

Challenges in Abstract Interpretation for Software Safety

Patrick Cousot

École normale supérieure, Paris, France

cousot@ens.fr www.di.ens.fr/~cousot

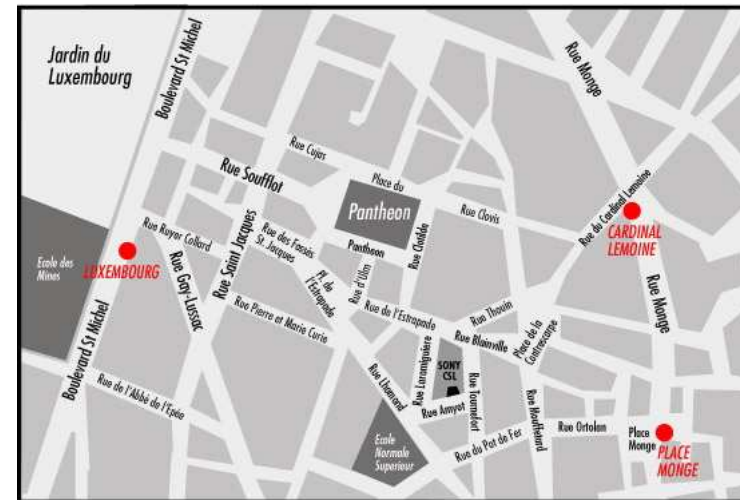
Franco-Japanese Workshop on Security

Keio University, Mita Campus, September 5–7, 2005

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Normale Sup. (ENS)



A few former students: Évariste Galois, Louis Pasteur, ...; Nobel prizes: Claude Cohen-Tannoudji, Pierre-Gilles de Gennes, Gabriel Lippmann, Louis Néel, Jean-Baptiste Perrin, Paul Sabatier, ...; Fields Medal holders: Laurent Schwartz, Jean-

Pierre Serre (1st Abel Prize), René Thom, Alain Connes, Pierre-Louis Lions, Jean-Christophe Yoccoz, Laurent Lafforgue; Fictitious mathematicians: Nicolas Bourbaki; Philosophers: Henri Bergson (Nobel Prize), Louis Althusser, Simone de Beauvoir, Emile Auguste Chartier “Alain”, Raymond Aron, Jean-Paul Sartre, Maurice Merleau-Ponty, Michel Foucault, Jacques Derrida, Bernard-Henri Lévy. . . ; Politicians: Jean Jaurès, Léon Blum, Édouard Herriot, Georges Pompidou, Alain Juppé, Laurent Fabius, Léopold Sédar Senghor, . . . ; Sociologists: Émile Durkheim, Pierre Bourdieu, . . . ; Writers: Romain Rolland (Nobel Prize), Jean Giraudoux, Charles Péguy, Julien Gracq, . . . ;

The software safety challenge for next 10 years

State of Practice in Software Engineering

- Present-day software engineering is **almost exclusively manual**, with very few automated tools;
- Trust and confidence in specifications and software can no longer be entirely based on the **development process** (e.g. DO178B in aerospace software);
- In complement, quality assurance must be ensured by **new design, modeling, checking, verification and certification tools based on the product itself.**

