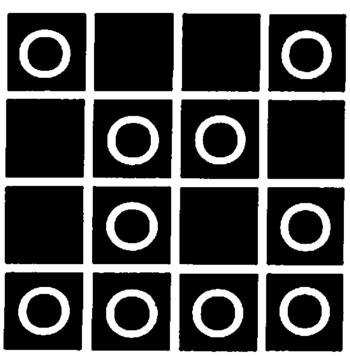
PuzzleCorner

This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (1, 9, 9, and 5) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 1994 yearly problem is in the "Solutions" section.

Problems

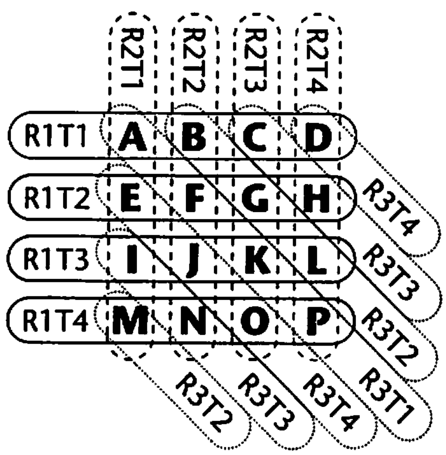
Y1995 How many integers from 1 to 100 can you form using the digits 1, 9, 9, and 5 exactly once each and the operators +, -, x (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1, 9, 9, and 5 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign does count as an operator.

JAN 1.



In the figure above the 10 circles are placed so that there are 10 rows, columns, and diagonal that each contain an even number (2 or 4) of circles. In 1928 Sam Loyd asked how to place the 10 circles so as to obtain the maximum number of rows, columns, and diagonals containing an even number of circles. Nob Yoshigahara, in the spirit of today's "less is more" generation, asks you to place the 10 circles so as to obtain the minimum number of rows, columns, and diagonals containing an even number of circles.

JAN 2. Jerome Dausman, a former U.S. Monopoly champion, was asked by Parker Brothers to judge a regional championship, which consists of three rounds of play each with four tables. When there are 16 contestants the following pairings are used.



(The oval R3T2 contains the players who are to sit at table 2 during round 3.) This solution has no player playing the same opponent more than once. There were 16 contestants at the championship Dausman was to judge so the decision was made to have two three-player tables and two four-player tables in each of the three rounds. An added requirement was that each player would sit at at least one three-player table and at least one four-player table during the three rounds. Find a set of pairings so that the number of players to meet an opponent more than once (a "second pairing") is minimized.

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO: ALLAN GOTTLIEB
NEW YORK UNIVERSITY
715 BROADWAY, 10TH FLOOR
NEW YORK, N.Y. 10012,
OR TO: GOTTLIEB@NYU.EDU

Speed Department

Speedy Jim Landau wants to know why manhole covers are always round.

Solutions

Y1994. The following solution is from Roger Claypoole:

