Static Analysis of Embedded Control/Command Software by Abstract Interpretation

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Kestrel Technology, Palo Alto, CA, Nov. 7th, 2005

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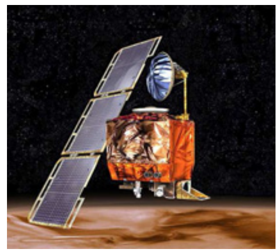
Motivation

All Computer Scientists Have Experienced Bugs

Ariane 5.01 failure (overflow)

Patriot failure (float rounding)



Mars orbiter loss (unit error)

It is preferable to verify that mission/safety-critical programs do not go wrong before running them.

Static Analysis by Abstract Interpretation

Static analysis: analyze the program at compile-time to verify a program runtime property

Undecidability $\longrightarrow$

Abstract interpretation: effectively compute an abstraction/ sound approximation of the program semantics,

- which is precise enough to imply the desired property, and
- coarse enough to be efficiently computable.

Syntax of programs

X	variables $X \in \mathbb{X}$
T	types $T \in \mathbb{T}$
E	arithmetic expressions $E \in \mathbb{E}$
B	boolean expressions $B \in \mathbb{B}$
$D ::= T\ X$;	
$\mid\ T\ X$; D'	
$C ::= X$ = E;	commands $C \in \mathbb{C}$
$\mid$ `while` $B\ C'$	
$\mid$ `if` $B\ C'$ `else` C''	
$\mid$ { $C_1 \ldots C_n$ }, $(n \geq 0)$	
$P ::= D\ C$	program $P \in \mathbb{P}$

Abstract Interpretation, Reminder using a simple example

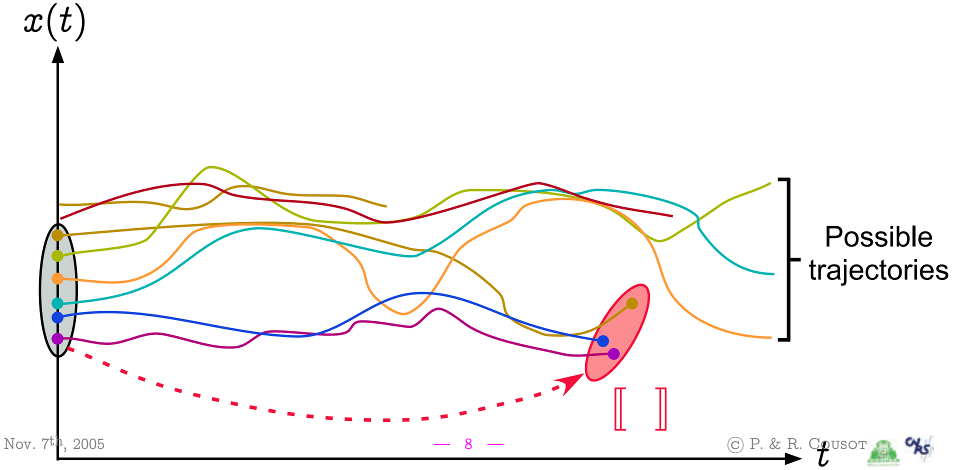
Reference

[POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In 4^{th} *ACM POPL*.

[Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.

[POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In 6^{th} *ACM POPL*.

Postcondition semantics



States

Values of given type:

$$\mathcal{V}[\![T]\!] \; : \; \text{values of type } T \in \mathbb{T}$$

$$\mathcal{V}[\![\texttt{int}]\!] \stackrel{\text{def}}{=} \{z \in \mathbb{Z} \mid \texttt{min\_int} \leq z \leq \texttt{max\_int}\}$$

Program states $\Sigma[\![P]\!]$ [1]:

$$\Sigma[\![D\ C]\!] \stackrel{\text{def}}{=} \Sigma[\![D]\!]$$

$$\Sigma[\![T\ X\,;]\!] \stackrel{\text{def}}{=} \{X\} \mapsto \mathcal{V}[\![T]\!]$$

$$\Sigma[\![T\ X\,;\ D]\!] \stackrel{\text{def}}{=} (\{X\} \mapsto \mathcal{V}[\![T]\!]) \cup \Sigma[\![D]\!]$$

Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[\![P]\!] \stackrel{\text{def}}{=} \wp(\Sigma[\![P]\!]) \qquad \text{sets of states}$$

i.e. program properties where $\subseteq$ is implication, $\emptyset$ is false, $\cup$ is disjunction.

