Perhaps my upcoming 40th MIT reunion has inspired this nostalgic introduction. I just received a 750-gigabyte external hard disk. There is nothing remarkable about this $267 device. Indeed, I hear that one can now buy a disk that holds a full terabyte (10^{12} characters). But it is remarkable to me!

Early in my career at NYU, we were working with NASA's Institute for Computer Applications in Science and Engineering in Maryland, and I was invited there for a day. A highlight was seeing the MPP parallel processor and the institute's enormous online storage facility. It occupied a large room with plenty of air conditioning. When I arrived back at NYU, I bragged to everyone who would listen that I had just seen this awe-inspiring storage facility.

Its capacity was … one terabyte.

PROBLEMS

J/A 1. Arthur Wasserman offers a problem from the *Bermuda Bowl Bulletin.* South is to make 4S against any defense and (almost) any distribution: hearts are not 8-0, trumps are 4-1, and the opening lead is a heart. The hands are

x x x
A x x x
A Q J
A x x

A K Q J 10
x
10 9 8 x
x x x

J/A 2. The following is from the collection of "logical hat problems" that Richard Hess prepared for the "Gathering for Gardner 6." In these problems, logicians wear hats with numbers on them. Each can see the hats of the others but not his or her own. Here, two logicians, A and B, have been told that two integers greater than 1, but not necessarily distinct, have been chosen; their product is written on A's hat and their sum on B's. A and B then have the following dialogue:

A: "There is no way you can know the number on your hat."
B: "I now know my number."
A: "I now know my number, and the sum of our numbers is under 300."

What numbers are on A's and B's hats?

J/A 3. Phil Lally's goat is tethered with a 15π-foot rope to a point on the outside of a silo 30 feet in diameter. What area can the goat traverse? You can treat the goat as a point and assume it cannot penetrate the silo. Phil would prefer a solution without calculus.

SPEED DEPARTMENT

Ermanno Signorelli reports that Frank Morgan knows a common English word whose meaning you can make plural by prepending an *a*. Can you find it?

SOLUTIONS

M/A 1. Jorgen Harmse wonders, What is the correct play of the following hand for declarer after an opening lead of the diamond ace?

♠ K J 10 8 6 2
♥ 9 4
♦ 9 7 2
♣ 7 2

♠
♥ A K Q J 10 7 6 3
♦
♣ A K J 6 3

The bidding was

S	W	N	E
7H	All pass		

Tom Harriman notes that generally a sequence of plays keeping you alive to try the next chance is better than a sudden-death finesse. He believes this principle applies to Harmse's problem and writes,

"For a first-order approximation, assume a club split of no worse than 4-2 (see below for 5-1). After ruffing the ace, lead the ace and king of clubs, followed by the 3 to be ruffed.

"1. If the queen drops under the king, you ruff with the 9, pull trump, and claim.

"2. If West ruffs, dummy overruffs, and you repeat the play, winning unless West holds the 8 and 5.

"3. If, instead, West plays either a small club or the queen, dummy ruffs with the 9 to prevent an overruff. This play either drops the last club, and you claim, or

"4. discloses the original 4-2 distribution. You need East to have neither the trump 8 nor 5 so that you can ruff another club with the 4, dropping the queen.

"Finally, with a 5-1 club split, a 15 percent probability, the queen would be singleton 1/6 of the time and not combined with the 8 or 5 of trump about 1/4 of that, resulting in an increment of .15*(1/24), giving about a 68 percent overall probability of success."

M/A 2. Richard Hess offers us a problem similar to the yearly offering every January. He wants you to use the digits 2, 3, and 4 once and only once to form arithmetic expressions that equal the whole numbers from 1 to 20. You may use addition, subtraction, multiplication, division, powers, and decimal points.

Many readers will agree with Glenn Stith, who successfully solved the problem, that 16 was the trickiest to find. Indeed, every complete solution used the same expression for 16. (Recall that the proposer stated which operations were allowed, so using square root, factorial, or reciprocal was not permitted. Also stated was the requirement that all three digits be used for each number.) The following solution is from the Marinan team of Mark, Emily, and Kathleen.

$1 = 2 + 3 - 4$
$2 = 2 \times (4 - 3)$
$3 = 2 - 3 + 4$
$4 = 4 \times (3 - 2)$
$5 = 4 + 3 - 2$
$6 = 4 \times 3/2$
$7 = (3 - .2)\ /.4$
$8 = 4^{(3/2)}$
$9 = 2 + 3 + 4$
$10 = 2 \times 3 + 4$
$11 = 2 \times 4 + 3$
$12 = 2^3 + 4$
$13 = 4^2 - 3$
$14 = 2 \times (3 + 4)$
$15 = 2 \times 3\ /\ .4$
$16 = (.2 + .3)^{-4}$
$17 = 4\ /.2 - 3$
$18 = 3 \times (2 + 4)$
$19 = 4^2 + 3$
$20 = (2 + 3) \times 4$

M/A 3. Gordon Stallings notes that just as there are scalene right triangles with integer-length sides, there are also scalene triangles with a 60° angle and integer-length sides. Indeed, Stallings wants you to show that for any triangle of the first class with perimeter P, you can find a corresponding triangle of the second class with perimeter $1.5P$. For example, corresponding to 3-4-5, we have 3-7-8.

The following solution is from Dan Sidney:

"Every primitive Pythagorean triple (a, b, c)—where $a^2 + b^2 = c^2$—can be generated by the well-known method $(a, b, c) = (m^2 - n^2, 2mn, m^2 - n^2)$, where m and n are positive integers and $m > n$. (Nonprimitive Pythagorean triples—i.e., triples in which a, b, and c all share a common factor—can be generated by multiplying the appropriate primitive triple by that common factor. Solving the problem for primitive solutions is in practice sufficient for solving it for all solutions.)

"If we take a survey of primitive Pythagorean triples and 60° triples (x, y, z)—where sides x and y have a 60° angle between them, and thus by the law of cosines $z^2 = x^2 + y^2 - xy$—it appears that every primitive Pythagorean triple with perimeter $P = a + b + c = 2(m^2 + mn)$ has a corresponding 60° triple with perimeter $x + y + z = 1.5P = 3(m^2 + mn)$, and the 60° triple has the additional property that $x = a = m^2 - n^2$. Making this last assumption, then, for any allowed (m, n) we can find three equations with the three unknowns (x, y, z):

1. $x + y + z = 3(m^2 + mn)$
2. $x = m^2 - n^2$
3. $x^2 + y^2 - xy - z^2 = 0$

"Substitute equation 2 into 1, then solve 1 for z and substitute that into 3. After algebraic manipulation, a linear equation for y results, and the ultimate solution is $(x, y, z) = (m^2 - n^2, m^2 + 2mn, m^2 + mn + n^2)$."

BETTER LATE THAN NEVER

2006 J/A 2. Sergey Ioffe's genetic algorithm has now found

$$\left(1.4^{5-(20.8-.3)^{9^{-6}}}\right)^{-.7} \text{ and } 3 + 5^{-(7^{.1})} + \frac{.49^{8}}{2^{6}}.$$

The first uses zero; the second does not. These two expressions differ from π by 8.8×10^{-14} and 3.8×10^{-13}, respectively.

N/D 2. Victor Barocas believes that C would also be able to identify his 60 on round three and not before if the numbers were 25-35-60, 18-42-60, or 16-44-60.

Y2006. Alan Taylor notes that $4 = 6 - 2 + 0 + 0$ uses fewer operators.

2007 M/A SD. Roland Jansbergs and Bruce Dan point out that Tom Swifties must end in adverbs. Dan Sidney goes further and offers "Oops, I've dropped my toothpaste," said Tom aimlessly.

OTHER RESPONDERS

Responses have also been received from R. Ackerberg, R. Bishop, R. Byard, A. Cangahuala, D. Church, R. Ellis, O. Estes, J. Feil, E. Foster, C. Garrett, R. Giovanniello, O. Helbok, R. Hess, H. Ingraham, T. Jabine, J. Kenton, P. Markstein, A. Mintz, Z. Moledina, A. Ornstein, K. Rosato, R. Rosensweig, J. Rudy, E. Sard, M. Seidel, E. Sheldon, C. Swift, D. Watson, and B. Wolf.

PROPOSER'S SOLUTION TO SPEED PROBLEM

A + "yes" gives "ayes."

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.