Problem 1
Answer whether each of the following languages is decidable, and justify your answer. You may find Rice’s Theorem useful for some parts.
1. \{⟨M, w, t⟩ : M halts on w in t steps\}
2. \{⟨M⟩ : ε ∈ L(M)\}
3. \{⟨M⟩ : M halts on ε\}
4. \{⟨M⟩ : M halts on some input\}
5. \{⟨M⟩ : L(M) is context-free\}

Problem 2
For each of the following statements, state whether it is TRUE or FALSE, and justify your answer.
1. \exists\ constants c<d such that \(n^d = O(n^c)\)
2. \(10^{10} \cdot n^{1000} = O(2^{0.001n})\)
3. \(n^{10} = O(2^{\log^2 n})\)
4. \(2^{\sqrt{\log n}} = O(\sqrt{n})\)
5. \(n^{\log n} = O(2^{\sqrt{n}})\)

Problem 3
Show that P is closed under the star operation (Hint: Use dynamic programming.) Recall that for a language \(L\),

\(L^* = \{x_1x_2...x_k \mid k \geq 0, \ x_i \in L \ \forall 1 \leq i \leq k\}\)

Problem 4
Let DOUBLE-SAT = \{⟨φ⟩ | φ is a boolean formula that has at least two satisfying assignments\}. Show that DOUBLE-SAT is NP-complete.

Problem 5
Problem 7.22 on Page 273 of Sipser (Problem 7.24 on page 296 in the new edition. This is the problem about \texttt{\neg}-SAT problem).

Problem 6
Problem 7.23 on Page 273 of Sipser (Problem 7.25 on page 296 in the new edition. This is the problem about MAX-CUT problem).