1 States $\rho \in \Sigma[\![P]\!]$ of a program P map program variables X to their values $\rho(X)$

Concrete Reachability Semantics of Programs

$$\mathcal{S}[\![X = E;]\!]R \stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in R \cap \mathsf{dom}(E)\}$$

$$\rho[X \leftarrow v](X) \stackrel{\text{def}}{=} v, \qquad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y)$$

$$\mathcal{S}[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{B}[\![\neg B]\!]R$$

$$\mathcal{B}[\![B]\!]R \stackrel{\text{def}}{=} \{\rho \in R \cap \mathsf{dom}(B) \mid B \text{ holds in } \rho\}$$

$$\mathcal{S}[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{S}[\![C'']\!](\mathcal{B}[\![\neg B]\!]R)$$

$$\mathcal{S}[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \mathsf{lfp}^{\subseteq}_{\emptyset}\, \lambda \mathcal{X} \cdot R \cup \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C_n]\!] \circ \ldots \circ \mathcal{S}[\![C_1]\!]R \quad n > 0$$

$$\mathcal{S}[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C]\!](\Sigma[\![D]\!]) \quad \text{(uninitialized variables)}$$

Not computable (undecidability).

Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \bot,\ \sqcup \rangle$$

such that:

$$\langle \mathcal{D}[\![P]\!],\ \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq \rangle$$

i.e.

$$\forall X \in \mathcal{D}[\![P]\!], Y \in \mathcal{D}^\sharp[\![P]\!] : \alpha(X) \sqsubseteq Y \iff X \subseteq \gamma(Y)$$

hence $\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \bot,\ \sqcup \rangle$ is a complete lattice such that $\bot = \alpha(\emptyset)$ and $\sqcup X = \alpha(\cup\, \gamma(X))$

Example 1 of Abstraction

Set of traces: set of finite or infinite maximal sequences of states for the operational transition semantics

$\stackrel{\alpha}{\rightarrow}$ Strongest liberal postcondition: final states s reachable from a given precondition P

$$\alpha(X) = \lambda P \cdot \{s \mid \exists \sigma_0 \sigma_1 \ldots \sigma_n \in X : \sigma_0 \in P \wedge s = \sigma_n\}$$

We have (Σ: set of states, $\dot{\subseteq}$ pointwise):

$$\langle \wp(\Sigma^\infty), \subseteq \rangle \xrightleftharpoons[\alpha]{\gamma} \langle \wp(\Sigma) \stackrel{\cup}{\longmapsto} \wp(\Sigma), \dot{\subseteq} \rangle$$

Example 2 of Abstraction

Set of traces: set of finite or infinite maximal sequences of states for the operational transition semantics

$\stackrel{\alpha_0}{\rightarrow}$ Trace of sets of states: sequence of set of states appearing at a given time along at least one of these traces

$$\alpha_0(X) = \lambda i \cdot \{\sigma_i \mid \sigma \in X \wedge 0 \leq i < |\sigma|\}$$

$\stackrel{\alpha_1}{\rightarrow}$ Set of reachable states: set of states appearing at least once along one of these traces (global invariant)

$$\alpha_1(\Sigma) = \bigcup \{\Sigma_i \mid 0 \leq i < |\Sigma|\}$$

$\stackrel{\alpha_2}{\rightarrow}$ Partitionned set of reachable states: project along each control point (local invariant)

$$\alpha_2(\{\langle c_i, \rho_i \rangle \mid i \in \Delta\}) = \lambda c \cdot \{\rho_i \mid i \in \Delta \wedge c = c_i\}$$

$\stackrel{\alpha_3}{\rightarrow}$ Partitionned cartesian set of reachable states: project along each program variable (relationships between variables are now lost)

$$\alpha_3(\lambda c \cdot \{\rho_i \mid i \in \Delta_c\}) = \lambda c \cdot \lambda \mathtt{X} \cdot \{\rho_i(\mathtt{X}) \mid i \in \Delta_c\}$$

$\stackrel{\alpha_4}{\rightarrow}$ Partitionned cartesian interval of reachable states: take min and max of the values of the variables [2]

$$\alpha_4(\lambda c \cdot \lambda \mathtt{X} \cdot \{v_i \mid i \in \Delta_{c,\mathtt{X}}\} = \lambda c \cdot \lambda \mathtt{X} \cdot \langle \min\{v_i \mid i \in \Delta_{c,\mathtt{X}}\}, \max\{v_i \mid i \in \Delta_{c,\mathtt{X}}\} \rangle$$

α_0, α_1, α_2, α_3 and α_4, whence $\alpha_4 \circ \alpha_3 \circ \alpha_2 \circ \alpha_1 \circ \alpha_0$ are lower-adjoints of Galois connections