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An example among many others (Matlab code)

```
> h=get(gca,'children');  
  
apple.awt.EventQueueExceptionHandler Caught Throwable : java.lang.ArrayIndexOutOfBoundsException: 2 >= 2  
java.lang.ArrayIndexOutOfBoundsException: 2 >= 2  
at java.util.Vector.elementAt(Vector.java:431)  
at com.mathworks.mde.help.IndexItem.getFilename(IndexItem.java:100)  
at com.mathworks.mde.help.Index.getFilenameForLocation(Index.java:706)  
at com.mathworks.mde.help.Index.access$3100(Index.java:29)  
at com.mathworks.mde.help.Index$IndexMouseMotionAdapter.mouseMoved(Index.java:768)  
at java.awt.AWTEventMulticaster.mouseMoved(AWTEventMulticaster.java:272)  
at java.awt.AWTEventMulticaster.mouseMoved(AWTEventMulticaster.java:271)  
at java.awt.Component.processMouseMotionEvent(Component.java:5211)  
at javax.swing.JComponent.processMouseMotionEvent(JComponent.java:2779)  
at com.mathworks.mswing.MJTable.processMouseMotionEvent(MJTable.java:725)  
at java.awt.Component.processEvent(Component.java:4967)  
at java.awt.Container.processEvent(Container.java:1613)  
at java.awt.Component.dispatchEventImpl(Component.java:3681)  
at java.awt.Container.dispatchEventImpl(Container.java:1671)  
at java.awt.Component.dispatchEvent(Component.java:3543)  
at java.awt.LightweightDispatcher.retargetMouseEvent(Container.java:3527)  
at java.awt.LightweightDispatcher.processMouseEvent(Container.java:3255)  
at java.awt.LightweightDispatcher.dispatchEvent(Container.java:3172)  
at java.awt.Container.dispatchEventImpl(Container.java:1657)  
at java.awt.Window.dispatchEventImpl(Window.java:1606)  
at java.awt.Component.dispatchEvent(Component.java:3543)  
at java.awt.EventQueue.dispatchEvent(EventQueue.java:456)  
at java.awt.EventDispatchThread.pumpOneEventForHierarchy(EventDispatchThread.java:234)  
at java.awt.EventDispatchThread.pumpEventsForHierarchy(EventDispatchThread.java:184)  
at java.awt.EventDispatchThread.pumpEvents(EventDispatchThread.java:178)  
at java.awt.EventDispatchThread.pumpEvents(EventDispatchThread.java:170)  
at java.awt.EventDispatchThread.run(EventDispatchThread.java:100)  
>
```

Abstract Interpretation

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Reference

- [POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In *4th ACM POPL*.
- [Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.
- [POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In *6th ACM POPL*.

Syntax of programs

X	variables $X \in \mathbb{X}$
T	types $T \in \mathbb{T}$
E	arithmetic expressions $E \in \mathbb{E}$
B	boolean expressions $B \in \mathbb{B}$
$D ::= T X;$ $T X ; D'$	
$C ::= X = E;$ while $B C'$ if $B C'$ else C'' $\{ C_1 \dots C_n \}, (n \geq 0)$	commands $C \in \mathbb{C}$
$P ::= D C$	program $P \in \mathbb{P}$

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States

Values of given type:

$$\mathcal{V}[T] : \text{values of type } T \in \mathbb{T}$$

$$\mathcal{V}[\text{int}] \stackrel{\text{def}}{=} \{z \in \mathbb{Z} \mid \text{min_int} \leq z \leq \text{max_int}\}$$

Program states $\Sigma[P]$ ¹:

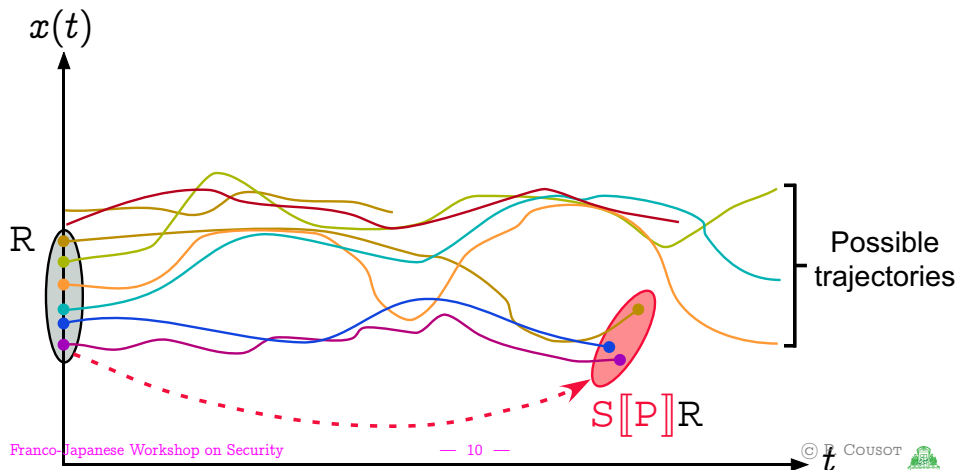
$$\Sigma[D C] \stackrel{\text{def}}{=} \Sigma[D]$$

$$\Sigma[T X ;] \stackrel{\text{def}}{=} \{X\} \mapsto \mathcal{V}[T]$$

$$\Sigma[T X ; D] \stackrel{\text{def}}{=} (\{X\} \mapsto \mathcal{V}[T]) \cup \Sigma[D]$$

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Postcondition semantics



Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[P] \stackrel{\text{def}}{=} \wp(\Sigma[P]) \quad \text{sets of states}$$

i.e. program properties where \subseteq is implication, \emptyset is false, \cup is disjunction.

