V22.0453-001: Honors Theory of Computation

Problem Set 4

Due Dec 9, 2005

All problems are worth 10 points.

Problem 1

Answer whether each of the following languages is decidable, and justify your answer. You may find Rice's Theorem useful for some parts.

1. $\{\langle M, w, t\rangle : M$ halts on w in t steps$\}$
2. $\{\langle M\rangle : \varepsilon \in L(M)\}$
3. $\{\langle M\rangle : M$ halts on $\varepsilon\}$
4. $\{\langle M\rangle : M$ halts on some input$\}$
5. $\{\langle M\rangle : L(M)$ is context-free$\}$

Problem 2

For each of the following statements, state whether it is TRUE or FALSE, and justify your answer.

1. $\exists$ constants $c < d$ such that $n^d = O(n^c)$
2. $10^{10} \cdot n^{1000} = O(2^{0.001n})$
3. $n^{10} = O(2^{\log^2 n})$
4. $2^{\sqrt{\log n}} = O(\sqrt{n})$
5. $n^{\log n} = O(2^{\sqrt{n}})$

Problem 3

Show that P is closed under the star operation (**Hint**: Use dynamic programming.) Recall that for a language L,

$$L^* = \{x_1 x_2 \dots x_k \mid k \geq 0,\ x_i \in L\ \forall 1 \leq i \leq k\}$$

Problem 4

Let DOUBLE-SAT $= \{\langle\phi\rangle \mid \phi$ is a boolean formula that has at least two satisfying assignments$\}$. Show that DOUBLE-SAT is NP-complete.

Problem 5

Problem 7.22 on Page 273 of Sipser (Problem 7.24 on page 296 in the new edition. This is the problem about $\neq$-SAT problem).

Problem 6

Problem 7.23 on Page 273 of Sipser (Problem 7.25 on page 296 in the new edition. This is the problem about MAX-CUT problem).