Example 3: Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D}, \subseteq \rangle \xrightleftharpoons[\alpha_1]{\gamma_1} \langle \mathcal{D}_1^\sharp, \sqsubseteq_1 \rangle \text{ and } \langle \mathcal{D}, \subseteq \rangle \xrightleftharpoons[\alpha_2]{\gamma_2} \langle \mathcal{D}_2^\sharp, \sqsubseteq_2 \rangle$$

the reduced product is

$$\alpha(X) \stackrel{\text{def}}{=} \sqcap \{\langle x, y \rangle \mid X \subseteq \gamma_1(x) \wedge X \subseteq \gamma_2(y)\}$$

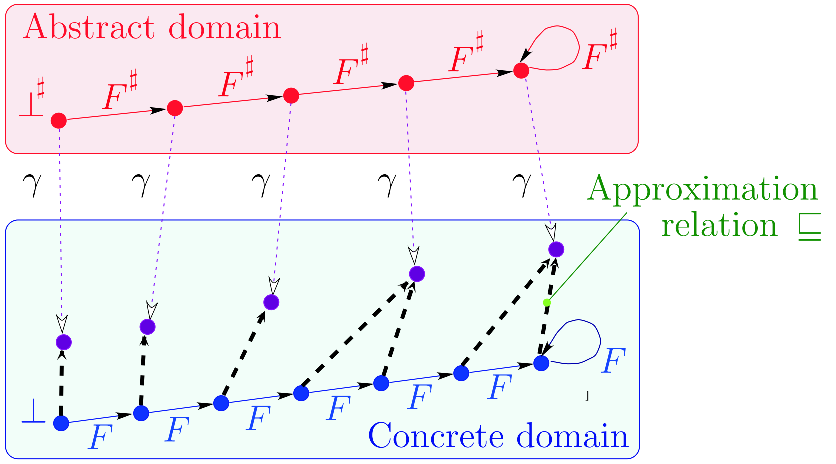
such that $\sqsubseteq \stackrel{\text{def}}{=} \sqsubseteq_1 \times \sqsubseteq_2$ and

$$\langle \mathcal{D}, \subseteq \rangle \xrightleftharpoons[\alpha]{\gamma_1 \times \gamma_2} \langle \alpha(\mathcal{D}), \sqsubseteq \rangle$$

Example: $\mathtt{x} \in [1, 9] \wedge \mathtt{x} \bmod 2 = 0$ reduces to $\mathtt{x} \in [2, 8] \wedge \mathtt{x} \bmod 2 = 0$

[2] assuming these values to be totally ordered.

Approximate Fixpoint Abstraction



$$F \circ \gamma \sqsubseteq \gamma \circ F^\sharp \;\Rightarrow\; \mathsf{lfp}\, F \sqsubseteq \gamma(\mathsf{lfp}\, F^\sharp)$$

Abstract Reachability Semantics of Programs

$$\mathcal{S}^\sharp[\![X = E;]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$

$$\mathcal{B}^\sharp[\![B]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$

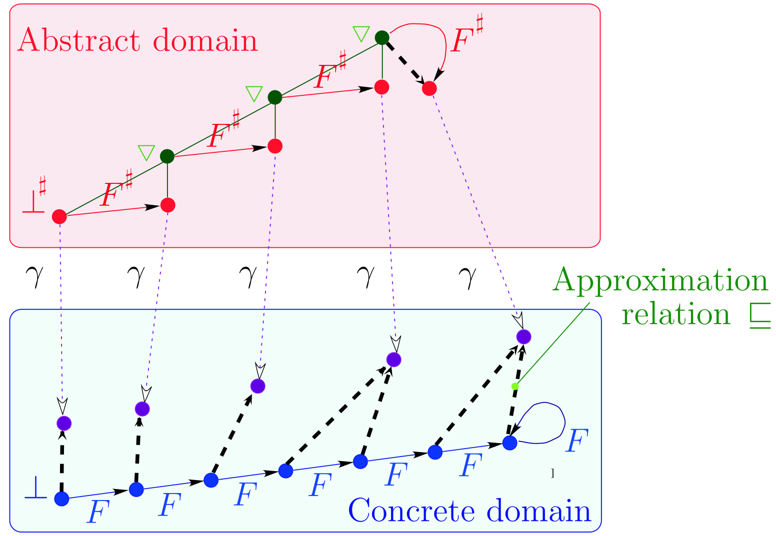
$$\mathcal{S}^\sharp[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \mathsf{lfp}^{\sqsubseteq}_{\perp}\, \lambda \mathcal{X} \cdot R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X}) \quad \text{in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}^\sharp[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!]R \quad n > 0$$

$$\mathcal{S}^\sharp[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

Convergence Acceleration with Widening



Abstract Semantics with Convergence Acceleration [3]

$$\mathcal{S}^\sharp[\![X = E;]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$

$$\mathcal{B}^\sharp[\![B]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$

$$\mathcal{S}^\sharp[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X} \cdot \text{let } \mathcal{Y} = R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X})$$
$$\text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \mathbin{\nabla} \mathcal{Y}$$
$$\text{and } \mathcal{W} = \mathsf{lfp}^{\sqsubseteq}_{\perp}\, \mathcal{F}^\sharp \qquad \text{in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}^\sharp[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!]R \quad n > 0$$

$$\mathcal{S}^\sharp[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

[3] Note: $\mathcal{F}^\sharp$ <u>not</u> monotonic!

