 66.42 ± 1.26, 81.52 ± 0.54
- 1x1 features: 63.61 ± 0.88, - | 83.41 ± 0.57
- 8 pixel grid resolution
  - SP + sparse codes + avg pool + kernel SVM: 62.85 ± 1.22, 70.27 ± 1.29, 83.15 ± 0.35
  - SP + sparse codes + max pool + kernel SVM: 64.62 ± 0.94, 71.81±0.96, 84.25 ± 0.35
  - SP + sparse codes + max pool + linear: 64.71 ± 1.05, 71.52 ± 1.13, 83.78 ± 0.53

Macrofeatures + Finer grid resolution
- SP + sparse codes + max pool + kernel SVM | 69.03±1.17 | 75.72±1.06 | 84.60 ± 0.38
- SP + sparse codes + max pool + linear | 68.78 ± 1.09 | 75.14 ± 0.86 | 84.41 ± 0.26
Small NORB dataset

- 5 classes and up to 24,300 training samples per class
NORB Generic Object Recognition Dataset

- **50** toys belonging to 5 categories: **animal, human figure, airplane, truck, car**
- **10** instance per category: **5 instances used for training, 5 instances for testing**
- **Raw dataset:** **972** stereo pair of each object instance. **48,600** image pairs total.

For each instance:
- **18 azimuths**
  - 0 to 350 degrees every 20 degrees
- **9 elevations**
  - 30 to 70 degrees from horizontal every 5 degrees
- **6 illuminations**
  - On/off combinations of 4 lights
- **2 cameras (stereo)**
  - 7.5 cm apart
  - 40 cm from the object

Training instances  │  Test instances
Small NORB dataset

Two-stage system: error rate versus number of labeled training samples

- No normalization
- Random filters
- Unsup filters
- Sup filters
- Unsup+Sup filters
Learning To Approximate Sparse Coding
Karol Gregor, Yann LeCun
Sparse Coding with ISTA (Iterative Shrinkage and Thresholding Algorithm)

ISTA/FISTA: converges to optimal sparse code
Time unrolling of ISTA

ISTA/FISTA: converges to optimal sparse code
LISTA vs ISTA/FISTA

![Graph showing error vs iter for LISTA and FISTA]
LISTA with partial mutual inhibition matrix

error

dim reduction (4x)
elements removal (4x)
dim reduction (1x)
elements removal (1x)
LcoD (iteration = number of updated components)
Learning Complex Cells with Invariance Properties

[Kavukcuoglu et al. CVPR 2008]
Unsupervised PSD ignores the spatial pooling step.

Could we devise a similar method that learns the pooling layer as well?

Idea [Hyvarinen & Hoyer 2001]: group sparsity on pools of features
- Minimum number of pools must be non-zero
- Number of features that are on within a pool doesn't matter
- Pools tend to regroup similar features
Learning the filters and the pools

Using an idea from Hyvarinen: topographic square pooling (subspace ICA)

1. Apply filters on a patch (with suitable non-linearity)
2. Arrange filter outputs on a 2D plane
3. Square filter outputs
4. Minimize sqrt of sum of blocks of squared filter outputs

Overall Sparsity term: \[ \sum_{i=1}^{K} \sqrt{v_i^2} \]

\[ v_1^2 = \sum_{j \in P_1} (w_j z_j)^2 \]
\[ v_k^2 = \sum_{j \in P_k} (w_j z_j)^2 \]

Units in the code \( Z \) Define pools and enforce sparsity across pools
Learning the filters and the pools

- The filters arrange themselves spontaneously so that similar filters enter the same pool.
- The pooling units can be seen as complex cells.
- They are invariant to local transformations of the input.
  - For some it's translations, for others rotations, or other transformations.
Pinwheels?
Invariance Properties Compared to SIFT

- Measure distance between feature vectors (128 dimensions) of 16x16 patches from natural images
  - Left: normalized distance as a function of translation
  - Right: normalized distance as a function of translation when one patch is rotated 25 degrees.

