

ASTRÉE: A Static Analyzer for Large Safety-Critical Software

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Automatic Program Verification by Abstract Interpretation

Result:

- ♦ Can produce **zero or very few false alarms** while checking **non-trivial properties** (absence of Run-Time Error);
- ♦ **Does scale up.**

How ?

- ♦ We **specialize** the abstract interpreter for a **family of programs** (which correctness proofs would be similar).
- ♦ The abstract domains are **generic** invariants **automatically** instantiated by the analyzer (to make these proofs).

Considered Programs and Semantics

Which Programs are Considered ?

- ♦ Embedded avionic programs;
- ♦ Automatically generated from a proprietary graphical system control language (à la Simulink);
- ♦ Synchronous real-time critical programs:

```
declare volatile input, state, and output variables;  
initialize state variables;  
loop forever  
  read volatile input variables,  
  compute output and state variables,  
  write to volatile output variables;  
  wait for next clock tick  
end loop
```

Main Characteristics of the Programs

Difficulties:

- ♦ Many global variables and arrays (> 10 000);
- ♦ A huge loop (> 75 000 lines after simplification);
- ♦ Each iteration depends on the state of the previous iterations (state variables);
- ♦ Floating-point computations (80% of the code implements non-linear control with feed-back);
- ♦ Everything is **interdependent** (live variables analysis, slicing ineffective);
- ♦ Abstraction by elimination of any variable is too imprecise.

Simplicities:

- ♦ All data is **statically allocated**;
- ♦ Pointers are restricted to call-by-reference, **no pointer arithmetics**;
- ♦ Structured, **recursion-free** control flow.

Semantics

- ♦ The standard **ISO C99 semantics**:
 - arrays should not be accessed out of their bounds, ...

restricted by:

- ♦ The **machine semantics**:
 - integer arithmetics is 2's complement,
 - floating point arithmetics is IEEE 754-1985,
 - int and float are 32-bit, short is 16-bit, ...

restricted by:

- ♦ The **user's semantics**:
 - integer arithmetics should not wrap-around,
 - some IEEE exceptions (invalid operation, overflow, division by zero) should not occur, ...

Goal of the Program Static Analyzer

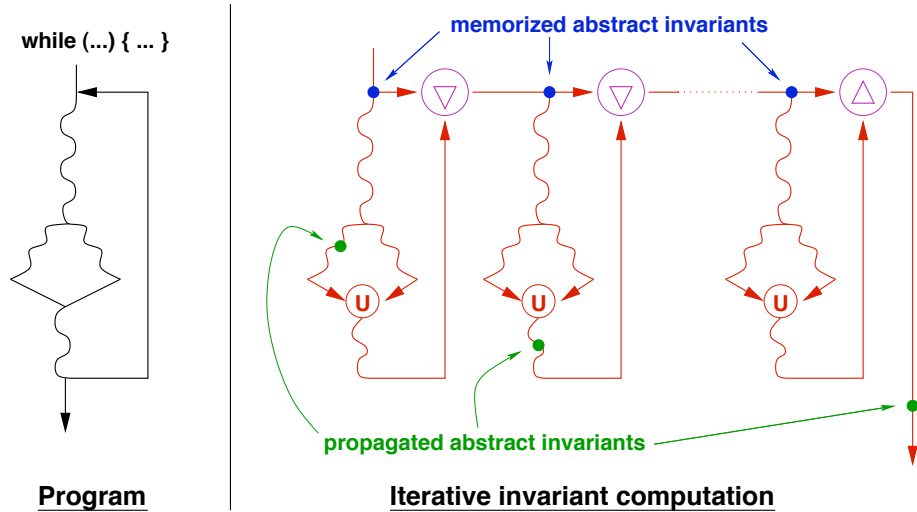
♦ Correctness verification.

- ♦ Nothing can go wrong at execution:
 - no integer overflow or division by zero,
 - no exception, NaN, or $\pm\infty$ generated by IEEE floating-point arithmetics,
 - no out of bounds array access,
 - no erroneous type conversion.
- ♦ The execution semantics on the machine **never reaches an indetermination or an error case** in the standard / machine / user semantics.

Information about the Program Execution Automatically Inferred by the Analyzer

- ♦ The analyzer effectively computes a **finitely represented, compact** over-approximation of the **immense** reachable state space.
- ♦ The information is **valid for any execution** interacting with **any possible environment** (through undetermined volatiles).
- ♦ It is inferred **automatically** by abstract interpretation of the collecting semantics and convergence acceleration (∇ , Δ).

Iterations to Over-Approximate the Reachable States



Abstract Domains

Choice of the Abstract Domains

Abstract Domain:

- ◆ Computer representation of a **class** of **program properties**;
- ◆ **Transformers** for propagation through expressions and commands;
- ◆ Primitives for **convergence acceleration**: ∇ , Δ .

