PuzzleCorner

Readers who enjoy combinatorial and other math problems having a chess theme (like Problem 1 in this issue) may be interested to know that Mario Velucci, in collaboration with Alessandro Castelli, is working on a new edition of *Math-Chess Bibliography* and seeks references of all kinds on related material. You can reach him at Via Emilia 106, I-56121 Pisa Italy or <velucchi@cli.di.unipi.it>.

Problems

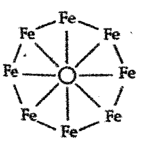
M/J 1. Mario Velucci wants you to place N queens on an N by N chess board so that the maximum number of vacant squares are Unattacked. For N=1, there are no vacant squares, and, for N=2 and N=3, all vacant squares are attacked. But, for N=4, you can leave 1 vacant square un-attacked and, for N=5, you can leave 3. How about N=10 and N=20 or possibly even N=30?

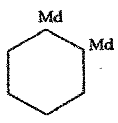
M/J 2. Just before mailing this problem to me in March 93, Eugene Sard purchased a bunch of 29-cent and 23-cent postage stamps and was surprised to note that the total was a (non-zero) whole number of dollars. What is the smallest number of stamps that Sard could have been purchasing?

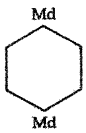
M/J 3. Nob Yoshigahara wants you to factor 123456789 into two five digit numbers.

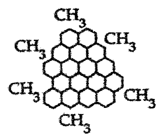
Speed Department

Winslow Hartford wants you to name the following "chemicals."









SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO: ALLAN GOTTLIEB NEW YORK UNIVERSITY 715 BROADWAY, 10TH FLOOR NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU

Solutions

1995 A/S 2. This problem as corrected in January reads:

ACROSS
1. A multiple of this number is obtained by removing the first digit and placing it after the last digit.
7. The year in the 20th century when Easter is earliest.
11. Divisible by 7, 11, and 13.
12. Multiple of 30 Down.
13. When added to 16 Across is equal to the sum of 23 Down and 25 Down.
14. See 26 Across.
15. A multiple of 9.
16. See 13 Across.
17. This number has the same first and last digits.
18. A multiple of 3.
19. Ten times 31 Across plus five times 13 Across.
21. Factorial 9.
24. Multiple of 28 Across.
26. Sum of 3 Down and 14 Across.
27. See 8 Down.
28. See 24 Across.
29. See 4 Down.
31. See 19 Across.
32. Equal to 22 Down.
33. $10^5 \times \pi$ to the nearest integer.

DOWN
1. The cube of a prime number.
2. A multiple of 17 Across.
3. A multiple of 7.
4. Sum of twice 21 Across and 29 Across
5. See 10 Down.
6. This number is equal to the sum of the cubes of its digits.
7. A cube number.
8. The sum of 15 Across and 27 Across.
9. See 20 Down.
10. A multiple of 5 Down.
19. A square number.
20. Ten times 9 Down plus 1.
22. Equal to 32 Across.
23. See 13 Across.
25. See 13 Across.
30. Factor of 12 Across.

[1]2	[2]8	[3]5	[4]7	[5]1	[6]4	[7]1	[8]9	[9]1	[10]3
[11]5	0	0	5	[12]3	0	8	[13]8	0	9
[14]7	8	6	[15]8	3	7	[16]7	4	1	9
[17]1	0	1	[18]8	1	[19]4	9	[20]1	5	5
[21]3	[22]6	[23]2	8	8	0	[24]9	0	[25]5	4
[26]5	8	4	7	[27]1	4	7	[28]1	8	5
[29]3	3	1	[30]2	7	0	[31]4	5	1	1
[32]6	8	3	8	[33]3	1	4	1	5	9

The following solution is from John Goodman. Reasoning (A and D mean Across and Down; n is used for any unknown digit; p through z for specific ones):

1. The following are immediately calculable: 7A=1913; 21A=362880; 33A=314159.
2. Since 10D starts with a 3, 5D must start with a 1 to be a factor.
3. 1A could thus end in 12 (its rotated multiple being 24); 13 (39); 14 (42); 16 (64); or 19 (95). 12, 16, and 19 can be eliminated due to definition 6D (no three-digit numbers beginning with 2, 6, or 9 are the sum of the cubes of their digits). 13 can also be eliminated because 1A would have to start with a 9 and multiplying by 3 (13 x 3 = 39) would add a digit. Hence 1A ends in 14 and starts with a 2. Back calculating gives 285714 (x 3 = 857142).
4. 6D must be therefore be 407.
5. 1D, a cube of a prime, is now 2nnn3nn. Testing shows that only 137 cubed (=2571353) has the two required digits.
6. 11A must be a multiple of (7 x 11 x 13) so 5nnn = 5005.
7. 17A is now 1n1, and 3D = 50n1 must be multiple of 7, and thus 5061.
8. 2D = 80n must be multiple of 17A, 1n1. Only two possibilities: 808 = 8 x 101, or 805 = 5 x 161, making 14A 786 or 756 making 26A 5847 or 5817.
9. 22D = 32A making them 68n8.
10. By definition 4D, 725760 + 3nnnn = 75nn87 so 29a = 3nn27 and 30d = 28.
11. 10D = (3 x 5d) so 12A starts with 1, 2, or 3. Only multiples of 28 with center zero digit are 308, 504, and 700 hence 308.
12. 7D, 18nnnnn4, is a cube and must be 264 cubed = 18399744.
13. 15A, qr7, is a multiple of 9. Digit q must be at least 5 because of definition 4D (75qn87 + 725760 = 3nn27). Also, since 10D (39n5nn19) = 3 x 5D (13rn8n73), r must be less than 4. The only two possibilities are 837 and 927. But since 8D = 15A + 27A, 9sn = (837 or 927) + nn7. 15A must be 837 and 27A = 1n7 and 8D must end in 4.
14. We have 5D x 3 = 10D or 133n8n73 x 3 = 39n5nn19; only solution is 5D = 13318173 and 10D = 39954519.
15. From definition 13A, st9 + 7nu9 = 2nvw + xyz5 so w = 3 and 32A = 22D = 6838.
16. By 4D again, 758n87 = 725760 + 33v27 so 4D = 758887, v = 1, 29A = 33127, and 18A = 81.
17. By definition 19A, n91n5 = 4pz10 + (st9 x 5), so 19A must begin with a 4 since (13A x 5) can be at most 4995.
18. 19D, 40nn1, is a square number and must be 201 squared = 40401, making 27A = 147 and 8D, 9s4, equal to 837 + 147 = 984.
19. Now 19A, 491n5, = 4pz10 + (5 x 8t9). Regardless of t, product is 4nn5 so p = 5.
20. 20D, 1tu51, = (10 x 1tun) + 1, so 9D ends in 5 making 19A = 49155.