Applications of Abstract Interpretation

A few applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79] including a.o. **Dataflow Analysis** [POPL '79], [POPL '00], **Set-based Analysis** [FPCA '95], **Predicate Abstraction** [Manna's festschrift '03], ...
- **Syntax Analysis** [TCS 290(1) 2002]
- **Hierarchies of Semantics (including Proofs)** [POPL '92], [TCS 277(1–2) 2002]
- **Typing & Type Inference** [POPL '97]

A few applications of Abstract Interpretation (Cont'd)

- **(Abstract) Model Checking** [POPL '00]
- **Program Transformation** [POPL '02]
- **Software Watermarking** [POPL '04]
- **Bisimulations** [RT-ESOP '04]
- ...

All these techniques involve sound approximations that can be formalized by abstract interpretation

A Practical Application of Abstract Interpretation to the ASTRÉE Static Analyzer

Reference

[1] http://www.astree.ens.fr/ P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, D. Monniaux, X. Rival

Programs analysed by ASTRÉE

– Application Domain: large safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.

– C programs:

– with
  - basic numeric datatypes, structures and arrays
  - pointers (including on functions),
  - floating point computations
  - tests, loops and function calls
  - limited branching (forward `goto`, `break`, `continue`)

– without
  - `union` (new memory model in progress [4])
  - dynamic memory allocation
  - recursive function calls
  - backward branching
  - conflicting side effects
  - C libraries, system calls (parallelism)

[4] Thanks A. Miné

Concrete Operational Semantics

– International norm of C (ISO/IEC 9899:1999)

– *restricted by* implementation-specific behaviors depending upon the machine and compiler (e.g. encoding of integers, IEEE 754-1985 norm for floats and doubles)

– *restricted by* user-defined programming guidelines (such as no modular arithmetic for signed integers, even though this might be the hardware choice)

– *restricted by* program specific user requirements (e.g. volatile environment specified by a *trusted configuration file*, `assert`, execution stops on first runtime error [5],)

Implicit Specification: Absence of Runtime Errors

– No violation of the norm of C (e.g. array index out of bounds, division by zero)

– No implementation-specific undefined behaviors (e.g. maximum short integer is 32767, no float NaN)

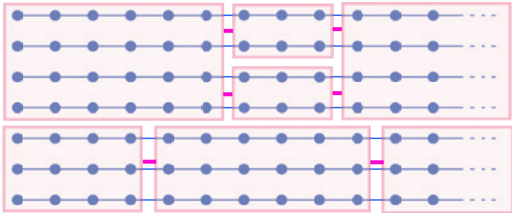
– No violation of the programming guidelines (e.g. static variables cannot be assumed to be initialized to 0)

– No violation of the programmer assertions (must all be statically verified).

[5] semantics of C unclear after an error, equivalent if no alarm

Abstraction

– Set of traces of relational state abstractions of subtraces for the concrete trace operational semantics



Requirements on the Abstract Semantics

– Soundness: absolutely essential for verification

– Precision: few or no false alarm[6] (full certification)

– Efficiency: rapid analyses and fixes during development

[6] Potential runtime error signaled by the analyzer due to overapproximation but impossible in any actual program run compatible with the configuration file.

Example of Industrial applications

– Primary flight control software of the Airbus A340 family/A380 fly-by-wire system

– C program, automatically generated from a proprietary high-level specification (à la Simulink/SCADE)

– A340 family: 132,000 lines, 75,000 LOCs after preprocessing, 10,000 global variables, over 21,000 after expansion of small arrays

– A380: $\times$ 3/7 (up to 1.000.000 LOCs)

The Class of Considered Periodic Synchronous Programs

```
declare volatile input, state and output variables;
initialize state and output variables;
loop forever
    - read volatile input variables,
    - compute output and state variables,
    - write to output variables;
  __ASTREE_wait_for_clock ();
end loop
```

Task scheduling is static:

– Requirements: the only interrupts are clock ticks;

– Execution time of loop body less than a clock tick [EMSOFT '01].

