# INTRODUCTION

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

In each issue I present three regular problems (the first of which is chess, bridge, go, or computer-related) and one “speed” problem. Readers are invited to submit solutions to the regular problems, and three issues later, one submitted solution is printed for each problem; I also list other readers who responded. For example, solutions to the problems you see below will appear in the February/March issue and the current issue contains solutions to the problems posed in May/June. Since I must submit the February/March column in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the “Other Respondents” section. Major corrections or additions to published solutions are sometimes printed in the “Better Late Than Never” section as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue’s speed problem is given below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the January issue of each year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

# PROBLEMS

**OCT 1**. We begin with a bridge problem from Dudley Church, who wants to know if South can make 6 Spades against an opening lead of the Heart King?

North

| | |
|---|---|
| S | K Q 3 |
| H | Void |
| D | J 6 5 |
| C | A K 10 8 5 4 2 |

West

| | |
|---|---|
| S | J 8 4 |
| H | A K 6 3 |
| D | K 10 9 3 |
| C | 9 6 |

East

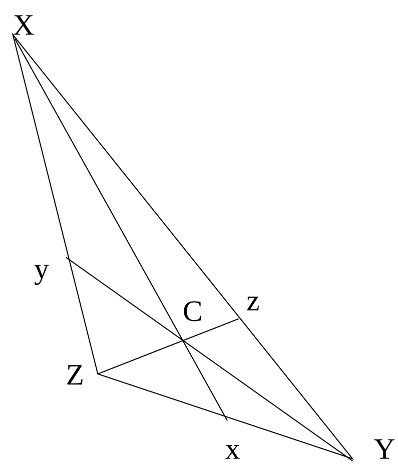
| | |
|---|---|
| S | 2 |
| H | 10 9 8 7 4 2 |
| D | 7 4 2 |
| C | Q J 7 |

South

| | |
|---|---|
| S | A 10 9 7 6 5 |
| H | Q J 5 |
| D | A Q 8 |
| C | 3 |

**OCT 2**. In the diagram below C is a point inside an arbitrary triangle XYZ. Nicholas Strauss has drawn a line segment from X through C and ending on the opposite side at a point we call x. Similarly, he has drawn lines starting at Y and Z and ending at y and z. Show that

$$\frac{XC}{Xx} = \frac{YC}{Yy} = \frac{ZC}{Zz}$$




**OCT 3**. Joseph and Charles Horton have inscribed a hexagon with side lengths 2,2,2,11,11,11 in a circle of radius r. What is the value of r?

## SPEED DEPARTMENT

Ken Rosato and his calculator have found a function f such that for x={0,1,2,3,4,5,6,7,8,9}, f(x)={6,1,3,2,0,0,0,4,1,3}. What is this function?

# SOLUTIONS

**M/J 1**. Mario Velucci wants you to place N queens on an N by N chess board so that the maximum number of vacant squares are Un-attacked. For N=1, there are no vacant squares, and, for N=2 and N=3, all vacant squares are attacked. But, for N=4, you can leave 1 vacant square un-attacked and, for N=5, you can leave 3. How about N=10 and N=20 or possibly even N=30?

Robert Bart notes that by bunching the queens in a corner, two triangular regions near the opposite corner are free, i.e. un-attacked. As N gets large, so does Free(N), the number of free squares. Your editor wonders if one can show that Free(N) grows at least linearly with N, i.e. Free(N)=Ω(N).

The diagrams and explanations below are from Matthew Fountain. The proposer supplies values for Free(N) for all N up to 30. His only disagreement with Fountain is Free(N)=22, but no diagram was supplied.

