Objet Dart

This June is my class's 25th reunion and I was asked to write a page of "Puzzle Corner" for the book they are producing. In preparing for the column I reread the introductions from the first seven years of "Puzzle Corner" and I must confess that it was fun to escape back to those earlier, more carefree days. I found some of my old words touching, some boring, and occasionally some were quite surprising. I guess we really do change. If any of you have little vignettes of your past life stored away, I recommend that, when no one else is looking, you take some private time with your former self.

Problems

F/M 1. Dave Wachsman sent us a hand he played (as South) with his wife that was reported in Truscott's column in the *New York Times*.

North
♠ 8 2
♥ A 7 2
♦ Q 7 5
♣ K Q 9 4 2

West
♠ 9 7 3
♥ J 8 6 5
♦ K J 10 3 2
♣ J

East
♠ 6 5
♥ 10 4
♦ 9 8 4
♣ A 10 8 6 5 3

South
♠ A K Q J 10 4
♥ K Q 9 3
♦ A 6
♣ 7

Both sides were vulnerable. The bidding:

South	West	North	East
1 ♠	Pass	2 ♣	Pass
3 ♠	Pass	3 N.T.	Pass
4 ♥	Pass	4 N.T.	Pass
6 ♠	Pass	Pass	Pass

How does Mr. Wachsman bring home the slam after West leads the club jack?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU

F/M 2. John Prussing believes that the following puzzle, which was actually on the 1989 Putnam exam, seems about right for "Puzzle Corner."

A dart hits a square dartboard. If any two points on the dartboard have the same probability of being hit, what is the probability that the dart will land nearer to the center of the board than it does to an edge?

F/M 3. Our last problem is from my NYU colleague, Dennis Shasha, and can be found in his book, *The Puzzling Adventures of Dr. Ecco.*

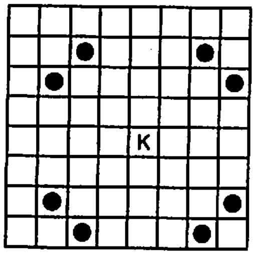
You are given 20 coins. Some are fake and some are real. If a coin is real, it weighs between 11 and 11.1 grams. If it is fake, it weighs between 10.6 and 10.7 grams. You are allowed 15 weighings on a scale (not a balance). You are to determine which coins are real and which are fake.

Speed Department

Speedy Jim Landau wants to know why the Kindom of Metrica chose to use a long, flat piece of wood as their Royal Standard of Length.

Solutions

OCT 1. We start with a chess problem (which may well be a computer problem) from Victor Barocas. It is well known that a knight can tour the chess board, touching each square once and only once, and beginning and ending on the same square. Consider now the generalized knight K(m,n), $m \leq n$, which moves m spaces along one axis of the board and n spaces along the other (the normal knight is K(1,2); also see diagram). For what values of m and n can the knight tour the board?



Moves available to K(2,3) at position e4.

The following solution is from Ken Kiesel:
Each time the knight moves one space in either direction, it moves to a square of the opposite color. Therefore, m + n must be odd, or the knight can reach only squares of one color.

A knight on one of the four center squares can move a maximum distance of 4,4. Thus, the possible solutions are 0,1 0,3 1,2 1,4 2,3 and 3,4.

K(0,1) is the only solution besides K(1,2), unless there is a bug in my program. K(0,3) obviously doesn't work.

Each corner square is accessible to only 2 squares. Therefore, one of these squares must be used to enter the corner, and the other to exit it. For K(3,4), diagonally opposite corners reach the same 2 squares; therefore it cannot tour the board. This leaves only K(1,4) and K (2,3).

The program I wrote found a solution to K(1,2) in about 7 seconds on my XT clone. It eliminated K(2,3) in less than that. In fact, K(2,3) can be shown not to work quite easily. Starting at a8, the possible first moves are to c5 or d6. Since the two are equivalent, choose d6. If the second move is not to g8, g8 will be left with only one accesible square, f5. It will be impossible to enter and exit it later. Therefore, the second move must be to g8. The only possible third move is to f5. If the fourth move is not to h7, then it will have only one entry square, f4. But if it is to h7, then b7 will have only one entry square, c4.

With two rapid results from the program, plus a test of the trivial K(0,1) and K(0,3) to further verify program operation, I launched it on K(1,4). It finished after about 80 hours, having found no solution.

OCT 2. Gordon Rice wonders how many Pythagorean triangles you can find in which one of the three sides is 1991.

The following is from Jerry Grossman, who has secret plans involving this problem:
There are five Pythagorean triangles one of whose sides is 1991: (10860,1991,11041), (1980,209,1991), (180180,1991,180191), (1982040,1991,1982041), and (16320,1991,16441). The "easiest" way to learn this is by asking a computer algebra package to solve (in integers) $x^2+y^2=1991^2$ and $u^2+1991^2=v^2$. I asked Maple and it gave me these answers. Here is how to do it more or less by hand:

Since 1991=11*181, we can look for three kinds of solutions: primitive solutions (i.e., no common factor to the three sides) in which one side is 1991, solutions in which one side is 11 (and then multiply all sides by 181 to obtain the desired triangle), and solutions in which one side is 181 (then multiply by 11).