Challenging aspects

- Size: > 100 kLOC, $> 10\,000$ variables
- Floating point computations
  including interconnected networks of filters, non linear control with feedback, interpolations...
- Interdependencies among variables:
  - Stability of computations should be established
  - Complex relations should be inferred among numerical and boolean data
  - Very long data paths from input to outputs

Characteristics of the ASTRÉE Analyzer

Static: compile time analysis ($\neq$ run time analysis Rational Purify, Parasoft Insure++)

Program Analyzer: analyzes programs not micromodels of programs ($\neq$ PROMELA in SPIN or Alloy in the Alloy Analyzer)

Automatic: no end-user intervention needed ($\neq$ ESC Java, ESC Java 2)

Sound: covers the whole state space ($\neq$ MAGIC, CBMC) so never omit potential errors ($\neq$ UNO, CMC from coverity.com) or sort most probable ones ($\neq$ Splint)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Multiabstraction: uses many numerical/symbolic abstract domains ($\neq$ symbolic constraints in Bane or the canonical abstraction of TVLA)

Infinitary: all abstractions use infinite abstract domains with widening/narrowing ($\neq$ model checking based analyzers such as VeriSoft, Bandera, Java PathFinder)

Efficient: always terminate ($\neq$ counterexample-driven automatic abstraction refinement BLAST, SLAM)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Specializable: can easily incorporate new abstractions (and reduction with already existing abstract domains) ($\neq$ general-purpose analyzers PolySpace Verifier)

Domain-Aware: knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in C Global Surveyor)

Parametric: the precision/cost can be tailored to user needs by options and directives in the code

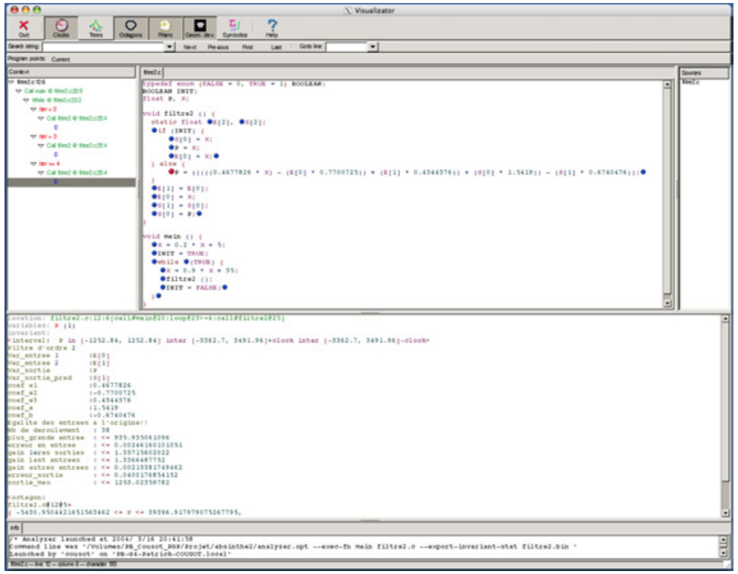
Characteristics of the ASTRÉE Analyzer (Cont'd)

Automatic Parametrization: the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

Modular: an analyzer instance is built by selection of O-CAML modules from a collection, each module implementing an abstract domain

Precise: very few or no false alarm when adapted to an application domain ⟶ it is a VERIFIER!

Example of Analysis Session



Benchmarks (Airbus A340 Primary Flight Control Software)

– 132,000 lines, 75,000 LOCs after preprocessing

– Comparative results (commercial software):

4,200 (false?) alarms,

3.5 days;

– Our results:

0 alarms,

40mn on 2.8 GHz PC,

300 Megabytes

⟶ A world première!

(Airbus A380 Primary Flight Control Software)

– 350,000 lines

– 0 alarms (Nov. 2004),

7h [7] on 2.8 GHz PC,

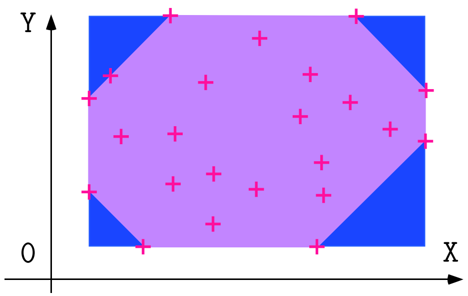
1 Gigabyte

⟶ A world grand première!

[7] We are still in a phase where we favour precision rather than computation costs, and this should go down. For example, the A340 analysis went up to 5 h, before being reduced by requiring less precision while still getting no false alarm.

