Before giving my annual description of the “Puzzle Corner” ground rules, let me pass along Tom Terwilliger enthusiastic recommendation of Rosenhouse and Taalman’s, *Taking Sudoku Seriously*.

Now for the rules. In each issue I present three regular problems, the first of which is normally related to bridge, chess or some other game, and one “speed” problem. Readers are invited to submit solutions to the regular problems, and two columns (i.e. four months) later one submitted solution is printed for each regular problem; I also list other readers who responded. For example, the current issue contains solutions to the regular problems posed in May/June.

I am writing this column in mid June and anticipate that the column containing the solutions will be due in mid October. Please try to send your solutions early to ensure that they arrive before my submission deadline. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the “Other Respondents” section. Major corrections or additions to published solutions are sometimes printed in the “Better Late Than Never” section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue’s speed problem is given below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that remained unsolved when first presented.

**Problems**

**S/O 1.** We begin with another wink problem from Rocco Giovan-niello, who this time wants you to start with a 4 × 6 board, with the square (4,3) empty and the others containing a wink. As usual you find a sequence of horizontal and vertical jumps so that only one wink remains.

**S/O 2.** In his “earlier years”, Alan Faller used to sail around Monheg-an Island in Maine each summer and wondered about the correct direction to tack.

In particular, as shown on the right, Alan wishes to travel from B to A against a head wind at 20 degrees. At what direction should he head initially assuming that he will change direction once and that the speed of the boat through the water is $v = V \sin(2 \pi \beta /360)$ where $V$ is a constant (depending on the wind speed, shape and size of the sails, etc) and $\beta$, the angle between the boat’s direction of progress and the wind’s direction, can be chosen from 0 to ±90°.

**S/O 3.** Tim Malony wants you to show that all solutions of the complex equation

$$e^z = \frac{z - 1}{z + 1}$$

lie on the imaginary axis.

**Speed Department**

Avi Ornstein asks for an “interesting property” satisfied by 5, 6, 25, 76, 376, 625, 9,376, and 90,625 and no other number below 100,000?

**Solutions**

**M/J 1.** I received several beautifully drawn solutions to Frank Rubin’s Coraline (for CORners And LINEs) puzzle. Here’s Ken Haruta’s solution to what Frank calls “a Corner Puzzle for the Puzzle Corner.”

**M/J 2.** I received a number of very fine solutions to Fred Tydeman’s sock problem. No one found a close form solution; indeed, it is not clear that one exists.

Jerrold Grossman derived recurrences and had Maple do the computations. He also had Maple simulate the process, and the analytic answers agree with the simulation data. The recurrences and the simulations are on the “Puzzle Corner” web site.

His answers for 2, 3, 4, and 5 socks are 5/3, 7/3 (although 35/15 seems like a better way to look at it—the denominators are all odd factorial numbers, i.e., products of the first $n - 1$ odd positive integers), 311/105 (about 2.96), and 3377/945 (about 3.57). For 10 socks and 20 socks, Grossman obtained respectively, 4248732053/654729075 (about 6.49) and 226261084752832183400743/18813587457228104165625 (about 12.03).

Grossman contacted Milton Eisner (apparently the originator of the problem) and Geoffrey Pritchard (who with Wenbo Li, wrote a paper on the problem) and the latter confirmed that asymptotically the distribution of the maximum number of socks on the bed is normal with mean $n/2$ and variance $n/4$.

Richard Hess also attacked the problem computationally. His values agree with Grossman’s above and then he gives some “non-exact” answers. In particular, for 100 pairs of socks that value is approximately 53.915.

The following solution from Donald Aucamp includes an example and a diagram to illustrate the technique.

Define state variables $i$ and $j$ based on a given realization, where at a given point $i$ is the number of socks drawn from the hamper and $j$ is the number of socks still on the bed. Define $U_{ij}$ and $D_{ij}$ as the transition probabilities of going up or down by one sock on the next draw if currently in state $(i,j)$. Then:
D_{ij} = j/(2n - i)
U_{ij} = 1 - D_{ij}

D_{ij} is based on the fact that there are 2n - i socks still in the basket and j socks are on the bed, so the number of socks on the bed will go down from j to j - 1 if the next sock chosen matches one of them. Now let p_{ij} be the probability of a realization reaching state (i,j). Then:

\[ p_{ij} = p_{i+1,j} + p_{i,j-1} U_{i-1,j} \]

Since \( p_{00} = 1 \) and all the U's and D's are known functions of i and j, then all the p's can be solved sequentially by incrementing i. This procedure can be manipulated to find E(x), as follows: Let P(x) be the probability of x (the maximum number of socks seen so far). Then:

\[ F(x) = p_{2n,x} \]

That is, F(4) = 2/3, p_{31} = 1 – D_{31} = 2/3, and so on. The following two proofs, from Richard Schoeller and Dan Katz, use (related) results from number theory and abstract algebra respectively.

Schooler writes. We number the surfaces from 0 to s, arbitrarily choosing one as zero and numbering the rest sequentially around the cross section.

The twist t maps surface p to \( p + t \mod s \). So surfaces \( (p + nt) \mod s \) are identified for all integers n. The set nt mod s is an ideal whose representative \( m \), or smallest number such that all elements are multiples, is gcd(t, s). For any smaller number \( 0 < q < m \), the surfaces p and \( p + q \) are disjoint by construction. So m is the number of disjoint surfaces.

That \( m = \gcd(t, s) \) follows from Bezout’s identify. The set nt mod s is the set \( (at + bs) \mod s \) for all integers a and b. And all \( at + bs \) are multiples of gcd(t, s). gcd(t, s) is also the complete intersection of factors of t and s, so any smaller candidate is missing one of those factors, and cannot be in the linear combinations of t and s.

An interesting special case is prime \( s \), in which case the gcd reduces to s for \( t = 0 \mod s \), otherwise 1.

Katz writes. Number the surfaces 0 to s – 1. When the ends are connected, each surface k “merges” with the surface k + t. So we need to identify each element k in the set \( \{0, \ldots, s - 1\} \) with the element \( k + t \), and see how many distinct elements remain.

Algebraically, this is equivalent to counting the elements of the quotient group \( G/H \), where G is the set of integers mod s, and H is the subgroup of G generated by the element t. It is well known in abstract algebra that \( H \) is the set of multiples of gcd(s, t), and the quotient group will be the set of integers mod gcd(s, t). This group has gcd(s, t) elements, which is the resulting number of surfaces.

Other Responders
Responses have also been received from J. Chandler, B. Currier, A. Goel, S. Gordon, J. Hardis, R. Hess, G. Perry, K. Rosato, E. Sheldon, W. Sun, T. Tewilliger, E. Underriner, and K. Zeger.

Proposer’s Solution to Speed Problem
If you square the number, the original appears at the end. For example 367² = 141376.