¹ States $\rho \in \Sigma[P]$ of a program P map program variables X to their values $\rho(X)$

Concrete Reachability Semantics of Programs

$$\begin{aligned}
 S[X = E;]R &\stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in R \cap \text{dom}(E)\} \\
 \rho[X \leftarrow v](X) &\stackrel{\text{def}}{=} v, \quad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y) \\
 S[\text{if } B \ C']R &\stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup \mathcal{B}[\neg B]R \\
 \mathcal{B}[B]R &\stackrel{\text{def}}{=} \{\rho \in R \cap \text{dom}(B) \mid B \text{ holds in } \rho\} \\
 S[\text{if } B \ C' \ \text{else } C'']R &\stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup S[C''](\mathcal{B}[\neg B]R) \\
 S[\text{while } B \ C']R &\stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_0^{\subseteq} \lambda \mathcal{X}. R \cup S[C'](\mathcal{B}[B]\mathcal{X}) \\
 &\quad \text{in } (\mathcal{B}[\neg B]\mathcal{W}) \\
 S[\{\}]R &\stackrel{\text{def}}{=} R \\
 S[\{C_1 \dots C_n\}]R &\stackrel{\text{def}}{=} S[C_n] \circ \dots \circ S[C_1] \quad n > 0 \\
 S[D \ C]R &\stackrel{\text{def}}{=} S[C](\mathcal{E}[D]) \quad (\text{uninitialized variables})
 \end{aligned}$$

Not computable (undecidability).

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Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\#[[P]], \sqsubseteq, \perp, \sqcup \rangle$$

such that:

$$\langle \mathcal{D}, \sqsubseteq \rangle \xleftrightarrow[\alpha]{\gamma} \langle \mathcal{D}^\#[[P]], \sqsubseteq \rangle$$

hence $\langle \mathcal{D}^\#[[P]], \sqsubseteq, \perp, \sqcup \rangle$ is a complete lattice such that $\perp = \alpha(\emptyset)$ and $\sqcup X = \alpha(\cup \gamma(X))$

Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D}, \sqsubseteq \rangle \xleftrightarrow[\alpha_1]{\gamma_1} \langle \mathcal{D}_1^\#, \sqsubseteq_1 \rangle \text{ and } \langle \mathcal{D}, \sqsubseteq \rangle \xleftrightarrow[\alpha_2]{\gamma_2} \langle \mathcal{D}_2^\#, \sqsubseteq_2 \rangle$$

the reduced product is

$$\alpha(X) \stackrel{\text{def}}{=} \cap \{ \langle x, y \rangle \mid X \subseteq \gamma_1(X) \wedge X \subseteq \gamma_2(X) \}$$

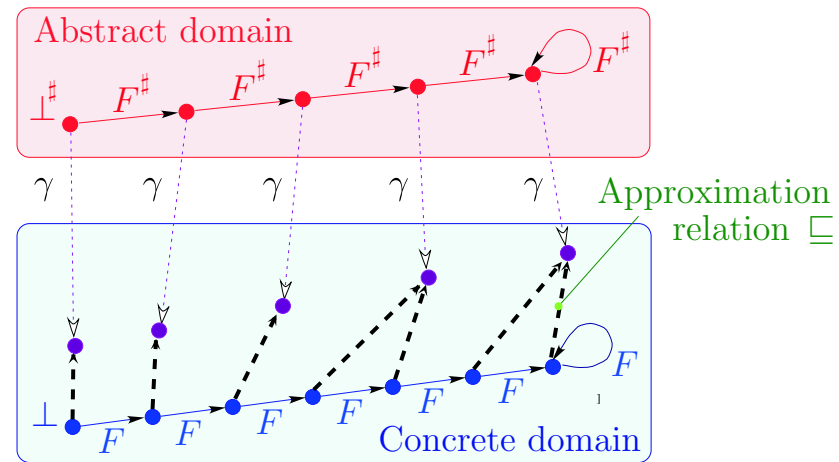
such that $\sqsubseteq \stackrel{\text{def}}{=} \sqsubseteq_1 \times \sqsubseteq_2$ and

$$\langle \mathcal{D}, \sqsubseteq \rangle \xleftrightarrow[\alpha]{\gamma_1 \times \gamma_2} \langle \alpha(\mathcal{D}), \sqsubseteq \rangle$$