The ten diagrams show the position of the N queens that allows the following number of free squares: 1 when N=4, 2 when N=5, 4 when N=6, 7 when N=7, 11 when N=8, 15 when N=9, 21 when N=10, 145 when N=20, 420 when N=30, and 841 when N=40. In cases of N<7 I found it best to arrange the free squares first. For example, when N=4 the free squares can be placed anywhere on the board. When placed on the edge, there are six squares available to place the 4 queens. In cases of N>6 I arranged the position of the queens first. I started by placing the queens in a compact group in one corner of the board. I selected the corner since then I had only one diagonal width of the group to minimize while I was also attempting to minimize the row and column widths of the group. Tight packing of the queens assured that certain squares would be attacked many times while leaving others free. As each queen attacks 3N-2+2E squares, where E is the distance in squares of the queen from the edge squares of the board, a few widely distributed queens can attack large numbers of squares. Five queens, properly placed, can attack every square of a N=11 board. Upon examining my placement of queens in the corner, I found, in some cases, their grouping could be made improved by relocating certain queens on the edge of the group so the group would have a slimmer appearance viewed along a row, column, or long diagonal of the group.

**Please place figure number 1 here.**

**M/J 2**. Just before mailing this problem to me in March 93, Eugene Sard purchased a bunch of 29-cent and 23-cent postage stamps and was surprised to note that the total was a (non-zero) whole number of dollars. What is the smallest number of stamps that Sard could have been purchasing?

The following solution is from Mark Seidel. The general equation to solve is 29n + 23m = 100k, for some integers k,m and n, such that n+m is minimized. Taking the general equation modulo 23 and then modulo 3 yields 2k mod 3 = 0 so k = 3r (for some positive integer r). Taking the general equation modulo 23 and then modulo 8 yields 6n mod 8 = 0, so n = 4p. Taking the general equation modulo 4 yields n+3m mod 4 = 0, so (because n=4p) m = 4q. Plugging these relations into the general equation yields the reduced equation 29p + 23q = 75r, which when taken modulo 23 yields 6(p-r) mod 23 = 0, i.e. p = r + 23s. Plugging this equation back into the reduced equation yields q = 2r -29s. Minimizing n+m means minimizing (n+m)/4 = p+q = 3(r-2s), which must be positive (and therefore 3 or greater). One obvious solution is s=0 leading to (p,q,r) = (1,2,1) making p+q=3, so this must be a minimal solution. The solution to the original problem is therefore four 29-cent stamps and eight 23-cent stamps costing 3 dollars.

**M/J 3**. Nob Yoshigahara wants you to factor 123456789 into two five digit numbers.

The following solution is from Leonard Nissim. The two factors must both be fairly close to the square root of the original (about 11111.111), since both are over 10000. Clearly 123456789 is divisible by 9, as the sum of its digits is 45. Since 123456789/9 = 13717421, we are reduced to finding two factors of 13717421 which are close to its square root (about 3703.704). Dividing it by primes close to that value yields $1371721 = 3607 \times 3803$, and the solution is three times each of these primes, namely $123456789 = 10821 \times 11409$.

## BETTER LATE THAN NEVER

**1995 A/S 2**. James Datesh and Bob Sackhein each found two errors. The entry for 16A should be 3419 and the entry for 25D should be 1815.

**1995 Jan 1**. This is quite amusing. We asked if there are infinitely many numbers that can be formed using their own digits in a non trivial way. Dave Pecora and Ethan Rappaport each responded with a family of solutions. The amusing part is that the base of Rappaport's solution is 117,648 and the base of Pecora's is 117,649.

Rapport notes that $117648 = (7^6 - 1^{48}) \times 1$ and hence $1176480 = (7^6 - 1^{48}) \times 10$, etc. Pecora starts with $7^6 \times 1^{149} = 117649$ and then adds six zeros to get $70^6 \times 1^{14900000} = 117649000000$, etc. Can anyone find a family where the number of nonzero digits grow without bound?

## OTHER RESPONDERS

Responses have also been received from J. Abbott, H. Amster, R. Ball, L. Bell, J. Bush, F. Carbin, D. Diamond, S. Feldmen, E. Friedman, A. Guttag, J. Harmse, T. Hartford, R. Hess, H. Huang, H. Ingraham Jr., J. Kenton, M. Lindenberg, C. Muehe, R. Nelson, A. Ornstein, M. Perkins, G. Perry III, D. Plass, R. Ruiz, D. Savage, A. Shagen, P. Silvenberg, K. Stahl, A. Taylor, D. Thresher, and C. Whittle.

