Puzzle Corner

INTRODUCTION

It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed in this column. Currently, I have a comfortable supply of regular problems and Bridge problems, but somewhat fewer speed problems.

PROBLEMS

May 1. Larry Kells wonders what is the fewest number of tricks a partnership can win in a full rubber and still win the rubber (using contemporary rubber Bridge scoring).

May 2. Norman Spenser has a triangle with sides of length *a*, *b* and *c*. What is the radius *R* of the circumscribing circle?

May 3. Steve Omohundro and Peter Blicher have performed a detailed analysis of the availability of Chicken McNuggets at McDonald's. They know that McNuggets come in packs of 6, 9 and 20 and wish to determine the largest number of McNuggets you *can't* buy (presumably so that they can order that many).

SPEED DEPARTMENT

David LaFrance-Linden wants to know the shortest way to express a real number in a nonobvious manner.

SOLUTIONS

Dec 1. Robert Bart notes that in the hand below South's opening 4-spade bid was passed out. He wonders whether, assuming best play on both sides, the contract will succeed or fail.

West	North	East
	♠ 9 3	
	♥ 10 7	
	♦ Q 10 7 4 2	
	♣ A K 6 5	
♠		♠ [10 8 5 4 2]
♥ K Q 9 8		♥ A J 5
♦ A J 8 6		♦ K 9 5 3
♣ J 10 7 4 2		♣ Q 9
	South	
	♠ A K Q J 10 5 2	
	♥ 6 4 3 2	
	♦	
	♣ 8 3	

For some reason East's hand was messed up a bit in production. I did receive a number of quite humorous "solutions" taking advantage of the duplicated cards, etc. I also received several real solutions in which the reader correctly figured out that East must hold the 8 7 6 4 of spades. Jonathan Hardis's follows:

With best play the contract fails. However, defense would require extraordinary luck and insight. In most actual play at the table, the contract should succeed.

As will be seen later, West must lead a club in order to defeat the contract.

If West leads a club or a diamond, declarer wins and plays a heart. Or perhaps West might lead a heart. In either case, in order to prevent declarer from ruffing a heart in dummy for his 10th trick, East must win and return a trump. Declarer wins and plays another heart. Again, in order to prevent a heart ruff in dummy, East (not West) must win and lead another trump. (Few Wests I know would show such restraint.)

At this point, South runs all but one of his trumps, squeezing West. Of the six cards that West must discard on the trumps, he can afford to discard no more than two clubs (else a low club in dummy becomes a winner) and one heart (else South's 6 of hearts becomes a winner). At this point, West has held on to either two hearts (the king and queen) or one heart and one diamond.

Every West I know would hold on to the ace of diamonds for dear life. However, if West does this and discards a high heart instead, South leads a low heart, eventually trumps a diamond with his last spade and cashes his fourth heart for the winning trick.

If West discards all his diamonds, declarer crosses to dummy with a club and leads the queen of diamonds for a ruffing finesse. However, in order to return to dummy to cash the winning 10 of diamonds, one more entry is needed. The other high club serves this purpose, unless it was lost on the first trick. Absent the second club entry, presuming that West holds on to a heart, South will lose the last two tricks, losing the contract.

Dec 2. Victor Barocas and Eric Lehman have a circle of radius $R >> 1$. Can they fit in more squares of area 1 or equilateral triangles of area 1?

No (mathematical) proof has been submitted, so the problem remains open. However, votes have been cast on both sides and supporting evidence provided. Michael Brill and Gardner Perry believe that more triangles can fit, Brill noting that in a certain sense one has more freedom with triangles. Richard Hess examined solutions composed of concentric rings and found that more squares can be packed. Charles Muehe considered certain symmetrical placements and, using a computer analysis, found more squares (for circles with integral radius around 100, there were about a half a percent more squares).

Dec 3. Norman Spencer has a 1-inch cube that he wants to give as a present. What is the area of the smallest rectangular piece of wrapping paper that can be folded to cover the cube? No cutting is permitted.

Mark Perkins showed that the area of the paper can be arbitrarily close to 6, the surface area of the cube. He writes that the general idea is to use a long, narrow strip of paper, covering one face at a time by doing a 180-degree turn at each edge—just the way I mow my lawn. It turns out the area of the "extra" paper needed for turning/folding/overlapping is O (width), so by making the rectangle narrower and narrower we get closer and closer to the theoretical minimum.

