The analytic semantics of weakly consistent parallelism

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Analytic semantics
Weak consistency models (WCM)

- **Sequential consistency:**
  reads read(p,x) are implicitly coordinated with writes w(q,x)

- **WCM:**
  No implicit coordination (depends on architecture, program dependencies, and explicit fences)

\[
w(q, x) \quad r(p, x) \quad rf(w(q, x), r(p, x))
\]
Analytic semantic specification

- **Anarchic semantics:**
  describes computations, no constraints on communications

- **cat specification (Jade Alglave & Luc Maranget):**
  imposes architecture-dependent communication constraints

- **Hierarchy of anarchic semantics:**
  many different styles to describe the same computations (e.g. interleaved versus true parallelism)
Example: load buffer (LB)

• Program:  
  \{ x = 0; y = 0; \}
  P0 | P1 ;
  r[] r1 x | r[] r2 y ;
  w[] y 1 | w[] x 1 ;
  exists(0:r1=1 /\ 1:r2=1)

• Example of execution trace \( t \in S^+[P] \):

  \[
  t = w(\text{start}, x, 0) w(\text{start}, y, 0) r(P0, x, 1) \uparrow f[w(P1, x, 1), r(P0, x, 1)] w(P0, y, 1) r(P1, y, 1)
  \]

  \[
  w(P1, x, 1) \uparrow f[w(P0, y, 1), r(P1, y, 1)] r(\text{finish}, x) \uparrow f[w(P1, x, 1), r(\text{finish}, x, 1)]
  \]

  \[
  r(\text{finish}, y, 1) \uparrow f[w(P0, y, 1), r(\text{finish}, y, 1)]
  \]

• Abstraction to cat candidate execution \( \alpha_{\Xi}(t) \)

  P0    P1
  \[
  a: Rx=1 \quad c: Ry=1
  \]
  \[
  b: Wy=1 \quad d: Wx=1
  \]
Example: load buffer (LB), cont’d

- **cat specification:**

\[
\text{acyclic } (\text{po} \mid \text{rf})^+ \]

The cat semantics rejects this execution \(\alpha_\Xi(t)\):

\[
\text{⇓\text{\[cat\]}} (\alpha_\Xi(t)) = \text{false}
\]
The WCM semantics

- Abstraction to a candidate execution:

\[ \alpha_{\Xi}(t) \triangleq \langle \alpha_e(t), \alpha_{po}(t), \alpha_{rf}(t), \alpha_{iw}(t), \alpha_{fw}(t) \rangle \]

\[ \alpha_{\Xi}(S) \triangleq \{ \langle t, \alpha_{\Xi}(t) \rangle \mid t \in S \} \]

- The cat semantics:

\[ \alpha_{\text{cat}}[\text{cat}](S) \triangleq \{ t \mid \langle t, \Xi \rangle \in S \land \text{\textup{\textcircled{cat}}}[\text{cat}](\Xi) \} \]

- The WCM semantics:

\[ \alpha_{\text{cat}}[\text{cat}] \circ \alpha_{\Xi}(S[P]) \]

**GC:**

\[ \langle \emptyset(C^{+\infty}), \subseteq \rangle \xrightarrow{\gamma_{\Xi}} \langle \emptyset(C^{+\infty} \times \Xi), \subseteq \rangle \xleftarrow{\gamma_{\text{cat}}[\text{cat}]} \langle \emptyset(C^{+\infty}), \subseteq \rangle \]
Definition of the anarchic semantics
Axiomatic parameterized definition of the anarchic semantics

- The semantics $S^\dagger[P]$ is a finite/infinite sequence of interleaved events of processes satisfying well-formedness conditions.

- Example: computation (local variable assignment)

  register assignment event by process $p$ in trace $\tau$

  unique event stamp $\theta$

\[
\forall p \in \mathcal{P} . \forall k \in \{0, 1, \ldots, |\tau|\} \cdot \forall \ell \in \mathbb{L}(p) . \forall v \in \mathbb{D} .
\]
\[
(\exists \theta \in \mathcal{P}(p) . \overline{\tau}_k = a(\langle p, \ell, r := e, \theta \rangle, v))
\]
\[
\implies (\ell \in N^p(\tau, k) \land \text{action}(p, \ell) = r := e \land v = E^p[e](\tau, k - 1)) .
\]
Axiomatic parameterized definition of the anarchic semantics

- Example: communication

- a read event is initiated by a read action:
  
  \[ \forall p \in \Pi . \forall k \in [1, 1 + |\tau|] . \forall \ell \in \mathbb{L}(p) . \]
  \[ (\exists \theta \in \mathbb{P}(p) . (\overline{\tau}_k = r(\langle p, \ell, r := x, \theta \rangle, x_\theta))) \]
  \[ \implies (\ell \in N^p(\tau, k) \land \text{action}(p, \ell) = r := x) . \]

- a read must read-from (rf) a write (fairness):

  \[ \forall p \in \Pi . \forall i \in [1, 1 + |\tau|] . \forall r \in \mathbb{Rf}(p) . \]
  \[ (\overline{\tau}_i = r) \implies (\exists j \in [1, 1 + |\tau|] . \exists w \in \mathbb{M}.i . \overline{\tau}_j = rf[w, r]) . \]
Axiomatic parameterized definition of the anarchic semantics