Examples of Abstractions

General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \le x \le 9 \\ 1 \le y \le 20 \end{cases}$$

Octagons [10]:

$$\begin{cases} 1 \le x \le 9 \\ x + y \le 77 \\ 1 \le y \le 20 \\ x - y \le 04 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program and analyzer) [POPL '77, 10, 11]

Floating-Point Computations

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951488.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

$$(x + a) - (x - a) \neq 2a$$

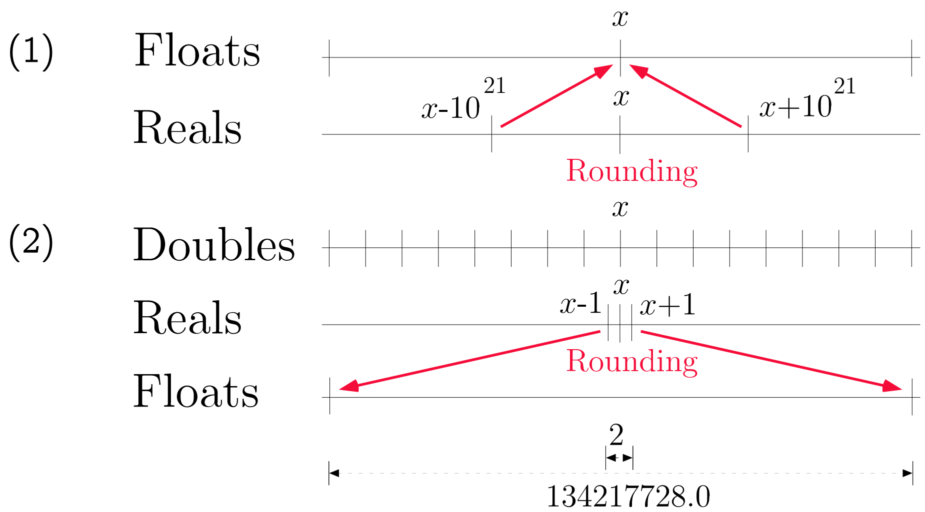
Floating-Point Computations

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951487.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
0.000000
```

$$(x + a) - (x - a) \neq 2a$$

Explanation of the huge rounding error



Floating-point linearization [11, 12]

- Approximate arbitrary expressions in the form
  $[a_0, b_0] + \sum_k([a_k, b_k] \times V_k)$
- Example:
  `Z = X - (0.25 * X)` is linearized as
  $\mathtt{Z} = ([0.749\cdots, 0.750\cdots]\times \mathtt{X}) + (2.35\cdots 10^{-38} \times [-1, 1])$
- Allows simplification even in the interval domain
  if X ∈ [-1,1], we get $|\mathtt{Z}| \leq 0.750\cdots$ instead of $|\mathtt{Z}| \leq 1.25\cdots$
- Allows using a relational abstract domain (octagons)
- Example of good compromize between cost and precision

Symbolic abstract domain [11, 12]

- Interval analysis: if $x \in [a, b]$ and $y \in [c, d]$ then $x - y \in [a - d, b - c]$ so if $x \in [0, 100]$ then $x - x \in [-100, 100]$!!!
- The symbolic abstract domain propagates the symbolic values of variables and performs simplifications;
- Must maintain the maximal possible rounding error for float computations (overestimated with intervals);

```
% cat -n x-x.c
     1  void main () { int X, Y;
     2            __ASTREE_known_fact(((0 <= X) && (X <= 100)));
     3            Y = (X - X);
     4            __ASTREE_log_vars((Y));
     5  }
```

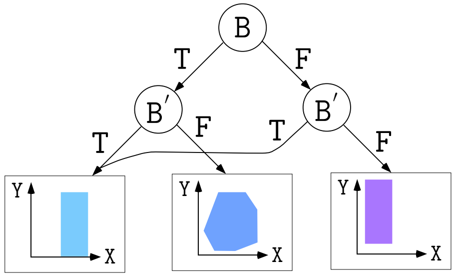
```
astree -exec-fn main -no-relational x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in [-100, 100]>
```

```
astree -exec-fn main x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in {0}> <symbolic: Y = (X -i X)>
```

Boolean Relations for Boolean Control

– Code Sample:

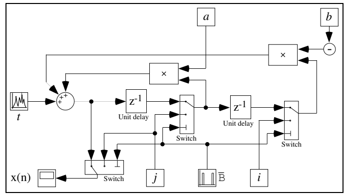
```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

Ellipsoid Abstract Domain for Filters

2^d Order Digital Filter:

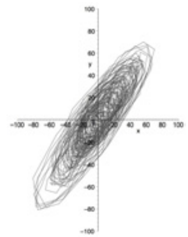


– Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$

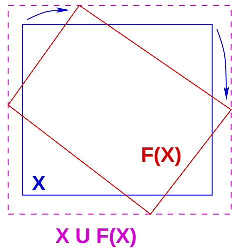
– The concrete computation is bounded, which must be proved in the abstract.