Example: $x \in [1, 9] \wedge x \bmod 2 = 0$ reduces to $x \in [2, 8] \wedge x \bmod 2 = 0$

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Approximate Fixpoint Abstraction



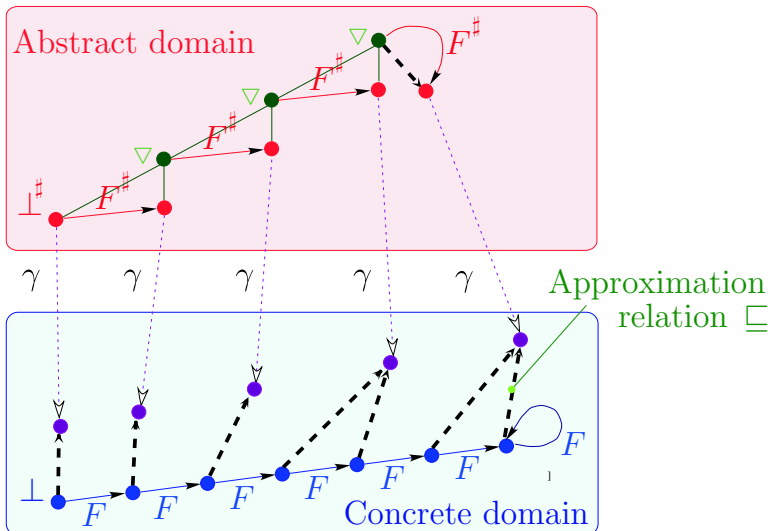
$$F \circ \gamma \sqsubseteq \gamma \circ F^\# \Rightarrow \text{lfp } F \sqsubseteq \gamma(\text{lfp } F^\#)$$

Abstract Reachability Semantics of Programs

$$\begin{aligned}
 S^\sharp[X = E;]R &\stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\}) \\
 S^\sharp[\text{if } B \ C']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup B^\sharp[\neg B]R \\
 B^\sharp[B]R &\stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\}) \\
 S^\sharp[\text{if } B \ C' \ \text{else } C'']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup S^\sharp[C''](B^\sharp[\neg B]R) \\
 S^\sharp[\text{while } B \ C']R &\stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \lambda \mathcal{X}. R \sqcup S^\sharp[C'](B^\sharp[B]\mathcal{X}) \\
 &\quad \text{in } (B^\sharp[\neg B]\mathcal{W}) \\
 S^\sharp[\{\}]R &\stackrel{\text{def}}{=} R \\
 S^\sharp[\{C_1 \dots C_n\}]R &\stackrel{\text{def}}{=} S^\sharp[C_n] \circ \dots \circ S^\sharp[C_1] \quad n > 0 \\
 S^\sharp[D \ C]R &\stackrel{\text{def}}{=} S^\sharp[C](\top) \quad (\text{uninitialized variables})
 \end{aligned}$$

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Convergence Acceleration with Widening



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Abstract Semantics with Convergence Acceleration²

$$\begin{aligned}
 S^\sharp[X = E;]R &\stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\}) \\
 S^\sharp[\text{if } B \ C']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup B^\sharp[\neg B]R \\
 B^\sharp[B]R &\stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\}) \\
 S^\sharp[\text{if } B \ C' \ \text{else } C'']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup S^\sharp[C''](B^\sharp[\neg B]R) \\
 S^\sharp[\text{while } B \ C']R &\stackrel{\text{def}}{=} \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X}. \text{let } \mathcal{Y} = R \sqcup S^\sharp[C'](B^\sharp[B]\mathcal{X}) \\
 &\quad \text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \nabla \mathcal{Y} \\
 &\quad \text{and } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \mathcal{F}^\sharp \quad \text{in } (B^\sharp[\neg B]\mathcal{W}) \\
 S^\sharp[\{\}]R &\stackrel{\text{def}}{=} R \\
 S^\sharp[\{C_1 \dots C_n\}]R &\stackrel{\text{def}}{=} S^\sharp[C_n] \circ \dots \circ S^\sharp[C_1] \quad n > 0 \\
 S^\sharp[D \ C]R &\stackrel{\text{def}}{=} S^\sharp[C](\top) \quad (\text{uninitialized variables})
 \end{aligned}$$

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Applications of Abstract Interpretation

² Note: \mathcal{F}^\sharp not monotonic!

Applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79] including **Dataflow Analysis** [POPL '79], [POPL '00], **Set-based Analysis** [FPCA '95], **Predicate Abstraction** [Manna's festschrift '03], . . .
- **Syntax Analysis** [TCS 290(1) 2002]
- **Hierarchies of Semantics (including Proofs)** [POPL '92], [TCS 277(1-2) 2002]
- **Typing & Type Inference** [POPL '97]

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Applications of Abstract Interpretation (Cont'd)

- **(Abstract) Model Checking** [POPL '00]
- **Program Transformation** [POPL '02]
- **Software Watermarking** [POPL '04]
- **Bisimulations** [RT-ESOP '04]

All these techniques involve **sound approximations** that can be formalized by **abstract interpretation**

Static Analysis

Reference

- [1] P. Cousot and R. Cousot. Static determination of dynamic properties of programs. In *Proceedings of the Second International Symposium on Programming*, pages 106–130. Dunod, Paris, France, 1976.

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State of the Art in Automatic Static Program Analysis

Static analysis tools

- Determine automatically from the program text **program properties** of a certain class that do hold at **runtime** (e.g. absence of runtime error);
- Based on the automatic computation of machine representable **abstractions**³ of all possible executions of the program in any possible environment;
- Scales up to hundreds of thousands lines;
- Undecidable whence **false alarms** are possible⁴

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Degree of specialization

- Specialization for a **class of runtime properties** (e.g. absence of runtime errors)
 - Specialization for a **programming language** (e.g. **PolySpace Suite** for **Ada**, **C** or **C++**)
 - Specialization for a **programming style** (e.g. **C Global Surveyor**)
 - Specialization for an **application type** (e.g. **ASTRÉE** for embedded real-time synchronous⁵ autocodes)
- ⇒ The more specialized, the less false alarms⁶!

³ sound but (in general) uncomplete approximations.

⁴ cases when a question on the program runtime behavior cannot be answered automatically for sure

⁵ deterministic

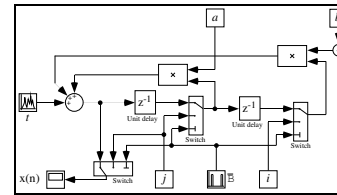
⁶ but the less specialized, the larger commercial market (and the less client satisfaction)!

The ASTRÉE static analyzer (www.astree.ens.fr)

- **ASTRÉE** is a **static program analyzer** aiming at proving the absence of **Run Time Errors** (started Nov. 2001)
 - **C programs**, no dynamic memory allocation and recursion
 - Encompass many (automatically generated) **synchronous, time-triggered, real-time, safety critical, embedded software**
 - automotive, energy and **aerospace applications**
- ⇒ e.g. No false alarm on the electric flight control codes for the A340 (Nov. 2003) and A380 (Nov. 2004) generated from SAO/SCADE.

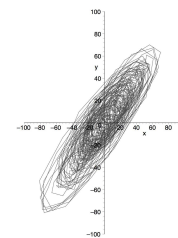
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2^d Order Digital Filter:

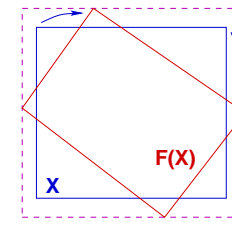


Ellipsoid Abstract Domain for Filters

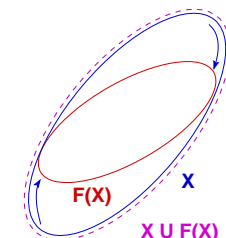
- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is **bounded**, which must be proved in the abstract.
- There is **no stable interval or octagon**.
- The simplest stable surface is an **ellipsoid**.



execution trace



$X \cup F(X)$
unstable interval



$X \cup F(X)$
stable ellipsoid

```

typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
    static float E[2], S[2];
    if (INIT) { S[0] = X; P = X; E[0] = X; }
    else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
                + (S[0] * 1.5)) - (S[1] * 0.7)); }
    E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
    /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
    while (1) {
        X = 0.9 * X + 35; /* simulated filter input */
        filter (); INIT = FALSE; }
}

