PuzzleCorner

As I mentioned a few issues ago I attended a conference in Italy during June and before going, I listened to a few Italian tapes. For about two months, I probably averaged 15–20 minutes per day. I must say that the tapes worked and considerably enhanced my enjoyment during the trip. In fact I was disappointed when a group of us decided on a restaurant and the waiter spoke (excellent) English. I recommend the practice to everyone. The tapes I used were called "Listen and Learn Italian," but I have no reason to believe they are better or worse than any others.

To repeat an announcement from last issue, we are still low on Bridge problems.

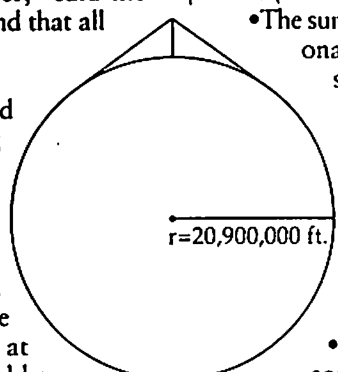
Problems

N/D 1. Here is an offering from Joseph Keilin, who writes:

On their way home from an evening of bridge, a couple was commiserating with each other. "Can you imagine that? We had 39 high-card points between the two of us, I played perfectly, and still we went down one at 3 no trump. And it wouldn't have made a difference if you had been declarer," said one. "The opponents played perfectly, also. Besides, it would have been even worse in any suit contract, again regardless of who was declarer," said the other. What was the hand that all four players held?

N/D 2. Perhaps Richard Hess is considering some flagpole-sitting in cold climates. He writes:

Imagine a rubber band stretched around the world and over the North Pole as shown at right. Given that the rubber band has had to stretch an extra foot to accommodate the North Pole, how high is the North Pole?



N/D 3. Warren Himmelberger just loves to have eight numbers add up to 260.

Write the numbers from 1 to 64 in the checkerboard square so that all columns and rows add up to 260, and also that the following groups of squares add up to 260:

+	O	+	/	\	+	O	+
O	+	/			\	+	O
+	/	□	*	*	□	\	+
/		*	□	□	*		\
\		*	□	□	*		/
–	\	□	*	*	□	/	–
O	–	\			/	–	O
–	O	–	\	/	–	O	–

- Any of the 70 arrangements of 2 by 4 squares.
- The sum of the 4 middle squares and 4 corner squares.
- The sum of any two adjacent half-diagonals (indicated by - - -).
- The sum of the interior half-diagonals (indicated by □).
- The sum of the top 3-square diagonals and the 2 corner squares enclosed (indicated by +), and the sum of the bottom 3-square diagonals and the 2 corner squares enclosed (indicated by –).
- The sum of the 4 2-square diagonals (indicated by o).
- The sum of the pairs of squares adjacent to the 4 middle squares (indicated by *).
- The sum of the pairs of squares midway between the edge squares and those pairs adjacent to the 4 middle squares (the 8 blank squares).

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO: ALLAN GOTTLIEB NEW YORK UNIVERSITY 715 BROADWAY, 10TH FLOOR NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU

Speed Department

Mike Auerbach has mechanized an old chestnut.

A condemned prisoner is placed in a cell with two unlocked doors and two computers. One door leads to freedom and the other to execution. Both computers know which door is which. One computer always tells the truth, the other always lies. The prisoner is allowed to ask only one question of either one of the computers. What should he ask?

Solutions

Jul 1. John Rudy wants to know how South is to make seven spades after West leads the diamond king.

North
♠ A K Q J
♥ A x x
♦
♣ Q J 10 9 8 7

West
♠
♥
♦
♣

East
♠
♥
♦
♣

South
♠ 10 9 x x x
♥
♦ A x x x x x
♣ A K

It does not seem possible to make the hand against all distributions. So Richard Thornton maximized the probability of success.