Here are the details. Let the width of the rectangle be $w = 1/N$ for large integer N. To paper the first face, we will run the paper up and down starting on the left side. Start in the lower left-hand corner with the paper running up the face parallel to the left edge. After reaching the top of the face, make a 45-degree fold so the paper turns 90 degrees and is now running parallel to the top edge of the face just beyond the edge of the face. Now make another 45-degree fold so the paper is running down the face just to the right of and along the paper already on the face. Repeat these steps, a total of N-1 times, until the face is completely covered and there are a bunch of little triangles hanging off the top and bottom of the face. There are actually N-1 of these triangles, but we'll pretend there are N to simplify the math. Each triangle has a width of $2w$ and height of w and is a double thickness of paper. Thus, N of these triangles would use an area of paper = $2N(1/2)w(2w) = 2Nw^2$. This represents a length of $2Nw$. The total length used to paper the face is thus $N + 2Nw$. But $Nw = 1$, so this is actually $N + 2$. Now we need to allow some extra paper to get to the starting point on the next face. Let's allow a length of 6—enough to traverse each face once. Thus the total length for papering the face is $N + 8$. Since there are six faces, we need a total length of paper of $6N + 48$. Now, the area of this length of paper is $w(6N + 48) = 6 + 48w$. Thus, by making N large (and thus w small), we can get as close to the theoretical minimum of 6 as we want—but we can never reach it.

Which reminds me of the classic engineer/mathematician joke that I first heard at MIT: The E and M are each put in a room opposite a beautiful woman and told that each minute they can close half the distance to the woman. The M walks off in a huff knowing he will never get to the woman, but the E knows he will get "close enough." Of course, today, perhaps, the engineer will also walk off in a huff, because she is not gay.

Oct 1. You may recall that space limitations in March prevented us from printing the Oct 1 solution. We have extra space this issue so a shortened version of Oct 1 is appended. The full solution remains at allan.ultra.nyu.edu/~gottlieb/tr.

The obvious way to play for slam is to take the trump finesse. However, since East's diamonds are only queen-high, it may be more likely that he has the spade king. If he holds Kx you can play for an endplay by taking the diamond ace, ruffing a diamond, cashing the heart king, leading to the ace of spades, and cashing two more hearts and the ace of clubs. Now lead the club king. If East is 2-3-7-1 he can ruff in but will have only diamonds left and has to yield a ruff-sluff. If he follows to the club king, he is 2-3-6-2 and you can lead a spade to throw him in; the same holds if he is 2-3-7-1 and doesn't ruff the club king.

The problem with this line of play is that it fails if East has three spades or fewer than three hearts. You have to find him with a singleton or doubleton king of spades as well as guess exactly how many hearts or clubs he has. This reduces the probability of success considerably below the odds of finding him with the king to begin with. Whereas, even if the chances of West holding the spade king are less than half, the finesse is straightforward and, if it works, the contract is almost assured. All things considered, the spade finesse must be your best shot.

But we are told to absolutely maximize our chances. There is one remaining vulnerability even if the spade king is onside. What if West holds all four spades? Then we cannot finesse him out of the king. Is there anything we can do about that? The only hope would be to execute a form of trump coup known as a Smother Play. Can we plan our finessing strategy to include this possibility?

The idea is to throw East in at trick 11 with nothing but diamonds left, with South holding ♠ J 10, West ♠ Kx and North ♠ A and a small side card (not a diamond). Then West's trump trick disappears. Under what conditions can this be done? As in all trump coups, South first has to shorten his trumps to be equal to West's. Since dummy's third diamond is to be the throw-in card, this means that South has to ruff a small diamond and heart in his hand. So West must have exactly two diamonds (with only one he will overruff if South tries to ruff a diamond, and with three he can prevent his partner from being thrown in with the third diamond) and at least four hearts (or he could overruff a heart). East must have at most two clubs, or you can't strip him and he will have a safe exit there. Therefore West must have at least three clubs. Tying this together, the only possible distribution for West that may allow a Smother Play is 4-4-2-3.

Because of limited entries to dummy, if we want to play for a possible Smother Play we must take the diamond ace and ruff a small diamond immediately. (There is a slight risk of an unnecessary overruff in a situation where West does not have all four trumps. However, in that case East would have eight diamonds and would have preempted to the four level at favorable vulnerability.) Then cash the heart king and finesse the spade 9. If East follows low, we are home. In the actual deal, East shows out. Now cash the ♥ A Q, ruff a heart, cash the ♣ A K and finesse the spade queen. Finally, lead the diamond 8 and discard the club 8. The defenders are helpless!

In summary, two things must be realized: the trump finesse has a better chance than an endplay. Preparation must be made to execute a Smother Play if trumps are 4-0.

OTHER RESPONDERS

Responses have also been received from R. Abes, C. A. Berg, K. Bernstein, D. Diamond, R. Dickinson, J. Feil, J. E. Fitzgerald Jr., E. Golembewski, J. Grossman, G. Hadley, E. F. Kurtz, R. L. Kyhl, M. Lindenberg, H. Mandeville, K. Rosato, E. W. Sard, W. A. Stiehl, and T. J. Wheeler.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Although "common," "shortest" and "nonobvious" are matters of opinion, my favorite is i^i, which equals $e^{-\pi/2}$.

Send problems, solutions and comments to Allan Gottlieb, New York University, 715 Broadway, 7th floor, New York NY 10003, or to gottlieb@nyu.edu.