Puzzle Corner

Challenges: yearly problem, bug walk, geometry cutout, timing the sun, bridge play, triangle calculations, prime puzzle.

INTRODUCTION

This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of digits of the new year (1, 9, 9 and 9) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 1998 yearly problem is in the "Solutions" section.

PROBLEMS

Y1999. How many integers from 1 to 100 can you form using the digits 1, 9, 9 and 9 exactly once each and the operators +, -, * (multiplication), / (division) and exponentiation? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 1, 9, 9, 9 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator. I should add that 1999 does not have many solutions and 2000 is worse!

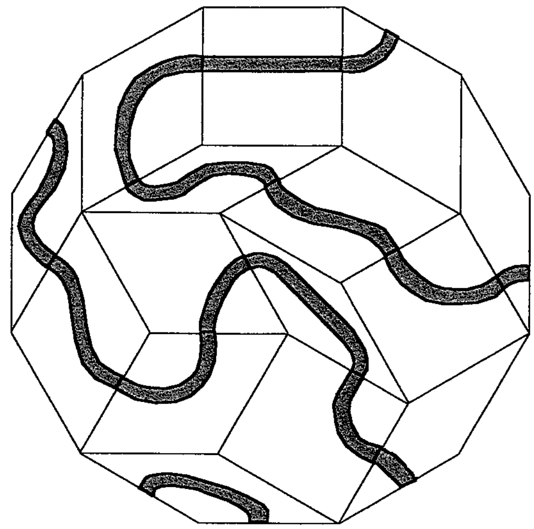
J/F 1. George Blondin has carefully observed "two bugs out for a spin" but needs help determining how fast they were moving.

A small, quick bug walks due north (relative to what it's walking on) at a constant speed. A second bug also walks due north at a constant speed, but it is laser-guided and stops if it gets off the beam. Each bug steps onto a 12-inch 33 1/3 rpm record at the 5 o'clock radial (12 o'clock is due north). The laser bug stops walking until it's back on the beam, while the other bug continues its march. The ride neither saves nor costs time for either bug time. How fast is each walking?

J/F 2. Nob Yoshigahara wants you to cut the figure at the top of the page into the indicated 15 rhombi, then rearrange them into a dodecagon with just one rope (the original has three ropes). He also asks what the maximum number of possible ropes is. Ropes can be cut at the periphery of the dodecagon, but all rope edges must be connected inside the dodecagon.

SPEED DEPARTMENT

Joseph Horton has repeatedly timed the setting sun and is confident that this event requires two minutes. What angle does the sun subtend from earth?



SOLUTIONS

Y1998. Avi Ornstein sent us the following solution.

$1=1^{998}$	54=(8-1)*9-9
2=91-89	55=(9-1)*8-9
3=(18+9)/9	62=9*8-1-9
4=8/(1+(9/9))	63=81-9-9
6=8-1-9/9	64=1-9+9*8
7=98-91	70=89-19
8=1*9-9+8	$71=9*8-1^9$
9=1+9-9+8	$72=(1^9)*9*8$
10=(89+1)/9	73=1*9*9+8 or 1^9+9*8
11=99/(1+8)	74=91-8-9
16=(1+9/9)*8	79=98-19
17=18-(9/9)	80=(19-9)*8
18=99-81	81=99-18
19=19*(9-8)	82=81+9/9
20=19+9-8	88=98-1-9
21=189/9	89=98-1*9
22=198/9	90=99-1-8
25=9+8+9-1	91=819/9
26=1*9+9+8	92=1+99-8
27=1+9+9+8	$97=98-1^9$
36=19+9+8	$98=99-1^8$
53=8*9-19	99=891/9

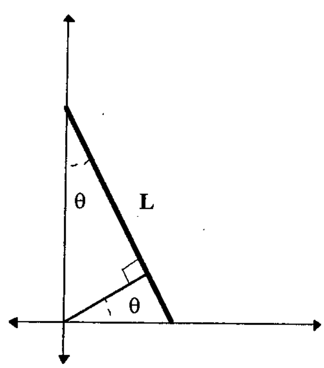
S/O 1. Larry Kells wants to know if it is possible to construct a deal in which no player has a void and (1) South as declarer can make eight or more tricks with some trump suit or no trump against the best defense, and (2) East as declarer can make seven or more tricks with the same trump suit or no trump against the best defense.

Robert Wake found a way. East and South can each make 3N as declarer on the following hand. Best defense in each case is to be passive and lead opponents' long suit. S as declarer is held to the seven clubs and two aces while E is able to endplay S for a ninth trick because the next-to-last diamond squeezes S one trick before the last diamond would have squeezed E. (Note: If West's singleton were in clubs, S could opening-lead the ace before exiting with a diamond to hold E to eight tricks, but an ace-of-spades lead results in the same endplay as a diamond opening with one fewer relevant suits.)