John Rudy's bridge problem requires unblocking clubs in a way that gives the highest probability of success. Although most readers will arrive at the correct solution using intuiton instead of calculation, readers of *Technology Review* should have no trouble with the calculation. Assuming that nothing about the distribution can be deduced from the bidding or other means (a peek?), the key is to remember that the numbers of combinations of m things taken n at a time is

$$C^m_n = \frac{m!}{(m-n)!n!}.$$

One strategy is to allow for a 4-0 trump split but admit failure if East has no diamonds. To guard against the 4-0 trump split South takes the first trick with the diamond ace, throwing a club from North. Then dummy is entered twice with a trump and two small hearts are trumped by South (using the 10 and 9). On the fourth lead of trumps North unblocks clubs by throwing a high club from South and then throws the remaining club on the ace of hearts. This allows

North to run four clubs for a win. This preferred strategy fails only if East has no diamonds, and the probability of such an event is

$$P = \frac{C_{13}^{18}}{C_{13}^{25}} = \frac{8568}{520030} = 0.0016$$

This calculation is based on the fact that the East hand has 13 of the 25 invisible cards and 18 of the 25 are not diamonds. Thus this stratedgy will fail only 0.16 percent of the time.

An alternate strategy is to trump the first trick and hope for a 3–1 or better trump split. In this case North has only enough trumps to lead them three times and South will have to lead the ace of clubs, before trumps are out, in order to unblock. This strategy will fail if the trumps are split 4–0, and after the initial lead the probability of such an event is computed as follows:

East has no trumps with probability

$$P_E = \frac{C_{13}^{21}}{C_{13}^{25}} = \frac{203490}{5200300} = 0.0391$$

West has no trumps with probability

$$P_W = \frac{C_{12}^{21}}{C_{12}^{25}} = \frac{293930}{5200300} = 0.0565$$

Since *both* East and West can not be void, the probability of one being void is P_E+P_W, so 9.56 percent of the time either East or West will be void in spades. The strategy will also fail if the clubs split 5–0 and the hand with no clubs has at least 2 trumps, an event that will occur about 4 percent of the time.

Conclusion: it is more than 100 times more likely for the second strategy to fail then for the first strategy to fail, but even the second strategy will work about 85 percent of the time. In this problem the probabilities are so different that it is intuitively obvious which strategy is most favorable, but in other cases the ability to calculate the best strategy is helpful, though not as helpful as a peek.

Jul 2. Chris Svenasgaard wants you to figure out who played whom and what the scores were in each game. Note that all games are intragroup and that (W, T, L, F, A, P) = (Wins, Ties, Losses, goals For, goals Against, 2W+T).

TEAM	GROUP A W	T	L	F	A	P
Sampdoria	1	1	0	2	0	3
Panathinaikos	0	2	0	0	0	2
Red Star	1	0	1	3	4	2
Anderlecht	0	1	1	2	3	1

TEAM	GROUP B W	T	L	F	A	P
Barcelona	1	1	0	3	2	3
Sparta Prague	1	0	1	4	4	2
Dynamo Kiev	1	0	1	2	2	2
Benifica	0	1	1	0	1	1

The following solution is from James Sinclair:

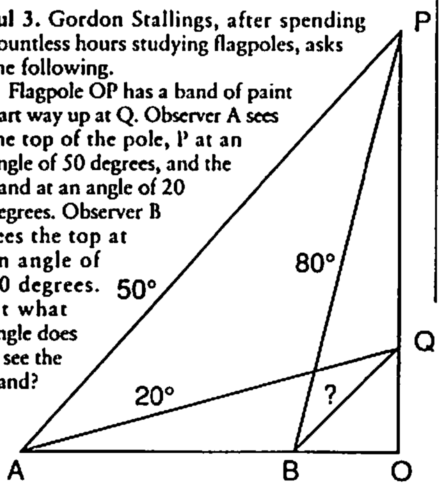
I refer to the teams by their names' first two letters, to shorten things a bit.