1 = 1^994	51 =
2 = 1+(9/9)^4	52 =
3 = 94-91	53 =
4 = 1^99*4	54 = 9*(4+1)+9
5 = 1^99+4	55 = 19+9*4
6 = 19-9-4	56 =
7 = (19+9)/4	57 = 49-1+9
8 = 9-1^49	58 = 99-41
9 = 9*1^49	59 = 49+1+9
10 = 9+1^49	60 =
11 = (9-1)/4+9	61 =
12 = 9-1^9+4	62 =
13 = 14-9/9	63 =
14 = 19-9+4	64 =
15 = 14+9/9	65 =
16 = (1+9/9)^4	66 =
17 = 9*4-19	67 = 9*9-14
18 = 9*(4-1)-9	68 = 49+19
19 = 1^4+9+9	69 =
20 = (9*9-1)/4	70 =
21 = 9+9+4-1	71 = (9+9)*4-1
22 = 1*(9+9+4)	72 = 1*(9+9)*4
23 = 41-9-9	73 = 1+(9+9)*4
24 = 19+9-4	74 =
25 = (1+99)/4	75 = 94-19
26 = 9*4-9-1	76 = (1+9+9)*4
27 = 4*9-9*1	77 = 1*9*9-4
28 = 1-9+9*4	78 = 91-4-9
29 =	79 =
30 = 49-19	80 = 9*9-1^4
31 = (1+9)*4-9	81 = 1^4*9*9
32 = 19+9+4	82 = 1^4+9*9
33 = 99/(4-1)	83 =
34 =	84 = 94-1-9
35 = 9*4-1^9	85 = 99-14
36 = 1^9*9*4	86 = 1-9+94
37 = 1^9+9*4	87 =
38 =	88 =
39 = 49-1-9	89 =
40 = (19-9)*4	90 = (1+4)*(9+9)
41 = 1-9+49	91 =
42 = 91-49	92 =
43 =	93 = 94-1^9
44 = 9*4+9-1	94 = 1^9*94
45 = (14-9)*9	95 = 1^9+94
46 = 1+9+9*4	96 = 1+99-4
47 =	97 =
48 = 49-1^9	98 = 99-1^4
49 = 1^9*49	99 = 99*1^4
50 = 49+1^9	100 = 99+1^4

A/S 1. In a high-stakes game of rubber bridge with N-S vulnerable, West leads the Spade King against 6NT. Jorgen Harmse wonders what dummy should play to the first trick?

♠ A 3 2
♥ A K Q 7
♦ 7 6 5
♣ 5 4 2
N

S
♠ 5 4
♥ 6 3
♦ A K Q 4
♣ A K Q J 6

Oh, my. The "expected" answer is that you duck to set up all sorts of squeezes (the technical term used by responders was to "rectify the count"). But Matthew Fountain shows that giving up the potential overtrick is too high a price to pay to decrease the (already low) odds of being set. He writes:

Dummy should play the ace, when defenders' diamonds split 3-3 (probability = (6!/3!)(20!/10!/10!)/(26!/13!/13!) = .3553 and then neither defender has five clubs (probability = 1-(2)(15!/5!/10!)/(20!/10!/10!)=.9675), North's spade A and hearts A, K, Q plus South's four diamonds and five clubs add up to 13 tricks. The probability of N-S making one overtrick worth 200 points is (.3553)(.9675) = .344.

When one defender has five clubs (probability = (2)(21!/8!/13!)(26!/13!/12!) = .03913) and then diamonds do not split 3-3 (probability = 1-(6!/3!/3!)(15!/10!/5!)/(21!/13!/8!) = .7049), N-S has 11 sure trick winners. The worst that can happen is that N-S will be set one trick, for which the penalty is 100 points. This set has the probability of (.03913)(.7049) = .0276. Let M be the value in points to N-S for making 6NT with no overtrick. The expected value in points to N-S for making 6NT with no ace is played first is (1-.0276)M+(22)(.344)-(100)(.0276) = 66.04+(.9724)M. This expected value exceeds M if M is less than 2392. But M is no more than 1640, the sum of 190 points for six tricks beyond six, 750 points bonus for vulnerable slam, and 700 points for winning a rubber in two games. In short, there is no need to consider whether a play other than the ace will be safer in respect to making the bid. The value of the frequent overtrick offsets the rare loss of making the bid.

A/S 2. Frederick Furland wants you to show that two WRONGs can add up to a RIGHT (at least cryptarithmetically).

Eugene Sard makes the (moral?) assertion that "there are far too many WRONGs that add up to a RIGHT" and proceeds to list the following 21 solutions.

