Question 1 What is the order-reversing dual of “If \( X, Y \in \wp(S) \) then \( X \subseteq Y \Leftrightarrow X \cap Y = X \)”?

Question 2 If \( X, Y \in \wp(S) \) then prove that \( X \subseteq Y \Leftrightarrow X \cap Y = X \)

Question 3 The analysis of the program

\[ x:=10; \quad \text{while } (x<>1) \{ x := x-1 \} \]

would yield \( x \in [-\infty, 10] \) since the iterates would be successively \([10, 10], [10, 10] \lor [9, 10] = [-\infty, 10]\) which is stable.

Imagine another definition of the widening to get \([1, 10]\).

Question 4 Prove that the question of whether a variable is assigned more than two different values during program execution is undecidable.

Recall the following version of Tarski’s fixpoint theorem: “Let \( F, G \in \wp(S) \mapsto \wp(S) \) be \( \subseteq \)-increasing and \( A \in \wp(S) \) such that \( A \subseteq F(A) \). Then \( \text{lfp}_A^\subseteq F = \bigcap \{ X \in \wp(S) \mid A \subseteq X \land F(X) \subseteq X \}. \)”

Question 5 Let \( F, G \in \wp(S) \mapsto \wp(S) \) be \( \subseteq \)-increasing such that \( F \subseteq G \triangleq \forall X \in \wp(S) : F(X) \subseteq G(X) \). Prove that if \( A \subseteq F(A) \) then \( \text{lfp}_A^\subseteq F \subseteq \text{lfp}_A^\subseteq G \).