## PROPOSER'S SOLUTION TO SPEED PROBLEM

f(x) = g(x)-x where g(x) is the number of LCD elements in x. [I was personally baffled by this answer until I realized that LCD does NOT mean least common denominator (think calculator)−ed.]

**There was an error at tech review. In the oct issue, they accidentally (re)printed the first page of the A/S issue together with the second page of the oct issue. As a result no problems labelled OCT ever appeared. Also only some of the solutions to M/J appeared (namely those that were on the second page of the column**

**In the nov/dec issue TR published the problems I sent in for oct (but labelled them nov/dec). They caught up with problems by printing the portion of M/J solutions not printed in OCT together with all the JUL solns.**

**The problems originally planned for nov/dec were moved to Feb/Mar 1997.**

## INTRODUCTION

I was pleased to receive a kind letter from an old time reader that serves as another reminder that it is very hard, if not impossible, to predict when and where some knowledge will pay off. The end of the note reads as follows.

‘‘... you may find it amusing that the solution to M/A5 of 1973 (which was a Pell equation in disguise) has just solved a small but important and very very frustrating piece of a research problem on matroids and arrangements of hyperplanes. (I found the solution via Sloane-Plouffe ‘Encyclopedia of Integer Sequences.’) Talk about late responses! Sincerely grateful, Tom Zaslavsky’’.

## PROBLEMS

**N/D 1**. The following bridge problem is from Tom Harriman who wants to know how you make the contract against best defense after an opening lead of the Diamond Jack.

| | North |
|---|---|
| S | Q 3 |
| H | A K Q 2 |
| D | 8 6 |
| C | A Q 10 4 2 |

| | West | | East |
|---|---|---|---|
| S | 10 9 8 | S | A 6 |
| H | 9 7 6 5 | H | J 10 8 4 |
| D | J 10 2 | D | A K 7 4 3 |
| C | 9 8 5 | C | J 7 |

| | South |
|---|---|
| S | K J 7 5 4 2 |
| H | 3 |
| D | Q 9 5 |
| C | K 6 3 |

| W | N | E | S |
|---|---|---|---|
| pass | 1 Club | 1 Diamond | 1 Spade |
| pass | 2 Hearts | pass | 2 Spades |
| pass | 3 Spades | pass | 4 Spades |
| pass | pass | pass | |

**N/D 2**. With the US tennis open underway, the following problem seems quite timely. Joe Shipman knows two tennis players who are evenly matched. That is, whoever is serving has a probability p ($0<p<1$) of winning the point. Find a value of p and a ‘‘game situation’’ where the player who is ‘‘ahead’’ under standard tennis scoring is a underdog to win the set. To clarify, an example game situation would be ‘‘server ahead 5 games to 2 and up 40 love in the present game’’ (however, in this example I do not believe there is a p satisfying the desired conditions).

**N/D 3**. Nob Yoshigahara wants you to arrange the nine cards below into a 3x3 square so that all twelve pentominoes appear in the interior of the square.

**Please place figure number 1 here.**

## SPEED DEPARTMENT

Speedy Jim Landau communicates the following from William Barker who alleges it to have been popular at the National Bureau of Standards in the 1960's. Given a string of four letters, you are challenged to find a common English word containing that string. For example if the challenge is ‘‘tege’’, a possible answer is ‘integer’. Here are three favorites from the NBS: ‘‘hkak’’, ‘‘houe’’, and ‘‘eonh’’.

## SOLUTIONS

**Jul 1**. In the hand below, submitted by Doug Van Patter, the bidding was short and sweet: South opened with 1H, West bid 2C, North bid 4H, and everyone then passed. How can South make the contract after opening lead of the King of Clubs?