The fundamental fact we need to use is that all primitive triangles are of the form ($2mn$, m^2-n^2, m^2+n^2), where m and n are relatively prime positive integers.

Let's start with the solutions in which one side is 11. Clearly 2mn cannot equal 11, nor can m^2+n^2. So the only possibility here is that m=6 and n=5, giving us the triangle (60,11,61), and hence the solution to the original problem (60,11,61)*181= (10860,1991,11041).

Next we look for solutions in which one side is 181. Again we cannot have 181=2mn, but we can have $181=m^2+n^2$, with m=10 and n=9 (this is the only way). This gives a triangle (180,19,181), yielding the second solution (1980,209,1991). We can also have $181=m^2-n^2$, only by letting m=91 and n=90. This gives us the triangle (16380,181,16381), and hence our third solution (180180,1991,180191).

Finally we look for primitive solutions with one side being 1991. Again, 1991 is not 2mn. A computer search shows that 1991 cannot equal m^2+n^2.

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Symposium on Kids and Computers" in late September. She delivered the keynote address for the Conference on Cultural Issues in Psycholanalytic Theory in New York in early October. . . . **Jessica Wang** and **Bruce Bimber**, STS graduate students, shared the first Siegel Prize for their essay on the relationship of science and technology to political events in the U.S. The prize was established in memory of Benjamin M. Siegel, '38, PhD '40, a pioneer in electron microscopy and until his death in 1990, professor emeritus of applied engineering and physics at Cornell. Wang's essay recounted the security problems of the scientist E.U. Condon during the loyalty investigations of the McCarthy period. Bimber's essay dealt with the development, philosophy, and politics of the Office of Technology Assessment. . . . **Barbara Masi**, '86 (II), SM '88, **Wade Roush**, and **Ken White** presented papers on "Boundaries and Paradigms in Artificial Intelligence Research" at the 4S meeting November 15 at the Cambridge Center Marriott.—Phyllis Klein, STS Program, MIT, Rm. E51-128, Cambridge, MA 02139.

Deceased

The following deaths have been reported to the Alumni/ae Association since the *Review* last went to press:

G. Hobart Stebbins, '17; September 26, 1991; Bellevue, Wash.
Henry R. Lacey, '18; March 13, 1991; Melbourne, Fla.
Webster W. Frymoyer, '21; October 5, 1991; Arlington Heights, Ill.
Eastman Smith, '22; September 18, 1991; Mountain Home, Ark.
Richard H. Frazier, '23; October 24, 1991; Winchester, Mass.
George H. Fuller, '25; May 30, 1991; Colonia, N.J.
Hyman Katz, '25; January 6, 1987; North Quincy, Mass.
Seward S. Merrell, '25; October 5, 1991; Saint Petersburg, Fla.
W. Alan Williamson, '26; September 19, 1991
Marion E. Knowles, '27; September 18, 1991; Akron, Ohio
Ralph W. Stober, '27; October 1, 1991; Newton Highlands, Mass.
Howard S. Root, '28; September 24, 1991; Harrington Sound, Bermuda