```

Filter Example

Reference
see <http://www.astree.ens.fr/>

Arithmetic-geometric progressions

- Abstract domain: $(\mathbb{R}^+)^5$ ⁷
- Concretization (any function bounded by the arithmetic-geometric progression):
 $\gamma \in (\mathbb{R}^+)^5 \mapsto \wp(\mathbb{N} \mapsto \mathbb{R})$
 $\gamma(M, a, b, a', b') =$
 $\{f \mid \forall k \in \mathbb{N} : |f(k)| \leq (\lambda x. ax + b \circ (\lambda x. a'x + b')^k)(M)\}$

Reference
see <http://www.astree.ens.fr/>

⁷ here in \mathbb{R}

Arithmetic-Geometric Progressions (Example 1)

```

% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
    R = 0;
    while (TRUE) {
        __ASTREE_log_vars((R));
        if (I) { R = R + 1; } ← potential overflow!
        else { R = 0; }
        T = (R >= 100);
        __ASTREE_wait_for_clock();
    }
}
% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.

```

Arithmetic-geometric progressions (Example 2)

```

% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

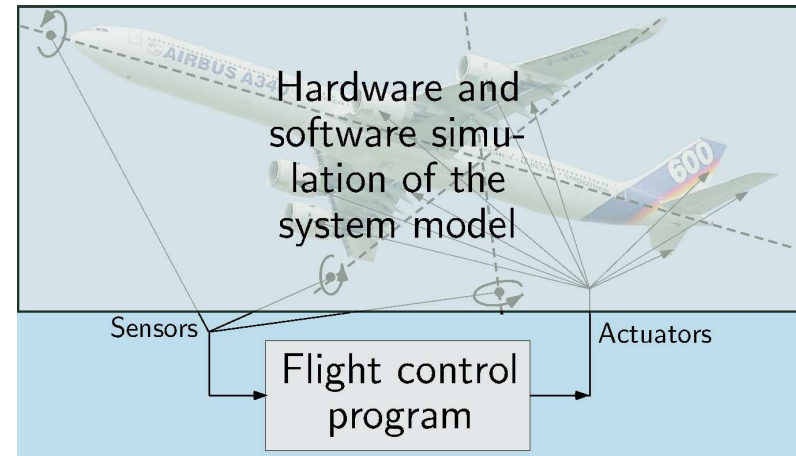
void dev()
{ X=E;
  if (FIRST) { P = X; }
  else
    { P = (P - (((2.0 * P) - A) - B)
          * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}

void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev();
    FIRST = FALSE;
    __ASTREE_wait_for_clock();
  }
}
% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
|P| <= (15. + 5.87747175411e-39
/ 1.19209290217e-07) * (1
+ 1.19209290217e-07)^clock
- 5.87747175411e-39 /
1.19209290217e-07 <=
23.0393526881

```


Towards System Verification Tools

Software test



Abstractions: program → none, system → precise

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- Very expensive
- Not exhaustive
- Extended during flight test period
- Late discovery of errors can delay the program by months (the whole software development process must be rechecked)

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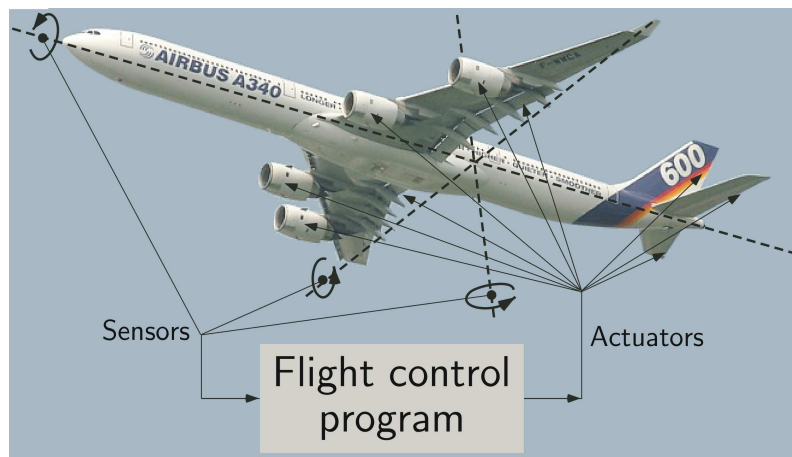
Reference

[2] P. Cousot. Proving Program Invariance and Termination by Parametric Abstraction, Lagrangian Relaxation and Semidefinite Programming, invited paper. In *Sixth International Conference on Verification, Model Checking and Abstract Interpretation (VMCAI'05)*, pages 1–24, Paris, France, January 17-19, 2005. Lecture Notes in Computer Science, volume 3385, Springer, Berlin.

