Problem 1
Answer whether each of the following languages is decidable, and justify your answer. You may find Rice’s Theorem useful for some parts.

1. \{ \langle M, w, t \rangle : M \text{ halts on } w \text{ in } t \text{ steps} \}
2. \{ \langle M \rangle : \varepsilon \in L(M) \}
3. \{ \langle M \rangle : M \text{ halts on } \varepsilon \}
4. \{ \langle M \rangle : M \text{ halts on some input} \}
5. \{ \langle M \rangle : L(M) \text{ is context-free} \}

Problem 2
For each of the following statements, state whether it is TRUE or FALSE, and justify your answer.

1. \( \exists \) constants \( c < d \) such that \( n^d = O(n^c) \)
2. \( 10^{10} \cdot n^{1000} = O(2^{0.001n}) \)
3. \( n^{10} = O(2^{\log^2 n}) \)
4. \( 2^{\log n} = O(\sqrt{n}) \)
5. \( n^{\log n} = O(2^{\sqrt{n}}) \)

Problem 3
Show that P is closed under the star operation (Hint: Use dynamic programming.) Recall that for a language \( L \),
\[ L^* = \{ x_1x_2 \ldots x_k \mid k \geq 0, \ x_i \in L \ \forall 1 \leq i \leq k \} \]

Problem 4
Let DOUBLE-SAT = \{ \langle \phi \rangle \mid \phi \text{ is a boolean formula that has at least two satisfying assignments} \}. Show that DOUBLE-SAT is NP-complete.

Problem 5
Problem 7.26 on Page 324 of Sipser (this is about the \( \neq \)-SAT problem).

Problem 6
Problem 7.27 on Page 325 of Sipser (this is about the MAX-CUT problem).