West		East
	North ♠ QJxxx ♥ QJxx ♦ xx ♣ xx	
♠ x ♥ xx ♦ AQxxxxxx ♣ xx		♠ Kxxxx ♥ Kxxxx ♦ x ♣ KJ
	South ♠ Ax ♥ Ax ♦ KJ ♣ AQxxxxx	

S/O 2. Roy Sinclair has a ladder of length L located in the first quadrant with its ends on the coordinate axes (see figure). He wants you to: (1) Find the equation of the boundary of the region swept out by the ladder as its foot is pulled away from the origin; (2) Find the area of this region; (3) Find the locus of the foot of the perpendicular from the origin to the ladder as the ladder moves.

Jackson Bross says that as a calculus teacher, he felt honor-bound to try these problems. I am pleased to report that he passed.



(1) The equation of the boundary of the region swept out by the ladder is (rather surprisingly, I thought) $x^{(2/3)} + y^{(2/3)} = L^{(2/3)}$.

(2) The area of this region is $3\pi/32$.

(3) The equation of the locus of the foot of the perpendicular is best given as a polar equation: $R = (^L/_2)\sin(2\theta)$, where θ is the angle the perpendicular makes with the X axis.

These are derived as follows:

(1) I chose to look at the family of all ladder positions and see where they intersected a particular vertical line. Whichever one intersects at the largest y-value gives us a point on the boundary curve. The line through (a,0) and (0,b) has equation $y=(-^b/_a)x + b$. Taking the derivative of this with respect to b, to find the maximum y value, gives us (after a little work) the point $(a^3/L^2, b^3/L^2)$ on the boundary. [Note: "a" is in fact a function of b, with $^{da}/_{db}=-^b/_a$, which is important when you actually take that derivative. "x" is being held constant, though.] A little bit of elbow grease and the formula $a^2 + b^2 = L^2$ gives us the fact that the points $(a^3/L^2, b^3/L^2)$ sit on the curve $x^{(2/3)} + y^{(2/3)} = L^{(2/3)}$.

(2) So now we "just" have to do the integral from 0 to L of $(L^{(2/3)}-x^{(2/3)})^{(3/2)}dx$. This is a bad but not impossible integral. The first thing I did was get rid of the cube roots by using $u=x^{(1/3)}$ (and hence $dx=3(u^2)du$) and $k=L^{(1/3)}$. This turns our integral into the integral from 0 to k of $3(u^2)(k^2-u^2)^{(3/2)}du$. As soon as you see a "$k^2-u^2$," you immediately think trig substitution: $u=k\sin(t)$, $du=k\cos(t)dt$. This substitution further rearranges the integral to become the integral from 0 to $\pi/2$ of $3(k^6)(\sin(t))^2(\cos(t))^4dt$. Not the best integral but a quick substitution of $(1-(\cos(t))^2)$ for $(\sin(t))^2$ gets it all in terms of powers of cosine. Note that k^6 is just L^2. Using reduction formulas for the cosine functions (or just looking up these fairly standard integrals), we get $(^{3\pi}/_{32})L^2$. Note that Mathematica or other calculation programs can do this integral for you.

(3) The vertical side of the large triangle has length $L\cos(\theta)$ because the top angle is also θ. But using the smaller triangle containing this side, the perpendicular must have length $L\cos(\theta)\sin(\theta)$. So the formula is $R=L\cos(\theta)\sin(\theta)$.

S/O 3. Richard Hess wonders: what is the largest prime that has no digit (from 0 to 9) used more than twice and only one repeated digit?

Matthew Fountain knows that a number is divisible by nine if and only if the sum of its digits is divisible by nine. From this he derived his answer as follows.

The largest prime is 98,876,532,401. The largest prime is to contain one each of the digits 0 through 9, plus one additional digit. That digit is not 9, as then the sum of the digits would be 54 which is divisible by 9. The largest prime is found by examining the numbers that can be formed, starting with the largest, and then, in turn, going to the next smaller, skipping those ending in even digits. Setting N to represent 988765, these numbers are N43201, N43021, N42301, N42103, N41203, N41023, N40231, N40213, N40123, N34201, N34021, N32401. The last is prime.

BETTER LATE THAN NEVER

S/O SD. Several readers note that since gold is much denser than lead and the gold coins are much lighter, one can identify the lead coins by just feeling the bags and picking the one with the bigger coins.

OTHER RESPONDERS

Responses have also been received from C. Balleisen, T. Barrows, M. Barton, J. Burke, V. Christensen, C. Dale, D. Dechman, J. Drumheller, J. Fell, T. Fell, B. Grosselfinger, J. Grossman, F. Huber, H. Ingraham, A. Kirmse, C. Muehe, A. Ostapenko, J. Papadopoulos, D. Pecora, C. Rife, K. Rosato, R. Royer, A. Russell, R. Santoro, E. Sard, D. Savage, B. Taft and A. Tracht.

PROPOSER'S SOLUTION TO SPEED PROBLEM

1/2 degree: Earth turns 360 degrees every 24 hours, or 1 degree every 4 minutes.

Send problems, solutions and comments to: Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York, NY 10012, or to gottlieb@nyu.edu. *Edited by Owen W. Ozier '98*