Composition of Abstract Domains:

- ◆ Essentially approximate **reduced product** (conjunction with simplification).

Design of Abstract Domains:

- ◆ Know-how;
- ◆ **Experimentation**.

Interval Abstract Domain

- ◆ Classical domain [Cousot Cousot 76];
- ◆ Minimum information needed to check the correctness conditions;
- ◆ **Not precise enough** to express a useful inductive invariant (thousands of false alarms);
- ◆ \Rightarrow must be refined by:
 - combining with existing domains through reduced product,
 - designing **new domains**, until all false alarms are eliminated.

Clock Abstract Domain

Code Sample:

```
R = 0;
while (1) {
  if (I)
    { R = R+1; }
  else
    { R = 0; }
  T = (R>=n);
  wait_for_clock ();
}
```

- Output T is true iff the volatile input I has been true for the last **n** clock ticks.
- The clock ticks every **s** seconds for at most **h** hours, thus **R is bounded**.
- To prove that **R cannot overflow**, we must prove that **R cannot exceed the elapsed clock ticks** (impossible using only intervals).

Solution:

- ♦ We add a phantom variable **clock** in the concrete user semantics to track elapsed clock ticks.
- ♦ For each variable X, we abstract **three intervals**: **X**, **X+clock**, and **X-clock**.
- ♦ If X+clock or X-clock is bounded, so is X.

Octagon Abstract Domain

Code Sample:

```
while (1) {
  R = A-Z;
  L = A;
  if (R>V)
    { ★ L = Z+V; }
  ★
}
```

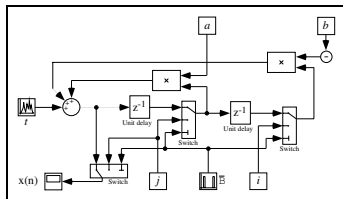
- At ★, the interval domain gives $L \leq \max(\max A, (\max Z) + (\max V))$.
- In fact, we have $L \leq A$.
- To discover this, we must know at ★ that $R = A - Z$ and $R > V$.

Solution: we need a numerical **relational** abstract domain.

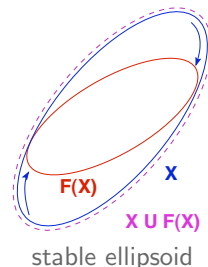
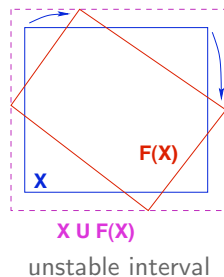
- ♦ The **octagon** abstract domain [Miné 03] is a good cost / precision trade-off.
- ♦ Invariants of the form $\pm x \pm y \leq c$, with $\mathcal{O}(N^2)$ memory and $\mathcal{O}(N^3)$ time cost.
- ♦ Here, $R = A - Z$ cannot be discovered, but we get $L - Z \leq \max R$ which is sufficient.
- ♦ We use many octagons on **small packs** of variables instead of a large one using all variables to cut costs.

Ellipsoid Abstract Domain

2^d Order Filter Sample:



- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is **bounded**, which must be proved in the abstract.
- There is **no stable interval or octagon**.
- The simplest stable surface is an **ellipsoid**.



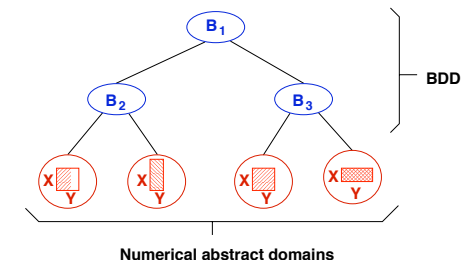
Decision Tree Abstract Domain

Synchronous reactive programs **encode control flow in boolean variables**.

Code Sample:

```
bool B1, B2, B3;
float N, X, Y;
N = f(B1);
if (B1)
  { X = g(N); }
else
  { Y = h(N); }
```

Decision Tree:



There are too many booleans (**4 000**) to build one big tree so we:

- ♦ limit the **BDD height** to 3 (analysis parameter);
- ♦ use a **syntactic criterion** to select variables in the BDD and the numerical parts.

Relational Domains on Floating-Point

Problems:

- ◆ Relational numerical abstract domains rely on a **perfect mathematical concrete semantics** (in \mathbb{R} or \mathbb{Q}).
- ◆ Perfect arithmetics in \mathbb{R} or \mathbb{Q} is costly.
- ◆ IEEE 754-1985 floating-point concrete semantics **incurs rounding**.

Solution:

- ◆ Build an **abstract mathematical semantics in \mathbb{R}** that over-approximates the concrete floating-point semantics, including rounding.
- ◆ Implement the abstract domains on \mathbb{R} **using floating-point numbers** rounded in a sound way.