- Topographic PSD features are more invariant than SIFT
Learning Invariant Features

Recognition Architecture

- HPF/LCN - filters - tanh - sqr - pooling - sqrt - Classifier

- Block pooling plays the same role as rectification
A/B Comparison with SIFT (128x34x34 descriptors)
32x16 topographic map with 16x16 filters
Pooling performed over 6x6 with 2x2 subsampling
128 dimensional feature vector per 16x16 patch
Feature vector computed every 4x4 pixels (128x34x34 feature maps)
Resulting feature maps are spatially smoothed

<table>
<thead>
<tr>
<th>Method</th>
<th>Av. Accuracy/Class (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>local norm</strong><em>{5\times 5} + boxcar</em>{5\times 5} + PCA_{3060} + linear SVM</td>
<td></td>
</tr>
<tr>
<td>IPSD (24x24)</td>
<td>50.9</td>
</tr>
<tr>
<td>SIFT (24x24) (non rot. inv.)</td>
<td>51.2</td>
</tr>
<tr>
<td>SIFT (24x24) (rot. inv.)</td>
<td>45.2</td>
</tr>
<tr>
<td>Serre et al. features [25]</td>
<td>47.1</td>
</tr>
<tr>
<td><strong>local norm</strong>_{9\times 9} + Spatial Pyramid Match Kernel SVM</td>
<td></td>
</tr>
<tr>
<td>SIFT [11]</td>
<td>64.6</td>
</tr>
<tr>
<td>IPSD (34x34)</td>
<td>59.6</td>
</tr>
<tr>
<td>IPSD (56x56)</td>
<td>62.6</td>
</tr>
<tr>
<td>IPSD (120x120)</td>
<td>65.5</td>
</tr>
</tbody>
</table>
A/B Comparison with SIFT (128x5x5 descriptors)
32x16 topographic map with 16x16 filters.

### Performance on Tiny Images Dataset

<table>
<thead>
<tr>
<th>Method</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPSD (5x5)</td>
<td>54</td>
</tr>
<tr>
<td>SIFT (5x5) (non rot. inv.)</td>
<td>53</td>
</tr>
</tbody>
</table>

### Performance on MNIST Dataset

<table>
<thead>
<tr>
<th>Method</th>
<th>Error Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPSD (5x5)</td>
<td>1.0</td>
</tr>
<tr>
<td>SIFT (5x5) (non rot. inv.)</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Learning fields of Convolutional Filters
Problem:

- With patch-level training, the learning algorithm must reconstruct the entire patch with a single feature vector.
- But when the filters are used convolutionally, neighboring feature vectors will be highly redundant.
**Convolutional Sparse Coding**

- Replace the dot products with dictionary element by convolutions.
  - Input Y is a full image
  - Each code component Zk is a feature map (an image)
  - Each dictionary element is a convolution kernel

- **Regular sparse coding**
  \[
  E(Y, Z) = \|Y - \sum_k W_k Z_k\|^2 + \alpha \sum_k |Z_k|
  \]

- **Convolutional S.C.**
  \[
  E(Y, Z) = \|Y - \sum_k W_k \ast Z_k\|^2 + \alpha \sum_k |Z_k|
  \]

"deconvolutional networks" [Zeiler, Taylor, Fergus CVPR 2010]
Problem with patch-based training: high correlation between outputs of filters from overlapping receptive fields.
Filters and Basis Functions obtained with 1, 2, 4, 8, 16, 32, and 64 filters.
Filters and Basis Functions obtained with 16, 32, and 64 filters.

- Smooth shrinkage encoder, coordinate gradient descent inference
Preliminary Results on C-101

C-101, 30 training samples/category

Single Stage: 64 filters, 9x9 → tanh or shrinkage → abs → local contrast normalization → 10x10 pooling → 5x5 subsampling → multinomial logistic regression

- With patch-level unsupervised training (tanh): 52.2%
- With convolutional unsupervised training (tanh): 56.0%
- With patch-level unsupervised training (shrinkage): 53.0%
- With convolutional unsupervised training (shrinkage): 57.0%

Two Stages:

- With patch-level unsupervised training (tanh): 65.5%
- With convolutional unsupervised training (tanh): 69.7%
Learning fields of Simple Cells and Complex Cells

[Gregor and LeCun, arXiv.org 2010]
Training Simple Cells with Local Receptive Fields over Large Input Images

Training on 115x115 images. Kernels are 15x15
Simple Cells + Complex Cells with Sparsity Penalty: Pinwheels

- Training on 115x115 images. Kernels are 15x15
119x119 Image Input
100x100 Code
20x20 Receptive field size
sigma=5

K Obermayer and GG Blasdel, Journal of Neuroscience, Vol 13, 4114-4129 (Monkey)

Michael C. Crair, et. al. The Journal of Neurophysiology Vol. 77 No. 6 June 1997, pp. 3381-3385 (Cat)
Same Method, with Training at the Image Level (vs patch)

Color indicates orientation (by fitting Gabors)
Deep Learning for Mobile Robot Vision
Getting a robot to drive autonomously in unknown terrain solely from vision (camera input).

