Honors Analysis of Algorithms

Problem Set 5

Collaboration is allowed, but you must write your own solutions. Proofs of correctness are a must.

Problem 1 (Randomized Algorithms)

Solve: [Kleinberg Tardos] Chapter 13, problem 1, page 782.

Problem 2 (Randomized Algorithms)

Solve: [Kleinberg Tardos] Chapter 13, problem 10, page 789.

Hint: Consider an agent such that there are k agents with a higher bid than her. What is the probability that her bid results in an update of b^*?

Problem 3 (Randomized Algorithms)

Solve: [Kleinberg Tardos] Chapter 13, problem 11, page 789.

Problem 4

Solve Problem 5 from the 2019 PhD exam:

https://cs.nyu.edu/home/phd/algorithms_exams/algorithms_2019fall_exam.pdf

Problem 5

In this problem, we explore the notion of *oracle reducibility*. If A is a language, then a *Turing machine with oracle* A is a Turing machine with a "magical" subroutine that decides membership in A. In other words, the subroutine, when given a string w, tells the machine whether or not $w \in A$. Let

$$\mathrm{HALT}_{\mathrm{TM}} = \{\langle M, x\rangle \mid M \text{ is a Turing machine that halts on } x\}.$$

Show that there is a Turing machine with oracle $\mathrm{HALT}_{\mathrm{TM}}$ that decides the following problem with only *two* questions to the oracle: Given three (machine, input) pairs $\langle M_1, x_1\rangle, \langle M_2, x_2\rangle, \langle M_3, x_3\rangle$, decide for each pair whether the Turing machine halts on the corresponding input.

Note: This is trivial if one allows *three* questions. Just ask the oracle whether $\langle M_i, x_i\rangle \in \mathrm{HALT}_{\mathrm{TM}}$ for $i = 1, 2, 3$.

Problem 6

Show that the collection of Turing-recognizable languages is closed under the operation of (a) union (b) concatenation (c) star and (d) intersection. What about complementation operation ?