– There is no stable interval or octagon.

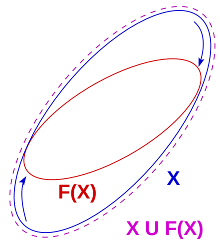
– The simplest stable surface is an ellipsoid.



execution trace



unstable interval



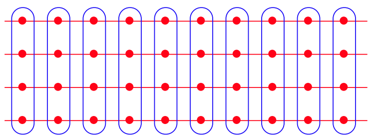
stable ellipsoid

Control Partitionning for Case Analysis

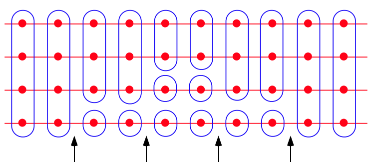
–Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

  ... found invariant −100 ≤ x ≤ 100 ...

  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}
```

Control point partitionning:



Trace partitionning:



Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

Filter Example [7]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
               + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}
```

Arithmetic-geometric progressions [8] [8]

– Abstract domain: $(\mathbb{R}^+)^5$

– Concretization:

$$\gamma \in (\mathbb{R}^+)^5 \longmapsto \wp(\mathbb{N} \mapsto \mathbb{R})$$

$$\gamma(M, a, b, a', b') = \{f \mid \forall k \in \mathbb{N} : |f(k)| \leq \left(\lambda x \cdot ax + b \circ (\lambda x \cdot a'x + b')^k\right)(M)\}$$

i.e. any function bounded by the arithmetic-geometric progression.

Arithmetic-Geometric Progressions (Example 1)

```
% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
  R = 0;
  while (TRUE) {
    __ASTREE_log_vars((R));
    if (I) { R = R + 1; }          ← potential overflow!
    else { R = 0; }
    T = (R >= 100);
    __ASTREE_wait_for_clock(());
  }}
% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.
```

[8] here in $\mathbb{R}$

Arithmetic-geometric progressions (Example 2)