Our team (NYU/Net-Scale Technologies Inc.) was one of 8 participants funded by DARPA

All teams received identical robots and can only modify the software (not the hardware)

The robot is given the GPS coordinates of a goal, and must drive to the goal as fast as possible. The terrain is unknown in advance. The robot is run 3 times through the same course.

Long-Range Obstacle Detection with on-line, self-trained ConvNet

Uses temporal consistency!
Obstacle Detection

Obstacles overlaid with camera image

Camera image
Detected obstacles (red)
Navigating to a goal is hard... especially in a snowstorm.

stereo perspective

human perspective

especially in a snowstorm.
Self-Supervised Learning

- Stereo vision tells us what nearby obstacles look like
- Use the labels (obstacle/traversable) produced by stereo vision to train a monocular neural network
- Self-supervised “near to far” learning
Pre-processing (125 ms)

- Ground plane estimation
- Horizon leveling
- Conversion to YUV + local contrast normalization
- Scale invariant pyramid of distance-normalized image “bands”
Convolutional Net Architecture

- Operates on 12x25 YUV windows from the pyramid

- 100 features per 3x12x25 input window

- Convolutions with 6x5 kernels
  - 20x6x5 input window

- Pooling/subsampling with 1x4 kernels
  - 20x6x20 input window

- Convolutions with 7x6 kernels
  - 3x12x25 input window

- Logistic regression 100 features -> 5 classes

YUV image band
20-36 pixels tall,
36-500 pixels wide
Convolutional Net Architecture

YUV input

3@36x484

CONVOLUTIONS (7x6)

20@30x484

MAX SUBSAMPLING (1x4)

20@30x125

CONVOLUTIONS (6x5)

100@25x121
Long Range Vision: 5 categories

Online Learning (52 ms)

- Label windows using stereo information – 5 classes

![Diagram showing five categories: super-ground, ground, footline, obstacle, super-obstacle]
Trainable Feature Extraction

- “Deep belief net” approach to unsupervised feature learning
- Two stages are trained in sequence
  - each stage has a layer of convolutional filters and a layer of horizontal feature pooling.
  - Naturally shift invariant in the horizontal direction
- Filters of the convolutional net are trained so that the input can be reconstructed from the features
  - 20 filters at the first stage (layers 1 and 2)
  - 300 filters at the second stage (layers 3 and 4)
- Scale invariance comes from pyramid.
  - for near-to-far generalization
Long Range Vision Results

- Input image
- Stereo Labels
- Classifier Output

- Input image
- Stereo Labels
- Classifier Output
Long Range Vision Results

Input image  | Stereo Labels  | Classifier Output
---|---|---

Input image  | Stereo Labels  | Classifier Output
Long Range Vision Results

Input image
Stereo Labels
Classifier Output

Input image
Stereo Labels
Classifier Output
Feature Learning for traversability prediction (LAGR)

Comparing
- purely supervised
- stacked, invariant auto-encoders
- DrLIM invariant learning

Testing on hand-labeled groundtruth frames – binary labels

Comparison of Feature Extractors on Groundtruth Data

<table>
<thead>
<tr>
<th>Extractor Type</th>
<th>Error Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rbf</td>
<td>25</td>
</tr>
<tr>
<td>supervised</td>
<td>22.5</td>
</tr>
<tr>
<td>autoencoder</td>
<td>20</td>
</tr>
<tr>
<td>autoenc + sup</td>
<td>22.5</td>
</tr>
<tr>
<td>DrLIM</td>
<td>15</td>
</tr>
<tr>
<td>DrLIM + sup</td>
<td>15</td>
</tr>
<tr>
<td>No learning</td>
<td>12.5</td>
</tr>
</tbody>
</table>

- Hand-labeled groundtruth frames
- Error rate (%)
Collaborators

**Current PhD students:**
- Y-Lan Boureau, Koray Kavukcuoglu, Pierre Sermanet

**Former PhD students:**
- Raia Hadsell, Fu-Jie Huang, Marc'Aurelio Ranzato

**Postdocs and Research Scientists**
- Clément Farabet, Karol Gregor, Marco Scoffier

**Senior Collaborators**
- Rob Fergus (NYU): invariant feature learning
- Eugenio Culurciello (Yale): FPGA/ASIC design
- Yoshua Bengio (U. Montreal): deep learning
- Leon Bottou (NEC Labs): handwriting recognition
- Jean Ponce (ENS/INRIA), Francis Bach (ENS/INRIA): sparse coding.
The End