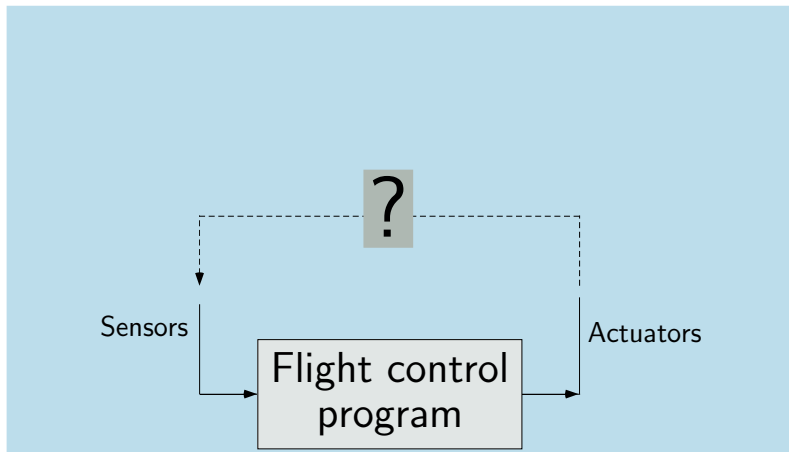
[APLAS'06] P. Cousot. Integrating Physical Systems in the Static Analysis of Embedded Control Software., invited talk In *APLAS'06*, Tokyo, Nov. 2005, to appear (LNCS).

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Computer controlled systems



Software analysis & verification with ASTRÉE



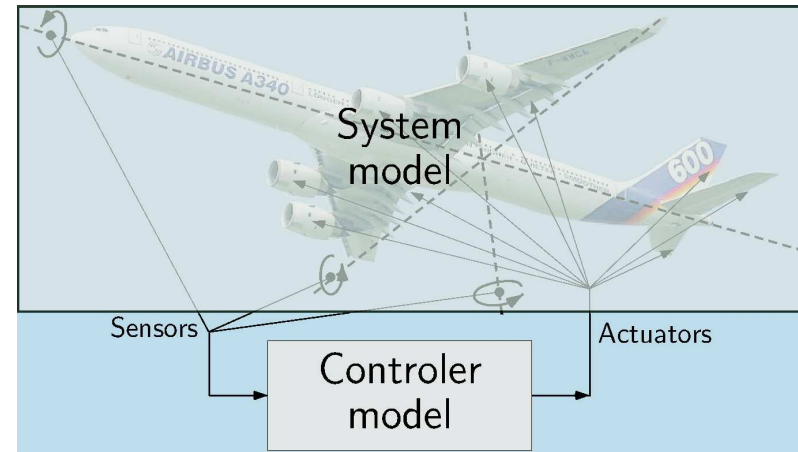
Abstractions: program → precise, system → coarse

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- Exhaustive
- Can be made precise by specialization⁸ to get no false alarm
- No specification of the controlled system (but for ranges of values of a few sensors)
- Impossible to prove essential properties of the controlled system (e.g. controllability, robustness, stability)

