Puzzle Corner

INTRODUCTION

Technology Review has made a splash in my (computer science) community with their "Freesoft vs. Microsoft" cover, and now (mid-February) we just had "Windows refund day." This seems like a timely moment to answer publicly a question I have received from several readers (and answered privately each time). I have been a longtime user of Linux and was responsible for its first "official" installation at NYU. I much prefer free software; I will not, however, be asking for a Windows refund. I often incorporate Powerpoint slides from others in talks and get Excel templates for budgets. Moreover, I confess to playing Diablo (when all my work is done, of course), and I no longer travel by car to a new location without having Streets98 up on my laptop.

PROBLEMS

M/J 1. We begin with a bridge problem from Larry Kells, who wonders about the difference between facing north or south (besides not getting sun in your eyes).

What is the largest possible variation between the number of tricks that South as declarer can take at a given suit contract, and the number of tricks that North would take in the same contract, assuming best play by both sides?

M/J 2. Here is one from Terry Langerdoen, who must work for a company with a very restrictive holiday policy.

Allyson's first job after graduation in 1996 is with a company whose paydays occur every other Friday, except when a payday Friday is a holiday (Independence Day, Christmas and New Year's Day), in which case it is the immediately preceding Thursday. Her last payday in 1996 is Friday, Dec. 20. Assuming she remains employed by this company and it doesn't alter its payday arrangements, what is the first year in which she will have 27 paydays? Similarly, if her last payday in 1996 is Friday, Dec. 27, what is the first year in which she will have 27 paydays?

M/J 3. Perhaps others have seen this one before or will find some quick solution. I, however, was amazed when I peeked at William Pulver's answer.

What 16 digit number, when multiplied by any single digit, will give a product that contains these same 16 digits? [A trivial solution is 0000000000000000. Pulver has a solution with considerably fewer leading zeros.—Ed.]

SPEED DEPARTMENT

Speedy Jim Landau asserts that Paul Hyman wants to know what you call the group of elderly men at the resort who always eat their meals together at the same table and spend the rest of their time sunbathing.

SOLUTIONS

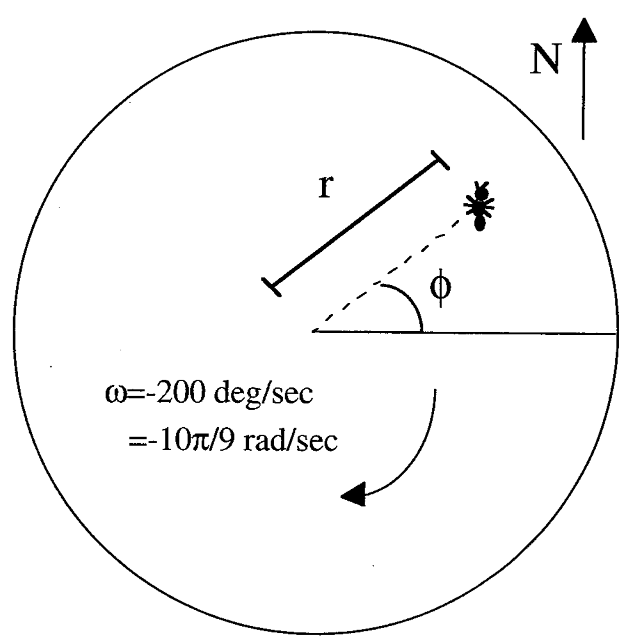
J/F 1. George Blondin has carefully observed "two bugs out for a spin" but needs help determining how fast they were moving.

A small quick bug walks due north (relative to what it's walking on) at a constant speed. A second bug also walks due north at a constant speed, but it is laser-guided and stops if it gets off the beam. Each bug steps onto a 12 inch 33 1/3 rpm record at the five o'clock radial (12 o'clock is due north). The laser (lazier??) bug stops walking until it's back on the beam, while the other bug continues its march. The ride neither saves nor costs time for either bug time. How fast is each walking?

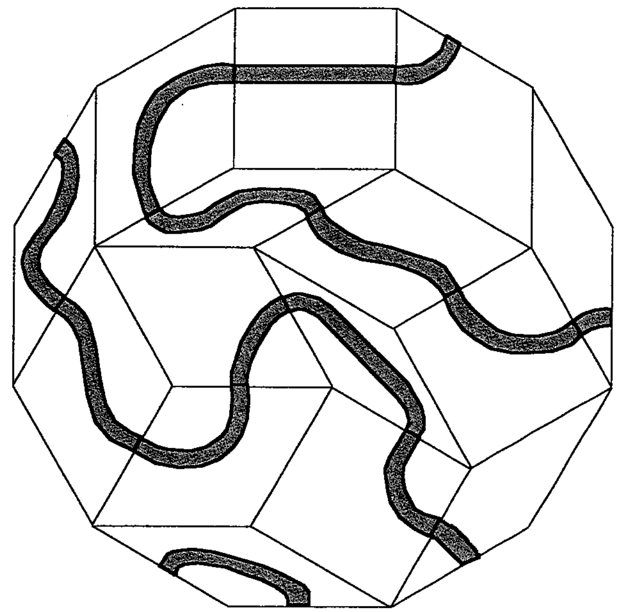
The following solution is from Matthew Fountain.

