Flash—we are running low on speed problems!

It's been a year since I reviewed the criteria used to select solutions for publication. Let me do so now.

As responses to problems arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred, as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or sent via e-mail, since these produce fewer typesetting errors.

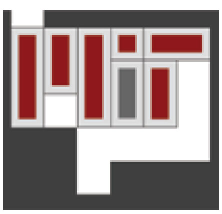
PROBLEMS

N/D 1. We start with a bridge problem from Larry Kells, who wants to know what the best chance of making seven spades is for a partnership that holds

♠ A	♠ K Q J 10 9 8
♥ A K Q	♥
♦ 5	♦ A K Q J 4 3 2
♣ J 9 7 6 5 4 3 2	♣

Declarer has the six-spade hand, and the opening lead is a spade, with East following suit. Assume there are no inferences to be had from the bidding or the lead, and that the opponents will make no mistakes for the rest of the play.

N/D 2. The MIT logo reminds David Hagen of a slider puzzle. He wants you to slide the tiles in the figure below so that the gray *I* escapes (at the top left, the only exit) without ever entering the black area. (If the colors of the tiles had been reversed, Hagan believes we would have had an ocular-medication problem: getting the red "I" out. Sorry.) As an added bonus, Hagen sent us a Word document (OpenOffice will also open it) with which you can actually slide the pieces and try to find the solution. I have made the document available on the Puzzle Corner Web page, http://cs.nyu.edu/~gottlieb/tr.



N/D 3. Rocco Giovanniello extends his 3-D tetrahedral game, which appeared here five years ago. Consider five equilateral triangles with side lengths five, four, three, two, and one so that the largest triangle is the base of an equilateral tetrahedron and the other three are parallel cross sections. On the five triangles, place sets of fifteen, ten, six, and three winks and one wink in the natural way. The base now looks like

$$\begin{matrix} w \\ w\ w \\ w\ w\ w \\ w\ w\ w\ w \\ w\ w\ w\ w\ w, \end{matrix}$$

and the smaller triangles look the same, but with one or more bottom rows deleted. Now remove the lone wink from the top triangle (leaving 34 winks in the tetrahedron) and play a checkers-like game in which a move consists of one wink's jumping over an adjacent wink and landing on a blank space and the removal of the jumped wink. (The jump must occur along a line; so the first jump must be by a corner wink on the triangle with six winks.) Since the number of winks decreases by one with each move, the longest possible sequence of jumps is 33. Can you find such a sequence?

SPEED DEPARTMENT

Paul Griffith has *composed* an *odd mix* for us *even* though his first Puzzle Corner offering (in 1968!) was mis-typeset.

Recall that a function o is odd if, for all x, $o(-x) = -o(x)$, and, similarly, e is even if, for all x, $e(-x) = e(x)$. (Sine is odd, but cosine is even.) Now let us call a function m mixed if it is neither odd nor even. What can you say about the composition of (a) two odd functions, (b) two even functions, (c) an odd and an even, (d) an even and an odd, (e) an odd and a mixed, and (f) an even and a mixed?

SOLUTIONS

J/A 1. The key is to get three diamond tricks. Richard Hess starts by taking the opening lead with the ace and drawing four rounds of trumps, throwing a heart from dummy. You then take the diamond ace and lead the queen.

If the defense ducks, lead the jack. If they don't take it, you have your three diamond tricks. If they take it, ruff a trick later and score the third diamond trick in your hand.

If the defense wins the diamond queen, their best play is to lead hearts. On the first two you play club losers from your hand. On the third you toss the diamond jack (unblocking) and ruff in your hand to score the 10 and 9 of diamonds, again giving you three diamonds and the contract.

J/A 2. Chris Hibbert enjoyed the problem, as it was related to a harder problem he is still working on. Joel Karnofsky notes that the problem actually requires an extra assumption about the 300 limit, e.g., that *A* knows that *B* knows this limit. Hibbert writes, "The two numbers are 4 and 13. Their product, 52, is on *A*'s hat, and their sum, 17, is on *B*'s hat.

"*A*'s first statement tells us that the sum can't be decomposed into two addends whose product is uniquely factorable into integers larger than 1. Nine can be decomposed into 2 and 7 (among other pairs), and these are the only factors of their product, 14. Eleven can be decomposed into 2 and 9, 3 and 8, 4 and 7, and 5 and 6. Each pair includes at least one composite, so their products don't have a unique pair of factors.

"This says that the sum can be any odd number that is two more than a composite number. All even numbers in this range can be written as sums of primes, so they're out. Any odd number that is two more than a prime is out, since it can be decomposed into 2 and a prime, which yield a uniquely factorable product. All the other decompositions of an odd number include an odd number and an even number, whose product can be factored in at least two ways.

"*B*'s first statement tells us two things: *A*'s first statement, along with the number on *A*'s hat, is sufficient to tell *B* what his number is, and *B* didn't know his number before *A*'s statement.

"So the product that *B* sees wasn't sufficient: it had multiple factorizations. But only one of the factorizations adds up to one of the potential sums. Only one number fits this description, and it's 52. Fifty-two is 4*13 or 2*26. Seventeen is an odd number that is two more than a composite, but 28 is not. All the other numbers with multiple factorizations and a sum less than 300 have either zero factorizations in the candidate list or more than one."

J/A 3. I am printing two solutions and posting a third. The first, from Luigi Iori, avoids calculus but asserts a number of nontrivial geometric facts; the others, from Robert Ackerberg and Frank Marcoline, do use calculus. John Prussing points out that "there are some round barns in Illinois, built by early German settlers, so they do really exist!" Al Cangahuala believes that the goat will eat the rope and wander freely. We start with Iori.

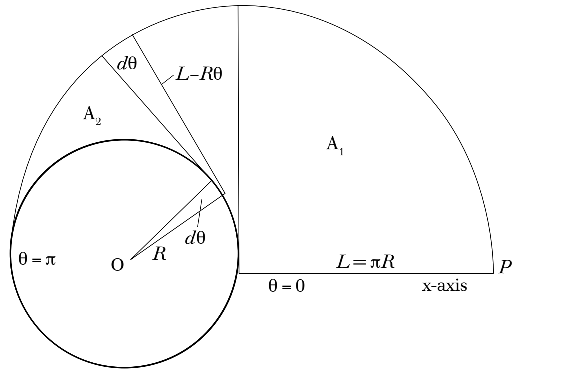
"Taking advantage of symmetry, we see that the area the goat can traverse is twice the area of a sector of a circle with an angle of $\pi/2$ and a 15π-foot radius and the area of the involute of the circle with an angle of π and a 15-foot radius.

"The area of a sector of a circle with an angle of ϕ and a radius of r is $\phi r^2/2 = (\pi/2)(15\pi)^2/2 = 56.25\pi^3$ square feet.

"The area of the involute of the circle with angle ϕ and radius r is $\phi^3 r^2/6 = \pi^3 15^2/6 = 37.5\pi^3$ square feet.

"The area the goat can traverse is $2(56.25\pi^3 + 37.5\pi^3) = 187.5\pi^3 = 5813.676878$ square feet."

Ackerberg writes, "Introduce polar coördinates with the center of the silo as origin and the x-axis extending to the right along the extended tether, which corresponds to $\theta = 0$. The length of the tether $L = \pi R$ is such that as the goat moves counterclockwise around the silo of radius R, it will end up on the silo surface at $\theta = \pi$. As the goat moves 90° counterclockwise from it original position P, an area A_1 of



a quarter-circle with radius L will be generated, with $A_1 = \pi L^2/4 = \pi^3 R^2/4$. As the goat continues counterclockwise, the tether is constrained and starts to wind along the silo wall. To determine this area we note that for a differential angle $d\theta$ traversed along the silo perimeter, the enclosed differential angle from the silo perimeter to the tether is also $d\theta$ because of the perpendicularity of the lines. The length of the tangent line from the silo perimeter to the tether $= L - R\theta$. The area of the differential triangle $dA = \frac{1}{2}(L - R\theta)^2 d\theta$. If this is integrated from $\theta = 0$ to $\theta = \pi$, we will determine the area A_2.

"Thus, $A_2 = \frac{1}{2}\int_0^\pi (L - R\theta)^2\, d\theta = \pi^3 R^2/6$ using straightforward integration, and $L = \pi R$.

"The maximum total area that can be traversed by the goat walking counterclockwise is the sum $A_1 + A_2 = (5/12)\pi^3 R^2$. Since the goat can also walk clockwise, this area has to be doubled, giving $(5/6)\pi^3 R^2 = (5/6)\pi L^2$."

Frank Marcoline's solution is posted on the Puzzle Corner website, http://cs.nyu.edu/~gottlieb/tr.

BETTER LATE THAN NEVER

2000 M/J 3. Frank Marcoline found an improved solution.

2007 M/A 3. Eugene Sard notes that there are additional restrictions for primitive Pythagorean triples, and as a result, there are primitive 60° triangles (e.g., 8, 15, 13) for which the corresponding Pythagorean triple (8, 6, 10) is nonprimitive

OTHER RESPONDERS

Responses have also been received from D. Aucamp, R. Bird, J. Chandler, D. Emmes, K. Hanf, T. Harriman, H. Ingraham, J. Karnofsky, D. Katz, L. Kyser, A. Maestri, Z. Moledina, E. Nelson-Melby, F. Pasterczyk, H. Snyder, and M. Strauss.

PROPOSER'S SOLUTION TO SPEED PROBLEM

(a) odd, (b) even, (c) even, (d) even, (e) mixed, (f) mixed.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.