ALLAN J. GOTTLIEB, '67

Cleaning Out the Problem Attic

In reviewing the May/June issue in preparation for the solution section below, I noticed that the following two personal contributions from the readers had inexplicably not appeared. Sorry for the delay. One of Mary Lindenberg's water colors appears in the 1990 American Press Calendar. She has kindly send a copy of the calendar to me; all three pictures, which depict various New England settings, are lovely. Congratulations.

Matthew Fountain writes that a house across the street has been purchased by a young engineer who, with the part time help of his three brothers and *seven* sisters, improved the foundation, planted trees, and put in a driveway and retaining wall. Knowing the demands put on the parents of just two boys, my hat goes off to the engineer's mother and father.

As we reported in July, "Puzzle Corner" is being phased out due to increased pressure for space in the Alumni section of *Technology Review* and the current issue contains the last installment of this column. As a result there are no new problems this month. I doubt that there are very many people with considerable experience in gracefully terminating a column that has been running continuously for 23 years; certainly I am not one. Let me conclude by thanking my faithful readers whose contributions have been the heart and soul of "Puzzle Corner" for all these years and also thank everyone who has written to me (and to the editors) after hearing that the column is to be discontinued. Thank you all.

Solutions

M/J 1. Doug Van Patter reports that most declarers in the 1989 Cherry Hill Regional failed to make six hearts on the deal shown below. West leads the jack of diamonds (his highest card). Is there a chance of making 12 tricks?

North	South
♠ A 8	♠ 5 3
♥ Q J 10 4 3	♥ A K 7 5 2
♦ K 10 6	♦ A 7 4
♣ A K 10	♣ Q J 5

The following solution is from Jonathan Hardis: Win the opening lead with the K of diamonds in dummy. Draw trump in 3 rounds, and play an additional round of hearts for a total of four. take the three quick-tricks in clubs, the ace of spades, and then play the low spades. After this point each player has five cards left, and East is marked for the queen of diamonds.

North	South
♠	♠
♥ x	♥ x
♦ 10 6	♦ A 7
♣	♣

If East takes this spade trick, you win. He is out of hearts, a club or spade return allows you to rough and to discard the diamond loser. A diamond return is a free finesse, since he is marked for the queen. If West takes the spade trick, you also win on a club or spade return. However, a diamond return requires some luck. Play the 10. If East lets it ride, you win. If East covers with the queen, take the trick with the ace. The remaining 7 of diamonds may or may not be a winner.

Also solved by Doug McMahon, Nike Agman, Richard Hess, Winslow Hartford, Eric Lund, Steven Feldman, Frederick Furland, Matthew Fountain, John Chandler, Daniel Loeb, Bill Huntington, Eugene Biek, and the proposer.

M/J 2. Randall Whitman proposes the following generalization of 1989 F/M 2. For each positive integer *n*, consider writing the integers from 1 to *n* inclusive and let *f*(*n*) be the number of times the digit 1 was used. For what values of *n* does *f*(*n*) = *n*?

John Chandler found them all and writes: Obviously, the first is n = 1. After that, they get scarce, and the next is n = 199,981. To see that, it is useful to count the 1's in each column separately, so that we can make use of the "clumping." For example, all the ten's digit 1's from 1-100 lie in 10-19, and there are exactly 10 of them. Thus, we see that f(9) = 1, f(99) = 10 × f(9) + 10 = 20, f(999) = 10 × f(99) + 100 = 300, and so on. It is easy to see that the ratio f(n)/n reaches a local minimum each time n reaches $10^x - 1$ (or $10^x - 1$ modulo 10^{x+1}) and then grows faster than average until n reaches $2 \times 10^x - 1$ (or modulo). From the values shown above, it is clear that the minimum is x/10 (plus a little) and that there will be no solutions for $n > 10^{10}$. Also, the local maximum at $2 \times 10^x - 1$ is (5 + x)/10 plus a little, so the first solution after n = 1 must be about 2×10^5. We get f(199,999) = 200,000, so we must back off past the last number ending in "1" to get f(n) = n and then back off again to get numbers with f(n) < n. Thus, all numbers from 199,981 through 199,990 are solutions, as well as 200,000 and 200,001. From there to 300,000, it is clear that f(n) = 200,000 + f(n-200,000), and so on up to 10^6, at which point, the ratio climbs again from the low of 0.6. Since f(999,999) = 600,000, and gains by 50,000 per 100,000, we find that f(1,599,999) = 1,600,000, and we pick up ten more solutions: 1,599,981 through 1,599,990. The ratio then continues to climb until n = 1,999,999