⁸ To specific families of properties and programs

System analysis & verification by control engineers



Abstractions: program → imprecise, system → precise

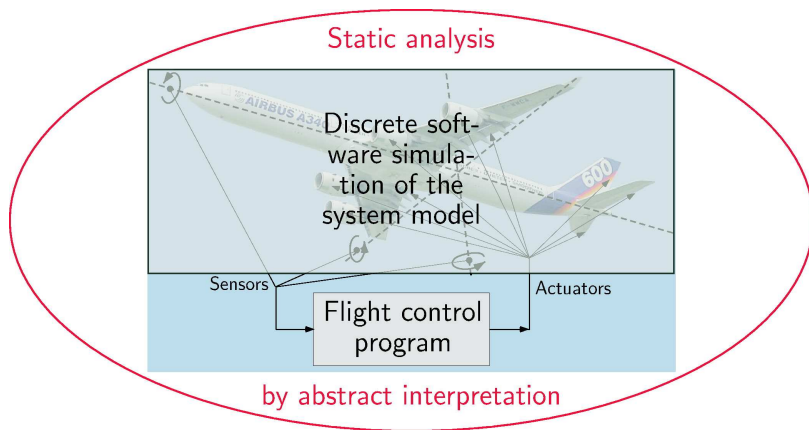
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- The controller model is a rough abstraction of the control program:
 - Continuous, not discrete
 - Limited to control laws
 - Does not take into account fault-tolerance to failures and computer-related system dependability.
- In theory, SDP-based search of system invariants (Lyapunov-like functions) can be used to prove reachability and inevitability properties
- Problems to scale up (e.g. over long periods of time)
- In practice, the system/controller model is explored by discrete simulations (testing)

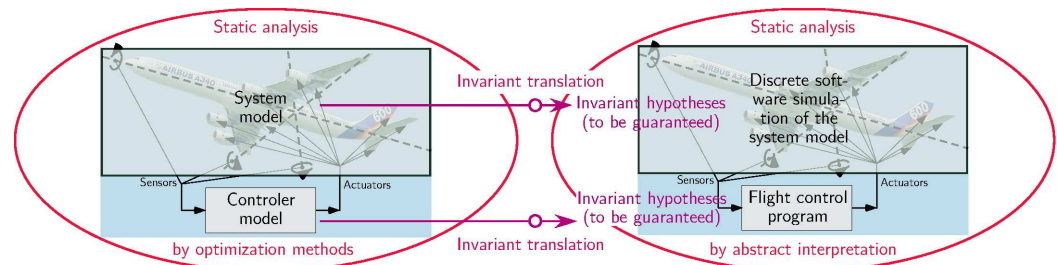
**Exploring new avenues
in static analysis**

- Exhaustive (contrary to current simulations)
- Traditional abstractions (e.g. polyhedral abstraction with widening) seem to be too imprecise
- Currently exploring new abstractions (issued from control theory like ellipsoidal calculus using SDP)
- Prototype implementation in construction!

System analysis & verification, Avenue 1



System analysis & verification, Avenue 2



Abstractions: program → precise, system → precise

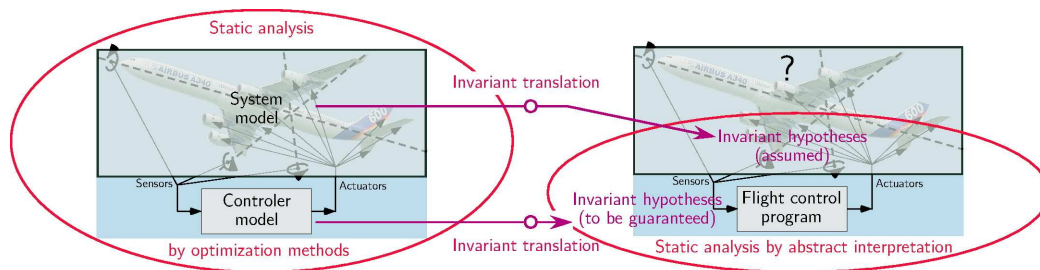
Abstractions: program → precise, system → precise

- Example of invariant translation: ellipsoidal \rightarrow polyhedral⁹
 - The static analysis is easier on the system/controller model using continuous optimization methods
 - The translated invariants can be checked for the system simulator/control program (easier than invariant discovery)
 - Should scale up since these complex invariants are relevant to a small part of the control program only
- The invariant hypotheses on the controlled system are assumed to be true
 - It remains to perform the control program analysis under these hypothesis
 - The results can then be checked on the whole system (as in case 2, but now using refined invariants on the control program!)
 - Iterating this process leads to *static analysis by refinement of specifications*

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System analysis & verification, Avenue 3



Conclusion

Abstractions: program \rightarrow precise, system \rightarrow precise

⁹ For which floating point computations can be taken into account

Scientific and technologic objective

To develop formal tools to answer questions about software:

- from control model design to software implementation,
- for a wide range of design and software properties, which would be general enough to benefit all software-intensive industries, and can be adapted to specific application domains.

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THE END, THANK YOU

More references at URL www.di.ens.fr/~cousot
www.astree.ens.fr.

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