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Puzzle

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21. 19A, 49155, = 45z10 + 4nn5. Only two possibilities are (809 x 5 = 4045) and (829 x 5 = 4245). So 13A is 809 or 829. Assume it is 829 (t = 2). Then 24A becomes 92 and 31A becomes 4501. 28A, a factor of 24A, would have to be 23 or 46 (u = 2 or 4). But by 13A, 829 + (7429 or 7449) = 2n13 + xy05 (z would be zero because of 19A 49155 = 45z10 + 4245). Both are impossible. Thus t = 0, 13A = 809, 24A = 90, and 31A = 4511.

22. By 13A, 809 + 74u9 = 2n13 + xy15, so u = 1, 16A = 7419, 9D = 1015, and 20D = 10151. Now 809 + 7419 = 8228 = 2n13 + xy15. Looking back at step 8, 23D must be 2113 or 2413. If it's 2113, 25D becomes 6115 and 28A becomes 11. But 11 is not a factor of 90, so 23D = 2413, 25D = 5815, 28A = 18, 26A = 5847, 14A = 786, 2D = 808 and 17A = 101.

1996 Jan 1. Now that you have just solved the yearly problem, take a crack at this variant from Philip Jacobs, who wants you to find numbers that can be formed using their own digits in a nontrivial way. That is, we do not want a trivial solution like

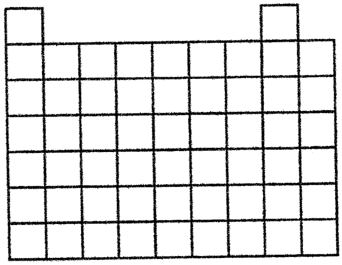
128 = 128

but do want the solution

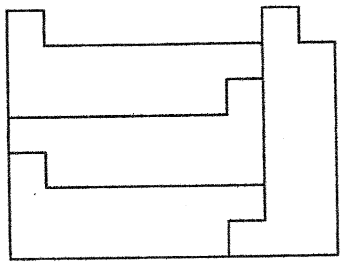
$128 = 2^{8\ 1}$

Several readers noticed that using exponents is a key and many claim that there are an infinite number of solutions (but I did not see any try a formal proof). Charles Rivers found the following string of 47 out of 48 consecutive numbers that can be so expressed, suggesting that there are indeed an infinite number of such solutions:

Jan 2. This problem appeared in Solomon Golomb's puzzle column in Johns Hopkins Magazine. You are to dissect the figure below into four congruent pieces.



Ken Rosato and John Boynton each found the following solution.



Other Responders

Responses have also been received from R. Anderson, C. Bahne, R. Bart, Rev. M. Buote, R. Campbell, D. Church, J. Datesh, M. Fountain, R. Hess, D. Hopkins, A. Ornstein, D. Plass, S. Portnoy, C. Rivers, J. Ryan, R. Sackheim, D. Savage, L. Schaider, A. Silva, and J. Varnick.

Proposer's Solution to Speed Problem

Ferrous wheel, Orthodox, Paradox, Hexamethylbathroomtile (what do you expect from a Course V man?).

117622	7^6-(2+1)^(2+1)	117638	7^6-	117654	7^6+1^15+4
117623	7^6+	117639	7^6-	117655	7^6+1^15+5
117624	7^6-	117640	7^6-	117656	7^6+1^15+6
117625	7^6-	117641	7^6-	117657	7^6+1^15+7
117626	7^6-	117642	7^6-	117658	7^6+1^15+8
117627	7^6-	117643	7^6-	117659	7^6+1^15+9
117628	7^6-	117644	7^6-	117660	7^6+1^6+10
117629	7^6-	117645	7^6-	117661	7^6+1^6+11
117630	7^6-	117646	7^6-	117662	7^6+1^6+12
117631		117647	7^6-	117663	7^6+1^6+13
117632	7^6-	117648	7^6-1^148	117664	7^6+1^6+14
117633	7^6-	117649	7^6+1^149	117665	7^6+1^6+15
117634	7^6-	117650	7^6+1^15+0	117666	7^6+1^6+16
117635	7^6-	117651	7^6+1^15+1	117667	7^6+1^6+17
117636	7^6-	117652	7^6+1^15+2	117668	7^6+1^6+18
117637	7^6-	117653	7^6+1^15+3	117669	7^6+1^6+19