Iteration Strategies for Fixpoint Approximation

Iteration Refinement: Loop Unrolling

Principle:

- ◆ Semantically equivalent to:
$$\text{while } (B) \{ C \} \implies \text{if } (B) \{ C \}; \text{ while } (B) \{ C \}$$
- ◆ More precise in the abstract:
 - **less** concrete execution paths are **merged** in the abstract.

Application:

- ◆ Isolate the **initialization phase** in a loop (e.g. first iteration).

Iteration Refinement: Trace Partitioning

Principle:

- ◆ Semantically equivalent to:
$$\begin{array}{l} \text{if } (B) \{ C1 \} \text{ else } \{ C2 \}; C3 \\ \quad \quad \quad \downarrow \\ \text{if } (B) \{ C1; C3 \} \text{ else } \{ C2; C3 \}; \end{array}$$
- ◆ More precise in the abstract:
 - concrete execution paths are **merged later**.

Application:

```
if (B)
{ X=0; Y=1; }
else
{ X=1; Y=0; }
R = 1 / (X-Y);
```

/ cannot result in a division by zero

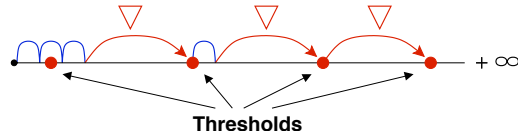
Convergence Accelerator: Widening

Principle:

- ♦ Brute-force widening:



- ♦ Widening **with thresholds**:



Examples:

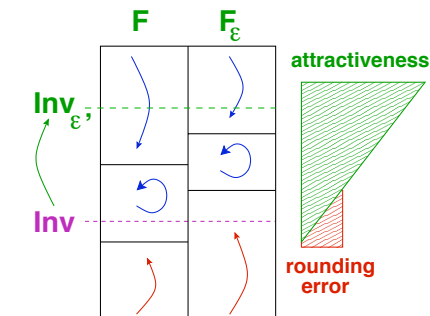
- ♦ 1., 10., 100., 1000., etc. for floating-point variables;
- ♦ maximal values of data types;
- ♦ syntactic program constants, etc.

Fixpoint Stabilization for Floating-point

Problem:

- ♦ Mathematically, we look for an abstract invariant **inv** such that $F(\text{inv}) \subseteq \text{inv}$.
- ♦ Unfortunately, abstract computation uses floating-point and incurs rounding: maybe $F_\epsilon(\text{inv}) \not\subseteq \text{inv}$!

Solution:



- Widen **inv** to **inv_{ε'}** with the hope to jump into a **stable zone** of **F_ε**.
- Works if F has some **attractiveness** property that fights against rounding errors (otherwise iteration goes on).
- ϵ' is an analysis parameter.

Results

Example of Analysis Session

```
Visualizator
quit clocks trees octagons filters help
Search string: TRUC[0] / Next Previous First Last Goto line: /
demo.c
else
{
    @P = coef1 * X + TRUC[0].e * coef2 + TRUC[1].e * coef3 +
    TRUC[0].s * coef4 + TRUC[1].s * coef5;@
}
*TRUC[1].e = TRUC[0].e;
@TRUC[0].e = X;
@TRUC[1].s = TRUC[0].s;
@TRUC[0].s = P;@
}
void coffee_machine_explosion()
info
/* Analyzer launched at 2003/ 6/ 2 11:45:43
-103.23142654533073426 <= X-P <= 166.32563104533073783
-67.325631045330709412 <= X+P <= 202.23142654533074847 }
>
<clock in {0}, <MACHIN in {0}>;
clock in {0};
<TRUC[0].e in {15.5}, TRUC[0].s in [-20.7485, 20.7485],
TRUC[1].e in {15.5},
TRUC[1].s in [-20.7485, 20.7485], X in {15.5}, P in [-2
coffee_machine_explosion 55:3 (ellipse)
L43 C21 c894
```

Results

♦ Efficient:

- tested on a **75 000** lines program (after reduction from 132 000 lines),
- Less than 100 iterations in the main loop i.e. 2h30 computation time on a 2.8GHz PC,
- **264 Mb** memory usage.

♦ Precise:

- **No** false alarm.

♦ Exhaustive:

- full control and data **coverage** (unlike checking, testing, simulation).

Conclusion

♦ Success story:

- we succeed where **a commercial abstract interpretation-based static analysis tool failed**
(because of prohibitive time and memory consumption and very large number of false alarms);

♦ Usable in practice for verification:

- **directly applicable** to other similar programs
by changing some analyzer parameters,
- approach **generalizable** to other program families
by including new abstract domains and specializing the iteration strategy.
(Work in progress: power-on self-test for a family of embedded systems.)

Reference

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