WRONG	RIGHT	WRONG	RIGHT	WRONG	RIGHT	WRONG	RIGHT
12734	25468	25173	50346	25734	51468	37806	75612
12867	25734	25193	50386	25867	51734	37846	75692
12938	25876	25418	50836	25938	51876	37908	75816
24153	48306	25438	50876	37081	74162	49153	98306
24765	49530	25469	50938	37091	74182	49265	98530
						49306	98612

He arrived at these solutions by expanding the equation 2(WRONG)=RIGHT in powers of 10, collecting like terms, and dividing by 20: N+10 O+1000 W=(T+10 H+98 G)/20+50 I+400 R. Since T+10 H+98 G must be a multiple of 20, there are 36 possible combinations of T, H, and G without duplication. Each of the 36 combinations resulting in 0, 1, 2, or 3 solutions. The only three-solution case is for G=8, T=6 and H=7, and completely illustrates the method. The original equation becomes N+10 O+1000 W=43+50 I+400 R. Therefore, N=3, and dividing by 10 yields O+100 W=4+5 I+40 R. Therefore, the two possible values of O are 4 and 9, yielding after division by 5: 20 W=I+8 R and 1+20 W=I+8 R, respectively. For O=4, the 5 non-duplicating available numbers are 0, 1, 2, 5, and 9, whereas for O=9, they are 0, 1, 2, 4, and 5. It is readily seen that for O=4, there is one solution W=2, R=5, and I=0, and for O=9, there are two solutions W=1, R=2, I=5 and W=2, R=5, I=1. Thus for this case the three WRONGs are 25438, 12938, and 25938, whereas the three RIGHTs are 50876, 25876, and 51876, respectively.

Avi Ornstein noted that, in addition, two RIGHTs can make a WRONG (in many ways).

A/S 3. Here's one from Jeff Kenton (and his mother?).

Suppose someone offers to play you a game with three specially made dice. He tells you that each die has from 1 to 6 spots on each of its 6 faces, but that the faces are not necessarily all different. The dice are "fair" in that each face has a 1/6 chance of being on top when its die rolled. If you agree to play (but not before) he will let you examine the dice and choose one. He will then choose a different one and pay you 6 dollars each time you roll a higher number than he does. If he rolls higher, you pay him 5 dollars. Should you ignore what your mother told you about betting against people with funny dice, and play the game?

The following solution is from Leonard Nissim:

The stranger can arrange the three dice to be non-transitive; that is, B will beat A, C will beat B, and A will beat C, even overcoming the payoff in your favor (win $6 when you win, lose only $5 when you lose).

Here is one such arrangement: Die A has (1, 4, 4, 4, 4, 4). Die B has (2, 2, 2, 5, 5, 5). Die C has (3, 3, 3, 3, 3, 6).

Note that if you choose A, then the stranger chooses B. He wins 21/36 of the time, while you win 15/36 of the time. Your expected payorr is (15/36) x (6) + (21/36) x (-5) = -5/12.

Note that if you choose B, then the stranger chooses C. He wins 21/36 of the time, while you win 15/36 of the time, for the same expected payoff of -5/12.

Note that if you choose C, then the stranger chooses A. Worse for you than the other cases, the stranger wins 25/36 of the time and you win only 11/36 of the time. The expected payoff to you is worse, -59/36.

With these dice, the best you can do is an expected payoff of -5/12. (Of the various solutions for the stranger with this value to you, -5/12, this one is pleasing in that no ties can happen when you play.)

It is amusing to note that the stranger need not use all the numbers from one to six to ensure that you have a negative expected payoff. Here is an example using only 1, 2, 3, 4, and 5; the best choice you can make gives you an expected payoff of -5/18: Die A has (1, 1, 3, 4, 4, 4). Die B has (2, 2, 2, 3, 5, 5). Die C need not even be rolled, as it has (3, 3, 3, 3, 3, 3).

Other Responders

Responses have also been received from M. Brennan, T. Bundy, F. Carbin, M. Cassidy, C. Dale, J. Drumheller, J. Dunham, R. Eiss, A. Fabens, S. Feldman, S. Goldstein, J. Grossman, R. Hansen, W. Hartford, R. Hess, A. Katzenstein, J. Keilin, L. Kells, P. Kramer, B. Layton, G. Perry, K. Rosato, J. Rosenthal, J. Rudy, J. Ryan, J. Salem, J. Shwimer, A. Tracht, and J. Walker.

Proposer's Solution to Speed Problem

It is not possible to drop a round manhole cover down a round manhole. □