- Predictive evaluation of media variables:

\[ V_p^{(32)}[x_\theta](\tau, k) \triangleq v \text{ where } \exists! i \in [1, 1 + |\tau|[ . (\bar{\tau}_i = r(p, l, r := x, \theta, x_\theta)) \land \\
\exists! j \in [1, 1 + |\tau|[ . (\bar{\tau}_j = rf[w(p', l', x := e', \theta'), v), \bar{\tau}_i]) \]

- Local path-based evaluation of an expression:

\[ E_p^{(30)}[r](\tau, k) \triangleq v \text{ if } k > 1 \land ((\bar{\tau}_k = a(p, l, r := e, \theta, v)) \lor \\
(\bar{\tau}_k = r(p, l, r := x, \theta, x_\theta) \land V_p[x_\theta](\tau, k) = v)) \]
\[ E_p^{(30)}[r](\tau, 1) \triangleq l[0] \]
\[ E_p^{(30)}[r](\tau, k) \triangleq E_p^{(30)}[r](\tau, k - 1) \]

i.e. \( \bar{\tau}_1 = \epsilon_{\text{start}} \) by \( \text{Wf}_{15}(\tau) \)

otherwise.
Abstractions of the anarchic semantics
Abstractions

- **Semantics:**

\[ S^\perp[P] \triangleq \lambda \langle B, \text{sat}, D, I, \mathcal{G}, V, E, N \rangle \cdot \{ \tau \in \mathcal{F}[P] | \bot | W_{15}(\tau) \land \ldots \land W_{29}(\tau) \} \]

- **Examples of abstractions:**
  - Choose data (e.g. ground values, uninterpreted symbolic expressions, interpreted symbolic expressions i.e. “symbolic guess”)
  - Bind parameters (e.g. how expressions are evaluated)
  - ...
The hierarchy of interleaved semantics

\[ \alpha_{\text{cat}}[\alpha] \circ \alpha_{\Xi}(S^\perp[P]) \]

\[ \alpha_{\text{cat}}[\alpha] \circ \alpha_{\Xi} \]

\[ S^{\text{vi}}[P] = S^{\text{vo}}[P] \]

\[ \alpha_u \]

\[ S^{\text{si}}[P] \]

\[ S^{\text{so}}[P] \]

\[ \alpha_i \]

\[ \alpha_i' \]

\[ \alpha_{\text{B}, \text{sat}, \text{D}, \text{l}} \]

\[ \alpha_{\text{B}_s, \text{sat}_u, \text{D}_s, \text{l}_s} \]

\[ \alpha_{\text{V}(34)} \]

\[ \alpha_{\text{V}(32)} \]

\[ \alpha_{\text{E}(30), \text{N}(31)} \]

\[ S^\perp[P] \]

\[ \alpha^{\text{inscrutable}} \]

\[ \alpha^{\text{unspecified predictability}} \]

\[ \alpha^{\text{predictive}} \]
True parallelism

• Extract from interleaved executions:
  • The subtrace of each process (sequential execution)
  • The rf communication relation (interactions between processes)

⇒ no more global time!
• At each point in a trace, the state abstracts the past computation history up to that point
• Example: classical environment (assigning values to register at each point $k$ of the trace):

\[ \rho^p(\tau, k) \triangleq \lambda r \in R(p) \cdot E^p[r](\tau, k) \]

\[ \nu^p(\tau, k) \triangleq \lambda x_\theta \cdot V^p_{(32)}[x_\theta](\tau, k) \]
Prefixes, transitions, ... 

- Abstract traces by their prefixes:

\[ \tilde{\alpha}(S) \triangleq \bigcup \{ \tilde{\alpha}(\tau) \mid \tau \in S \} \]
\[ \tilde{\alpha}(\tau) \triangleq \{ \tau[ j] \mid j \in [1, 1 + |\tau|] \} \]
\[ \tau[ j] \triangleq \langle \overrightarrow{T}_{i} \rightarrow \tau_{i} \mid i \in [1, 1 + j] \rangle \]

- and transitions: extract transitions from traces

\[ \Rightarrow \text{communication fairness is lost, inexact abstraction,} \]
\[ \Rightarrow \text{add fairness condition} \]
Effect of the cat specification on the hierarchy
Exactness and cat preservation

\[ \alpha \circ \alpha_\Xi(\alpha(S^\perp[P])) \]

WCM semantics

Concurrent execution semantics

Exact and cat-preserving semantics

Anarchic semantics

Semantics

\[ \alpha_\Xi(\alpha(S^\perp[P])) \]

\[ \varrho(\mathcal{E}^{+\infty}) \]

\[ \mathcal{E}^{+\infty} \]

\[ \mathcal{D} \]

\[ \gamma \]

\[ \alpha \Xi \]

\[ \alpha \]

\[ \varnothing \]
The cat abstraction

- The same cat specification $\alpha[\text{cat}]$ applies equally to any concurrent execution abstraction $\alpha$ of any interleaved/truly parallel semantics in the hierarchy.

- The appropriate level of abstraction to specify WCM:
  - No states, only marker (e.g. fence), r, w, rf(w,r) events
  - No values in events
  - No global time (only po order of events per process)
  - Time of communications forgotten (only rf of who communicates with whom)
Conclusion
Conclusion

• The hierarchy of anarchic semantics describe the same computations and potential communications in very different styles

• The cat semantics restricts communications to a machine/network architecture in the same way for all semantics in the hierarchy

• This idea of parameterized semantics at various levels of abstraction is useful for
  • Verification
  • Static analysis
The End