I am pleased to report that our long time “Puzzle Corner” contributor, Avi Ornstein, has just released a new book. I wish him success with “Sonia in Vert.”

PROBLEMS

M/A 1. Larry Kells wants to know the highest contract South, the declarer, can make with the following distribution of the four hands. The unusual aspect is that the opponents are to cooperate in this venture, i.e. the hand is to be played against “worst defense”.

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♥ 10 8 6
♦ 9
♠ 9 8 7 6 5 4 3 2
♣ A Q
♥ 9
♦ 8
♣ A Q
♥ K 10
♦ A K Q 8 6 4
♥ 5 4 3 2
♦ 10 6 4 2
♣ 10
♥ J 7 5 3 2
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M/A 2. Albert Mullin notes that 4159 is the first 4-digit prime to occur as consecutive digits of the decimal expansion of pi. He wonders what is the first 10-digit prime to occur this way.

M/A 3. Tim Barrows has sent us what looks to me to be a rather serious 3D geometry problem. He writes. An “osculating circle” is a circle which matches the slope and radius of curvature of another curve at some point. An osculating circle of an ellipse is shown in the diagram. In this example, the matching point is the intersection of the ellipse with its major axis. Let us call this a “major axis osculating circle.” Consider now the case in which an ellipse is created from the intersection of a plane and a right circular cone. The orientation is such that a line drawn from the tip of the cone to the nearest point on the ellipse is perpendicular to the plane, i.e. the plane is at a right angle to the side of the cone at that point. In other words, the tilt of the plane is equal to the half angle of the cone. Show that the center of one of the major axis osculating circles of this ellipse lies on the axis of symmetry of the cone.

SOLUTIONS

N/D 1. Robert Wake writes that it seems unlikely that N/S can get any unluckier than the 28-point hand shown below. At this range or above, there must be some contract where they can take 7 tricks if they get the chance, so E/W can only prevail if they can take 7 tricks off the top in all four suits and notrump. Seven tricks in notrump requires, at a minimum, either one long suit or (as below) two suits headed by at least the AQ. Relying on one long suit means trouble in suit contracts, because a suit that will run at notrump will pose too many complications for defeating contracts in the other three suits without enough high cards.

So this is the best pair of N/S hands I could find that is unable to make any contract from either side. Since the hands are totally symmetric, we can assume South is declarer without loss of generality. East-West take the club finesse, then the diamond finesse, then the second club finesse and East cashes one more high club. At clubs or notrump, East cashes the fourth club, plays a diamond, and they have eight tricks. Otherwise, East switches to diamonds, and the fourth diamond either is good (diamonds), is ruffed high by partner (hearts), or forces declarer to ruff and sets up West’s long trump as the seventh trick (spades).

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♥ 9 8
♦ A K Q J
♣ 7 6 5 4
♥ 10 4 3 2
♦ K J 9
♣ 7 6 5
♥ 10 4 3 2
♦ 3 2
♣ 9 8
♥ A K Q J
♦ K J 9
♣ 7 6 5 4
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N/D 2. There seems to be a question of scaling, but most solutions agreed that the rate of new restaurants should be proportional to the square root of the “death rate.” Ed Sheldonsent us a detailed solution, which appears on the Puzzle Corner website, cs.nyu.edu/~gottlieb/tr. Due to space limitations we present an abbreviated version here.

The problem is one of rates. Let us assume that the favorites die of at an average interval of $N_0$ (measured in meals eaten out). The sampling of new restaurants must be sufficient to produce, on average, one new favorite over the same interval. If we measure the enjoyment ($E$) on a scale of 0 to 1, the enjoyment of a new restaurant will be assumed to be a random value from 0 to 1. It will also be assumed that the pool of new restaurants is unlimited. Let us now assume the favorites have a value of $E_0$ or higher. Since the distribution is linear, the average enjoyment value of the favorites will be $E_f = (1+E_0)/2$ and the average value of the rejects in the pool (values of less than $E_0$) is $E_p = E_0/2$.

Now on average, for every $1/(1-E_0)$ samples, there will be one
sample above $E_0$, and this number of sampling visits must be taken in the interval $N_d$. Now for enjoyment purposes, one of the sampling visits was enjoyable, so the number of inferior members of the pool visited will be $N_d = E_0/(1 - E_0)$ and the average enjoyment will be $E_{av} = (E_0 \times N_d + E_0 \times (N_1 - N_d)) / N_d$. This can be simplified to

$$E_{av} = \frac{1}{2} \left\lfloor \frac{1 + E_0}{N_d(1 - E_0)} \right\rfloor.$$

The average value is thus a function of $E_0$ and $N_d$. Assuming the death rate is constant, the value of $E_0$ that will maximize average enjoyment can be found by differentiating, and setting the derivative equal to zero.

$$2 \times E_{av} = 1 + \frac{E_0}{N_d(1 - E_0)}.$$

which simplifies to $E_0 = 1 - 1/\sqrt{N_d}$.

The problem asked for the fraction of the time you should try new restaurants, which is $[1/(1 - E_0)]/N_d = 1/\sqrt{N_d}$.

Since $N_d$ is the interval between deaths, or the reciprocal of the death rate, the fraction is $\sqrt{\text{Deathrate}}$.

**Better Late Than Never**

2009 J/A 2. Aaron Ucko reports that I dropped an n from the expression for the general minimum number of touches, which should have been.

**N/D SD.** I normally do not print comments on speed problems but must this time as the solution given was wrong. We forgot that 25 and 50 have two factors of 5. As a result 52! ends in 12 zeros.

**Other Responders**


**Proposer’s Solution to Speed Problem**

The fair coin is slightly more likely. The likelihood of 4H6T is $p^4(1-p)^6$, which equals $9.52 \times 10^{-4}$ for the funny coin and $9.76 \times 10^{-4}$ for the fair one.

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Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.