The laser bug walks 8.66 inches/sec; the other bug travels 12.3662 inches/sec. In polar coordinates, one o'clock is (6,60) and five o'clock is (6,300).

The record spins at -200 degrees/sec. The laser beam intersects (6,60) and (6,300). The laser bug starts at (6,300), immediately leaves the laser beam, rides to (6,60) where he ends his ride. His ride takes (60-300)/(-200)=1.2 seconds. The distance from (6,300) to (6,60) is 2·6·sin(60). His walking speed is 12·sin(60)/1.2=8.66.



All angles that follow are in radians. The other bug walks north on the record with speed *s* inches/sec, its radial speed in space being $dr/dt = s\cdot\sin(\phi)$, its angular speed in space being $d\phi/dt = \omega + s\cdot\cos(\phi)/r$, and the record's rotational speed being $\omega = (-10/9)\pi$. Eliminating dt produces



$$dr/d\phi=s\cdot\sin(\phi)/(\omega+s\cdot\cos(\phi)/r),$$

which has the solution

$$r^2+2\cdot(s/\omega)\cdot\cos(\phi)\cdot r=k$$

where k is the constant of integration. For any given s,

$$k=6^2+2\cdot(s/\omega)\cdot\cos(5\pi/3)\cdot 6$$

to insure the walk starts at five o'clock. Solving for r yields

$$r=-\cos(\phi)/\omega + (k+s^2\cos^2(\phi)/\omega^2)^{1/2}$$

Since $\cos(\pi-a)=\cos(\pi+a)$, the bug's walk from five o'clock to nine o'clock is a mirror image of his walk from nine o'clock to one o'clock. He stops at one o'clock, regardless of his speed.

$$\cos(\phi)=(k-r)^2\cdot\omega/(2rs)$$
$$\sin(\phi)=(1/(2rs))\cdot(4r^2s^2-w^2\cdot(k^2-2kr^2+r^4))^{1/2}$$
$$dt=dr/(s\cdot\sin(\phi))=2rs\cdot dr/(4r^2s^2-\omega^2k^2+2\omega^2kr^2-\omega^2r^4)^{1/2}$$

Integrating this last equation, we get

$$t=(1/\omega)\sin^{-1}(\omega^2r^2-2s^2-k\omega^2)/(4+4ks^2\omega^2k)-1/2)+C$$

On the path $r=6$ at the start, and r reaches its minimum value when $\phi=\pi$ halfway through the path. To determine the desired speed, estimate s; calculate k for that s; calculate r at $\phi=\pi$ and then t using these values. Next calculate t using the same values of s and k but with $r=6$. The bug's time on the rotating record is twice the difference between the two ts. His time when not rotating is $2\cdot 6\cdot\sin(5p/3)/s$. Repeat with a better estimate of s until the times for the two types of walk are as close as desired. When $s=12.3662$, $k=14.7440$, r (at $\phi=\pi$)$=1.681756$, and the two $ts=0.45+C$ and $0.0298101+C$, the time on the record is ***0.840370.*** The time off the record is also ***0.840370.***

J/F 2. Nob Yoshigahara wants you to cut the figure at the top of the page into the indicated 15 rhombi, then rearrange them into a dodecagon with just one rope (the original has three ropes). He also asks what the maximum number of possible ropes is. Ropes can be cut at the periphery of the dodecagon, but all rope edges must be connected inside the dodecagon.

There were no responses, which surprised me greatly as Nob's problems normally trigger quite a number of replies. Indeed, even Nob didn't send an answer. Any takers?

BETTER LATE THAN NEVER

1998 J/A 1. Chatchawin Charoen-Rajapark found an alternative bidding scenario in which the final contract is simply one spade.

Y1998. Ewin Field and Richard Royer found four improvements:

$8=1^{99}\cdot 8$
$9=1^{99}+8$
$90=1-9+98$
$100=1^8+99$

1999 J/F SD. Peter Blicher notes that we are erroneously assuming the sun sets at a 90 degree angle to the horizontal.

1999 M/A SD. The ultimate answer was omitted: The representation of 17 in base 5 is **32.**

OTHER RESPONDERS

Responses have also been received from T. Barrows, R. Hess and F. Irons.

PROPOSER'S SOLUTION TO SPEED PROBLEM

The table of tan gents! [Groan...—Ed.]

Send problems, solutions and comments to: Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York, NY 10012, or to gottlieb@nyu.edu.

Edited by Owen W. Ozier '98