Henry D. Addison, '30; October 10, 1991; Northampton, Mass.
Robert A. Lytle, '30; June 14, 1991; Grosse Point, Mich.
Watson E. Slabaugh, '30; August 31, 1991; Mansfield, Ohio
D. Malcolm Fleming, '33; August 12, 1991; Rockville Centre, N.Y.
Charles N. Debes, '35; August 31, 1991; Rockford, Ill.
Joseph K. Raes, '35; August 3, 1991; Las Vegas, Nev.
Albert J. Del Favero, '36; September 18, 1991; Vista, Calif.
O. William Muckenhirn, '37; September 2, 1991; Toledo, Ohio
Robert D. Williams, '37; September 17, 1991; Lake Helen, Fla.
Robert R. Chase, '39; January 12, 1991; Austin, Tex.
Joseph W. Harrison, '39; October 14, 1991; New London, N.H.
Charles A. Lawrence, '39; July 12, 1991; Seattle, Wash.
Edward A. Ruckner, '41; September 12, 1991; Annapolis, Md.
David G. Edwards, '42; December 29, 1990; Pacific Grove, Calif.
Anthony P. Barbato, '44; July 17, 1991; Kettering, Ohio
Warren H. Howard, '44; September 29, 1991; Sunapee, N.H.
Louis H. Roddis, Jr., '44; September 15, 1991; Charleston, S.C.
Henry F. Lloyd, '46; June 12, 1991; St. Augustine, Fla.
Wilton M. Fraser, '47; February 14, 1991; Naples, Fla.
C. Gregory Bassett, Jr., '48; September 25, 1991; Hilton Head Island, S.C.
Aubert L. Mowry, '48; March 16, 1991; Northridge, Calif.
Warren W. Houghton, '49; October 20, 1991; Manchester, Mass.
Philip A. Lynn, '49; October 29, 1991; Reading, Mass.
William B. Martz, '50; May 2, 1991; Winchester, Mass.
Douglas Porter, '50; June 10, 1991; Watertown, Mass.
Morgan L. Foster, '51; January 11, 1990; Meadville, Pa.
Peter Bishop, '54; September 29, 1991; Falmouth, Maine
Francisco Torras, '54; October 3, 1991; Fairfield, Conn.
Joseph A. Kissinger, Jr., '55; June 1, 1990; La Habra, Va.
J. William A. Tyler, '55; February 1, 1990; Monroe, Ohio
John A. Welsh, '55; September 26, 1991; Richardson, Tex.
Herbert Curt Burrowes, Jr., '56; October 1, 1991; Concord, Mass.
Stanley L. Lopata, '56; October 9, 1991; Natick, Mass.
Charles V. Brown, '58; July 27, 1991; Bridge of Allan, Stirling, Scotland
Marvin H. Cantor, '59; May 16, 1991; Rockville, Md.
Irving Levinson, '61; February 26, 1991; Silver Spring, Md.
Frederik O. Jeppesen, '64; April 23, 1991; Denmark
Elsie P. Arnold, '65, February 25, 1991; Richmond, Ontario
George T. Onega, '70; March 23, 1990; Slidell, La.
Ronald G. Jackson, '71; June 1, 1991; Pymble, New South Wales, Australia
Daniel R. Siegel, '83; October 20, 1991; Wilmette, Ill.
Peter L. Armstrong, '84; August 27, 1991; Waban, Mass.

PUZZLE CORNER

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If $1991=m^2-n^2=(m+n)(m-n)$, then either we have $m+n=1991$, with $m-n=1$; or we have $m+n=181$ with $m-n=11$. This gives $m=996$, $n=995$ in the first case; $m=96$, $n=85$ in the second. The resulting triangles are our fourth and fifth solutions: (1982040,1991,1982041) and (16320,1991,16441).

OCT 3. That famous riverboat gambler, Bob High, was inspired by 1989 JUL 5 to ask a two-part question about shuffling cards. First, in a shuffled deck, what is the average (expected) number of cards occupying their original position? (This is to ask, for n = 52, what is the average number of fixed points of a permutation of n things.) Second, which is more likely in a random shuffle (permutation) of n things: exactly one fixed point, or exactly none?

Gordon Rice has a fine analytic proof, a copy of which can be obtained from Faith Hruby at *Technology Review*. Curiously, Rice was in the process of formulating a similar problem when he read OCT 3. The following shorter solution is from John Chandler, who believes he might be a 25-year veteran as a reader of "Puzzle Corner."

Consider any specific card. After a randomizing shuffle, its chance of being in its original position is simply 1/52. Obviously, the a posteriori probability of a second card being in *its* original position will depend on the actual position of the first card. Still, there is no preferred treatment of any of the cards, so the overall expectation of cards remaining in position after the shuffle must be just 52 x 1/52 = 1. For n << 52, this is easily proven by enumeration of all the permutations of n things. Moreover, it is simple to write down recursion formulas for the count of permutations with a given number of fixed points in terms of the counts for smaller n. For example, N(n,1) = N(n-1,0) * n, N(n,2) = N(n-2,0) * n * (n-1) / 2, and so on. The table begins:

n =	0	1	2	3	4	5	6	7
f=0	1	0	1	2	9	44	265	1854
1		1	0	3	8	45	264	1855
2			1	0	6	20	135	924
3				1	0	10	40	315
4					1	0	15	70
5						1	0	21
6							1	0
7								1

This suggests a further formula: $N(n,0) = N(n,1) + (-1)^n$, and that can, in fact, be proved by induction. The answer to the second question is, thus, that exactly one fixed point is (slightly) more probable if n is odd and exactly none if n is even.

Better Late Than Never

OCT SD. Dan Drucker notes a typo: S should be (D-1)/2 not (D+1)/2.

Other Responders

Responses have also been received from Matthew Fountain, Coe Wadelin, Mayer Wantman, Frank Carbin, Winslow Hartford, John Woolston, Eric Lund, Jim Landau, Ken Rosato, Steven Feldman, Rolph Person, William Waite, Ronnie Haige, Eugene Sard, Max Hailperin, Warren Jasper, Scott Berhenblit, Avi Ornstein, Thomas Lewis, Alan Friot.

Proposer's Solution to Speed Problem

Because a plank's constant.