SEND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012, OR TO: gottlieb@nyu.edu

Falmouth, Mass.
Andrew T. Regan, '33; May 15, 1990; Kingsport, Tenn.
Simeon I. Rosenthal, '33; December 28, 1989; Boynton Beach, Fla.
Stanley H. Walters, '33; November 8, 1989; East Sullivan, N.H.
Vito P. Battista, '34; May 24, 1990; Brooklyn, N.Y.
Julian A. Dorr, '34; March 24, 1990; Punta Gorda, Fla.
Joseph A. Serrallach, '34; December, 1989
John R. Whitney, '35; February 1, 1989; Pompano Beach, Fla.
Sydney R. Karofsky, '37; June 14, 1990; Weston, Mass.
David J. Whitney, '37; May 28, 1989; Bristol, N.H.
William A. Davis, '39; May 26, 1990; Lincoln, Mass.
Donald W. Waterman, '39; October 10, 1989; Easton, Conn.
Walter M. Foster, '40; 1985; Annapolis, Md.
Hrant Isbenjian, '40; May 15, 1990; Stamford, Conn.
Barton L. Weller, '40; May 25, 1990; Easton, Conn.
Charles N. Gilligan, '41; June 4, 1990; Arlington, Mass.
Knut J. Johnsen, '41; June 6, 1990; Newburgh, N.Y.
John E. Demoss, '42; April 19, 1990; Chester, Mass.
Charles R. Stempf, '42; August, 1988; Newport, Australia
John O'Meara, '43; April 26, 1990; St. Louis, Mo.
John Farley, '44; June 2, 1990; Westfield, N.J.
Robert J. Horn, '44; June 8, 1990; Concord, Mass.
Arthur F. Peterson, Jr., '44; January 3, 1989; Star Lake, N.Y.
Robert J. Reilly, '44; April 19, 1990; Riviera Beach, Fla.
John Upton, Jr., '44; May 24, 1990; Pittsburgh, Pa.
Margaret E. Knutzen, '47; 1976; Bronxville, N.Y.
Jerzy C. Kuczynski, '47; May 16, 1990; South Bend, Ind.
Donald Marshall, '48; April 22, 1990; La Jolla, Calif.
Joseph V. Yance, '48; December 7, 1989; Alexandria, Va.
David D. Gaillard, II, '49; June 3, 1990; Washington, D.C.
Stanley A. Murray, '49; April 11, 1990; Kingsport, Tenn.
Syed M.S. Alvi, '50; December, 1987; Woodland Hills, Calif.
Francis L. Fleming, Jr., '50; March 13, 1989; Manhattan Beach, Calif.
Richard S. Paull, '50; 1987
Fred J. Rayfield, Jr., '50; May 24, 1990; Jericho, Vt.
Federico G. Baptista, '51; March 26, 1981; Caracas, Venezuela.
Jurgen Elkan, '51; May 26, 1990; West Newton, Mass.
Richard R. Fidler, '51; June 13, 1990; Andover, N.H.
Robert J. Greaney, '51; May 17, 1990; Rockville, Md.
J. Arthur Tache, '52; August 30, 1989; St. Clair, Mich.
Sergio F. Valdes, '52; November 24, 1989; Canyon Country, Calif.
Robert N. Noyce, '53; June 3, 1990; Austin, Tex.
G. Lowell O'Daniel, '53; May 19, 1990; Vergennes, Vt.
David W. Dennen, '54; June 21, 1990; Cambridge, Mass.
Russell E. Long, '54; May 22, 1990; Holbrook, Mass.
William F. Stuart, Jr., '55; September 14, 1989; Alpharetta, Ga.
John O. Morin, '56; May 2, 1990; Hingham, Mass.
John D. Crowley, '57; April 21. 1990; New London, Conn.
Donald L. Jarrell, '57; December 3, 1989; McLean, Va.
Don W. Smith, '57; June 10, 1990; New York, N.Y.
William H. Moore, '58; 1989; Princeton, N.J.
James F. Hurley, III, '59; September 15, 1989; Chula Vista, Calif.
David I. Weisblat, '59; April 21, 1990; Galesburg, Mich.
Alfred J. Diefenderfer, '61; November 16, 1989; Fullerton, Calif.
Alan W. Carlson, '62; June 4, 1990; Beverly, Mass.
George Brydon, '64; October 27, 1989; Don Mills, Ontario, Canada
Paul M. Ledoux, Jr., '66; January 18, 1990; Arlington, Mass.
Nils G. Wahlstrom, '69; 1990; Lidingo, Sweden
Cyrus Behain, '72; September 2, 1989; San Clemente, Calif.
Joyce D. Waye, '75; September 25, 1989; Boston, Mass.