North
S J 8 6
H K 7 4
D K Q 10
C A 10 9 8

| West | | East | |
|---|---|---|---|
| S | K 10 3 | S | Q 7 5 4 2 |
| H | 10 3 | H | 9 8 |
| D | 8 4 | D | A J 7 6 5 |
| C | K Q J 7 5 2 | C | 3 |

South
S A 9
H A Q J 6 5 2
D 9 3 2
C 6 4

The solution to the problem lies in establishing a club as the 10th trick.

North wins the 1st trick with CA. South wins the next two tricks with HA and HQ leaving HK in the dummy. On trick-4, South plays its last club and losing to West's CJ (North plays C8).

If West leads a spade on trick-5, South wins with SA, crosses over to North's HK and discards S9 on North's C9 losing to West's CQ. South then concedes North's DK to East's DA, and uses North's DQ as the entry to use the now good CT for a diamond discard.

If West leads a diamond on trick-5, North plays DK:

- If East wins with DA and returns a spade (best), South wins with SA. North's HK is then used as the entry to discard South's S9 on North's C9. North's established CT is then used for a diamond discard using DQ as the entry.
- If East refuses to win trick-5 with DA, North then plays C9 and South discards a diamond on trick-6. The defense will get one more trick in diamonds. South's losing S9 will be discarded on North's CT using HK as the entry.

In all cases, N-S loses 2 club tricks and one diamond trick.

**Jul 2**. Phil Bonomo seems to like ‘‘space cadet’’ problems, especially those involving navigation satellites. In 93 he asked about their velocity, now he questions their altitude.

Geosynchronous orbits are generally taken to be the orbits of largest radius (highest altitude) for earth-orbiting spacecraft. These circular orbits, of radius $R_s$ ( ≈26,300 miles), are characterized by a 24 hour orbital period and are typically used by the equatorial, ‘‘stationary’’ class of communications spacecraft. Another class of communications spacecraft (the Russian ‘‘Molniya’’ class) employ inclined, highly eccentric orbits characterized by a 12 hour orbital period.

For what orbital conditions, if any, is the largest (apogee) radius of a Molniya orbit greater than the synchronous orbital radius $R_s$?

The solution below is from Andrew Mazzella. Note that it satisfies the constraint that the perigee radius must exceed the earth's radius. John Prussing, a Professor of Aero and Astro at Illinois, tells me that this constraint ‘‘is a prime recommendation of NASA’’.

**Please place figure number 2 here.**

**Jul 3**. Nob Yoshigahara wants you to figure out the area of X in the figure below.

**TR please insert figure from July**

An embarrassment of riches: I received a number of beautifully drawn solutions, all of which deserved publication. After some thought I decided on Kenneth Bernstein's.

**Please place figure number 3 here.**

## BETTER LATE THAN NEVER

**Jul SD**. I received several complaints about my answer that the central banks paid for the beer. The preferred answer is that patrons going the ‘‘wrong’’ way are paying. The argument being that central banks are too smart to permit official rates like these but the patrons would not check rates that carefully. I guess I was ascribing too much intelligence to beer drinkers and too little to central bankers (who presumably drink wine). My apologies.

## OTHER RESPONDERS

Responses have also been received from C. Bahne, R. Bart, A. Beris, G. Blondin, J. Bush, X. Dai, D. Detlefs, R. Dreselly, B. Durie, M. Egerton, E. Field, M. Fineman, M. Fountain, J. Grossman, J. Harmse, R. Hess, S. Hsu, P. Jung, L. Krakauer, J. Kusters, M. Lindenberg, G. Lum, N. Markovitz, D. Marron, J. Miller, C. Muehe, A. Peralta, J. Prussing, C. Rife, K. Rosato, E. Sard, L. Schaider, R. Scheidenhelm, E. Shung, N. Spencer, H. Stern, B. Thorburn, M. Veall, D. Wellington, J. Wright, and R. Yaseen.

## PROPOSER'S SOLUTION TO SPEED PROBLEM

Khaki, silhouette, pigeonhole.