GROUP A: Pa had 2 scoreless ties, which must have been with the only other tiers, Sa and An. Sa and Re had the only wins, and Re couldn't have beaten itself, so Sa beat Re (2–0, since Sa's tie was scoreless). Leaving: Re beat An, 3–2.

GROUP B: Ba and Be had a scoreless tie (since Be's total goals For = 0). So Ba's other game was a 3–2 win, which must have been against Sp because only Sp had more than 2 goals Against. So Sp's other game was a 2–1 win against the only remaining loser, Dy. Leaving: Dy beat Be 1–0.

Jul 3. Gordon Stallings, after spending countless hours studying flagpoles, asks the following.

Flagpole OP has a band of paint part way up at Q. Observer A sees the top of the pole, P at an angle of 50 degrees, and the band at an angle of 20 degrees. Observer B sees the top at an angle of 80 degrees. At what angle does B see the band?



Many readers used trig tables to find that the angle is *about* 60 degrees; Harvey Amster shows us why it *is* 60 degrees:

Each observer can calculate the height of the pole in terms of his distance from its base and the angle that he sees to its top. Equate these expressions:

$$OP = AO \tan 50° = BO \tan 80°$$

Do the same for the spot on the pole:

$$OQ=AO \tan 20°=BO \tan \theta,$$

where θ is the angle to be determined. Thus,

$$\tan \theta=(AO/BO)\tan 20°=(\tan 80°)(\tan 20°)/\tan 50°$$

Since $\tan \phi=1/\tan(90° - \phi)$ for any angle ϕ,

$$\tan \theta=(\tan 40°)(\tan 20°)/\tan 10°.$$

By using standard trigonometry tables, one can readily find that $\theta=60°$.

However, numerical tables do not show that this is the *exact* answer. For that, use is made here of the well-known multiple-angle formulas,

$$\tan 2A=2t/(1-t^2),\ \tan 3A=(3-t^2)t/(1-3t^2),$$
$$\tan 4A=4t(1-t^2)/(1-6t^2+t^4),$$

where $t=\tan A$. In our case, let $A=10°$ so that

$$\tan \theta=(\tan 4A)(\tan 2A)/t=8t/(1-6t^2+t^4).$$

Although we don't have an exact value of t to put into this expression, we *can find the exact value of the expression itself.*

Equate the multiple-angle formula for tan 3A to $1/\sqrt{3}$, the well-known value of tan 30°. Then rearrange the resulting equation to

$$t^3=\sqrt{3}t^2+3t-1/\sqrt{3}.$$

Multiply this equation by t, and then use this same equation again for the t^3 in the resulting expression. The result is

$$1-6t^2+t^4=(8/\sqrt{3})t,$$

which is the denominator in the above expression for $\tan \theta$. Substituting it in gives the result: $\tan \theta=\sqrt{3}$. Therefore, $\theta=60°$ *exactly.*

Other Responders

Responses have also been received from H. Amster, E. Anderson, G. Blondin, M. Boas, Rev. M. Buote, W. Coffey, A. Cowen, W. Deane, F. Desimone, D. Edmonds, S. Feld, S. Feldman, E. Field, M. Fountain, J. Grossman, J. Harmse, W. Hartford, W. Hartford, C. Hess, L. Kells, J. Landau, M. Lindenberg, N. Markovitz J. Maynard, D. Merkle, D. Merkle, B. Metcalfe, L. Nissim, L. Norman, A. Ornstein, R. Palacios, A. Palmer, D. Plass, J. Prussing, J. Prussing, C. Rappaport, K. Rosato, E. Sard, L. Schaider, A. Shagen, I. Shalom, T. Sim, N. Spencer, L. Steffens, H. Stern, F. Tydeman, F. Verhoorn, C. Wang, S. Wang, B. Wegerer, C. Whittle, J. Wright, B. Wurzburger, and R. Yassen.

Proposer's Solution to Speed Problem

"If I were to ask the other computer 'Which door leads to freedom?' what would it say?"