and then falls back so that f(2,599,999) = 2,600,000, and we pick up two more: 2,600,000 and 2,600,001. Similarly, the ratio hits another minimum at 10^7 and climbs up again to give a solution of 13,199,998. On the subsequent descent, we cross the break-even point at 35,000,000 and then repeat the initial pattern: 35,000,001, 35,199,981 through 35,199,990, 35,200,000, and 35,200,001. The next ascent gives 117,463,825. There's another break-even at 500,000,000, which repeats the initial pattern again: 500,000,001, 500,199,981 through 500,199,990, 500,200,000, 500,200,001, 501,599,981 through 501,599,990, 502,600,000, 502,600,001, 513,199,998, 535,000,000, 535,000,001, 535,199,981 through 535,199,990, 535,200,000, and 535,200,001. The next ascent gives a solution of 1,111,111,110—and that's the last one, since the following ascent reaches the minimum at 10^{10} without hitting a solution.

Also solved by Jonathan Hardis, Keith Price, Richard Hess, Daniel Loeb, Winslow Hartford, Ken Rosato, Jim McNamara, Matthew Fountain, Bill Huntington, Steven Feldman, Michael Baumann, Harry Zaremba, Bob High, Nob Yoshigahara (who has a different generalization), and the proposer.

M/J 3. Richard Hess has a drinking problem he wants us to help him solve. Consider three containers that hold 15 pints, 10 pints, and 6 pints. The 15-pint container is full and the other two are empty (15,0,0). Through transferring liquid among the containers you are to measure out exactly 2 pints to drink, drink the 2 pints, and continue transferring liquid to end up with 8 pints in the 10-pint container and 5 pints in the 6-pint container (0,8,5).

David Webster sends his answer, adding "We had fun solving this one, using beer as the liquid!"

15 pt	10 pt	6 pt	
15	0	0	
9	0	6	
9	6	0	
3	6	6	
3	10	2	drink
3	10	0	
3	4	6	
7	0	6	
7	6	0	
1	6	6	
1	10	2	
11	0	2	
11	2	0	
5	2	6	
5	8	0	
0	8	5	

Also solved by Doug McMahon, Keith Price, James Walker, Robert Moeser, George Wood, Thomas Black, Walter Cluett, Mary Lindenberg, Robert Bart, Richard Hess, Steven Feldman, Ken Rosato, Jim McNamara, Joel Shwimer, Dave Pinckney, Peter Tzanetos, Richard Kruger, Rong Ho, Ermanno Signorelli, Matthew Fountain, John Chandler, Daniel Loeb, Bill Huntington, