Puzzle Corner

INTRODUCTION

It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so now.

As responses to problems arrive, they are simply put in piles, with no regard to their date of arrival. When I write the column, I weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Often, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail. A particularly elegant solution is, of course, preferred, as are contributors whose solutions have not previously appeared. I also favor solutions that are neatly written, typed or sent via e-mail.

PROBLEMS

July 1. An update from Larry Kells follows.

With the winter rains ending, I took a stroll in the park and saw my friend with his new bride. They filled me in on their exploits at the bridge club since I last saw them.

It seems that they were playing against a couple who were good friends with his ex-wife; even though she no longer comes to the club, the bad blood was obvious. They kept trying to make his new wife as uncomfortable as possible. With both vulnerable, my friend's left-hand opponent dealt and bid one heart. Right-hand opponent responded one spade. LHO bid two diamonds. RHO made an invitational jump to three hearts. LHO paused, then decided not to accept the invitation and passed. "So," my friend says, "my sweetie decides to accept the invitation in his place, and bids four hearts!"

Naturally, the opponents doubled, expecting a slaughter. But she made four hearts with the help of a trump coup, and in fact no defense could have beaten her. The opponents were dumbfounded because both of them had bid reasonably; they quickly walked out and haven't been seen since.

Unfortunately, I couldn't elicit the details of the deal. Can you help me?

July 2. Jerry Grossman asks about a normal round robin tournament with 13 contestants in which each player plays one game against each of the other 12. He wants to consider the case in which each player wins six games and loses six. Jerry wonders how many triples of three players (A, B, C) there are such that A beat B who beat C who beat A.

July 3. Rocco Giovanniello has a three-dimensional game for us. Consider four equilateral triangles with side lengths 4, 3, 2, 1 so that the largest is the base of an equilateral tetrahedron and the other three are parallel cross sections. On the four triangles place 10, 6, 3, and 1 winks in the natural way. The base now looks like

w
w w
w w w
w w w w

and the smaller triangles look the same, but with one or more bottom rows deleted. Now remove the single wink from the top triangle and play a checkers-like game in which a move consists of one wink jumping over an adjacent wink to land on a blank space, and the removal of the jumped-over wink. The number of winks decreases by one with each move. What is the fewest number of winks you can end up with?

SPEED DEPARTMENT

Eight more "lighter side" conversion factors from Sanjay Palnitkar. What are the following?

Weight an evangelist carries with God
Basic unit of laryngitis
Shortest distance between two jokes
0.5 bathrooms
453.6 graham crackers
1 trillion microphones
1 million bicycles
365.25 days

SOLUTIONS

Mar 1. Jerry Grossman really liked this problem and writes:

Here is the solution to the *Technology Review* problem about four deuces.

The only way this is possible is for North-South to have bid and made exactly 3 NT; so the score is 600 points for North-South. I don't know why the players left so hurriedly.

First we show with a proof by contradiction that there can be no trump suit. Without loss of generality, assume that clubs is trump and that West is not the player who won with the deuces of spades, hearts or diamonds, which won tricks in that order. Then West must have been void of spades, hearts and diamonds at the point at which the two of diamonds was led, which means that he must have had clubs left, and so he trumped the two of diamonds, and it didn't win a trick after all.

So we can assume that the contract is no-trump. We next claim that the four deuces must have been led by four different players. If not, then there must be a player who didn't win any tricks with a deuce, and he must be out of all four suits when the final deuce is played, an impossibility.

Now, the opening leader can't win with a deuce at trick one, since then he'd have only one suit in his hand and be forced to win all 13 tricks. Furthermore, each person winning a trick with a deuce must have obtained the lead prior to winning his deuce (since high deuce can't beat whatever was led), so each player must win at least two tricks. That's four tricks for the defense, and so nine tricks (game at no-trump) is the maximum for the declarer.

The following hand shows that this actually can be done. I have made South the declarer.

	♠ Q	
	♥ A 10 9 8 7 6 5 4 3 2	
	♦ Q	
	♣ Q	
♠ A 10 9 8 7 6 5 4 3 2		♠ J
♥ K		♥ J
♦ K		♦ A 10 9 8 7 6 5 4 3 2
♣ K	♠ K	♣ J
	♥ Q	
	♦ J	
	♣ A 10 9 8 7 6 5 4 3 2	

Play goes as follows. West cashes the ace of spades and then the deuce, on which other players discard their tens. He then leads the diamond king to his partner's ace, and East cashes the deuce of diamonds, on which the other players discard their nines. That's book for the defense. East leads the club to declarer's ace, and South cashes the deuce of clubs, on which the other players discard their eights. Finally, South leads his heart to North, who overtakes and takes the rest of the tricks, including the deuce of hearts.

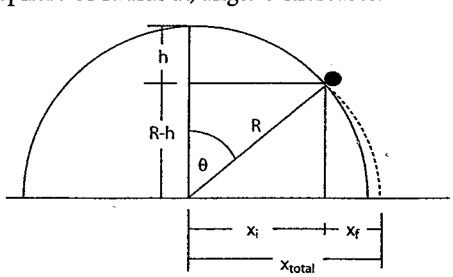
Mar 2. Edwin Field has sent us a fine solution as well as the following remarks on an extended problem.

I have solved the additional problem of how long (in time) the object stays on the hemisphere. For a one-foot radius hemisphere, placing the center of gravity of the object 0.01 foot (0.01 radian in this case) off dead center, it still takes 0.78 seconds for it to slide to the breakaway point. For 0.001 foot, the time is 1.19 seconds and for 0.0001 foot, 1.60 seconds. Every additional factor of ten closer to the center costs another 0.406 seconds (= (ln 10)/($\sqrt{g}$)). At that rate, even if you get to within an angstrom it still would take less than four seconds. You could get within a "googolth" of a foot and it would still take only 40 seconds. (I'm not sure Heisenberg would give you the position this exactly.)

Even if you could somehow position its center of mass directly above dead center, the gravitational attraction of your body would move it off this point by about an angstrom in 0.2 seconds or so, and then it's just a matter of another four seconds. A frictionless world would not be a simpler one.

Field's solution follows.

As a small body slides frictionlessly from very near the top of a hemisphere of radius R, angle θ increases:



1. When does the body leave the hemisphere?

The body accelerates down a "meridian" of the hemisphere. By energy conservation, when the body has dropped a vertical distance of h, its tangential velocity is given by:

$$v^2 = 2gh = 2\,g\,(R - R\cos\theta) = 2\,g\,R\,(1 - \cos\theta). \qquad (1)$$

Its radial acceleration is v^2/R. The body will leave the surface of the hemisphere when the normal (radial) component of its weight W, given by $W\cos\theta$, no longer exceeds the product of its mass times the radial acceleration. To use incorrect but prevalent parlance, this is the point where the normal force just equals the "centrifugal force":

$$W\cos\theta = (W/g)\,(v^2/R). \qquad (2)$$

Substituting (1) into (2) yields $\cos\theta = 2/3$. Thus, the particle leaves the surface when $h = R/3$. The body has, at that point, lost a third of its initial altitude above the table and has moved to the right by a distance x_i, such that $x_i/R = \sin\theta = \sqrt{5/3}$.

2. How far outside the hemisphere does it land?

As it leaves the hemisphere's surface, the body has a vertical velocity equal to sin θ times that given by (1) above, or $\sqrt{5/3}\,\sqrt{g\,R\,2/3}$. During the time T_f that it takes to "free-fall" to the table, its vertical velocity increases by $g\,T_f$. Since it must fall through a distance of (2/3) R, we write:

$$T_f = (2/3)\,R\,/\,[(\sqrt{5/3})\sqrt{g\,R\,2/3} + (g\,T_f/2)].$$

If we solve this quadratic, and then multiply the resulting answer for T_f by the constant horizontal velocity of [(2/3) $\sqrt{g\,R\,2/3}$], we find that during free fall the body moves to the right by a distance x_f, such that $x_f/R = (4/27)\,(\sqrt{23} - \sqrt{5})$. Adding this to the previously determined value $x_i/R = \sqrt{5/3}$ yields:

$$x_{total}/R = x_i/R + x_f/R = (5/27)\sqrt{5} + (4/27)\sqrt{23} = 1.12458.$$

Thus, the body lands 0.12458 R outside the hemisphere.

BETTER LATE THAN NEVER

2000 N/D 2. We are unable to print Philip Cassady's response due to space limitations. Read it at http://allan.ultra.nyu.edu/tr.

OTHER RESPONDENTS

Responses have also been received from R. V. Baum, T. M. Barrows, P. Cassady, T. J. Maloney, R. Marks, A. Ornstein, K. L. Rosato, E. Signorelli and R. A. Wake.

PROPOSER'S SOLUTION TO SPEED PROBLEM

One billigram; one hoarsepower; a straight line; one demijohn; one pound cake; one megaphone; two megacycles; one unicycle.

Send problems, solutions and comments to Allan Gottlieb, New York University, 715 Broadway, 7th floor, New York NY 10003, or to gottlieb@nyu.edu.