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P =  (P - ((((2.0 * P) - A) - B)
            * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock(());
  }}
% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
|P| <= (15.  + 5.87747175411e-39
/ 1.19209290217e-07) * (1
+ 1.19209290217e-07)^clock
- 5.87747175411e-39 /
1.19209290217e-07 <=
23.0393526881
```


(Automatic) Parameterization

– All abstract domains of ASTRÉE are parameterized, e.g.
  - variable packing for octagones and decision trees,
  - partition/merge program points,
  - loop unrollings,
  - thresholds in widenings, . . . ;

– End-users can either parameterize by hand (analyzer options, directives in the code), or

– choose the automatic parameterization (default options, directives for pattern-matched predefined program schemata).

The main loop invariant for the A340

A textual file over 4.5 Mb with

- 6,900 boolean interval assertions ($x \in [0;1]$)
- 9,600 interval assertions ($x \in [a;b]$)
- 25,400 clock assertions ($x + \mathrm{clk} \in [a;b] \wedge x - \mathrm{clk} \in [a;b]$)
- 19,100 additive octagonal assertions ($a \leq x + y \leq b$)
- 19,200 subtractive octagonal assertions ($a \leq x - y \leq b$)
- 100 decision trees
- 60 ellipse invariants, etc ...

involving over 16,000 floating point constants (only 550 appearing in the program text) $\times$ 75,000 LOCs.

Possible origins of imprecision and how to fix it

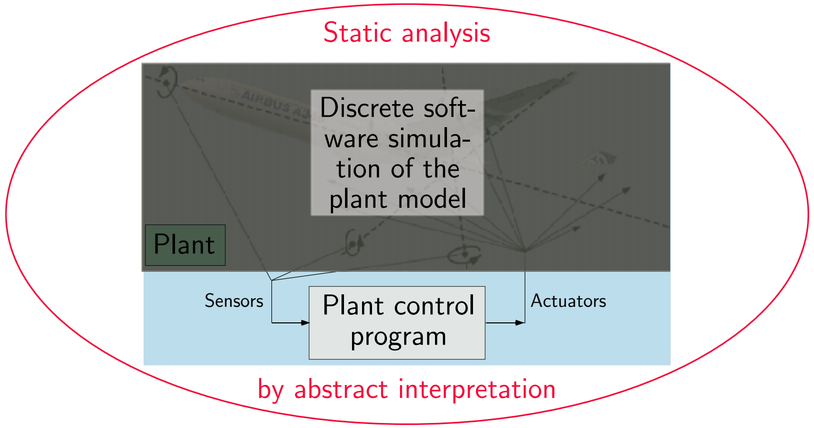
In case of false alarm, the imprecision can come from:

- Abstract transformers (not best possible) $\longrightarrow$ improve algorithm;
- Automatized parametrization (e.g. variable packing) $\longrightarrow$ improve pattern-matched program schemata;
- Iteration strategy for fixpoints $\longrightarrow$ fix widening [9];
- Inexpressivity i.e. indispensable local inductive invariant are inexpressible in the abstract $\longrightarrow$ add a new abstract domain to the reduced product (e.g. filters).

[9] This can be very hard since at the limit only a precise infinite iteration might be able to compute the proper abstract invariant. In that case, it might be better to design a more refined abstract domain.

Static analysis of systems

System analysis & verification, Avenue 1

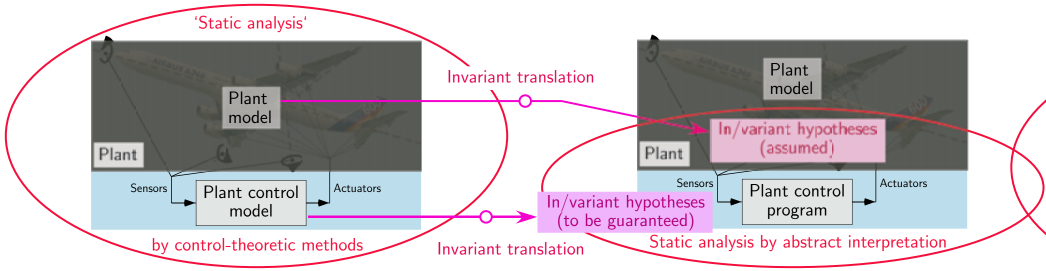


Abstractions: program $\rightarrow$ precise, system $\rightarrow$ precise

– Exhaustive (contrary to current simulations)
– The plant model discretization errors are similar to those of simulation methods (but for the use of the *actual* control program instead of a model!)
– In general, polyhedral abstractions are unstable or of very high complexity
– New abstractions have to be studied (e.g. ellipsoidal abstractions)!

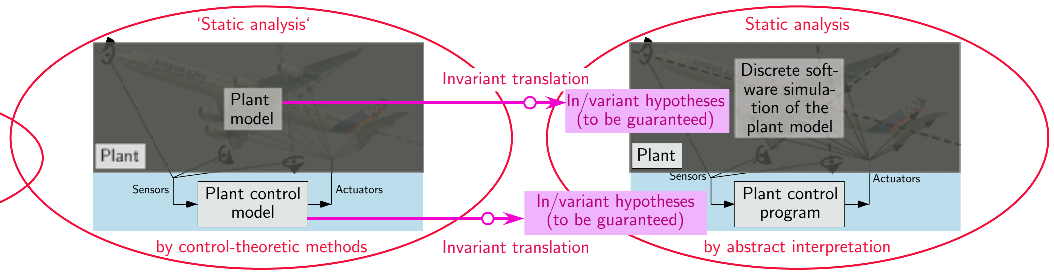
– The control-theoretic ‘static analysis‘ is easier on the plant/controller model using continuous optimization methods
– The in/variant hypotheses on the controlled plant are assumed to be true in the analysis of the plant control program
– It is now sufficient to perform the analysis analysis control program under these in/variant hypotheses
– The results can then be checked on the whole system (plant simulation + control program)

System analysis & verification, Avenue 2



Abstractions: program → precise, system → precise

System analysis & verification, Avenue 3



Abstractions: program → precise, system → precise

– The translated in/variants can be checked for the plant simulator/control program (easier than in/variant discovery)

– Should scale up (since these complex in/variants are relevant to a small part of the control program only [10])

Conclusion

[10] e.g. the plant model assumes perfect sensors/actuators/computers whereas the control program must be made dependable by using redundant failing sensors/actuators/computers

Conclusions

1. On soundness and completeness:
   – Software checking (e.g. [abstract] testing): unsound
   – Software static analysis (for a language): sound but unprecise
   – Software verification (for a well-defined family of programs): theoretically possible [SARA '00], practically feasible [PLDI '03]

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Conclusions (cont'd)

2. On specifications for static verification:
   – Implicit: e.g. from a language semantics (e.g. RTE) → extremely easy for engineers
   – Explicit:
     - By a logic → very hard for engineers
     - By a model → easy for engineers / hard for static analysis
     - By a program automatically generated from a model → easy for engineers / easy for static analysis

THE END, THANK YOU

More references at URL www.di.ens.fr/~cousot
www.astree.ens.fr.

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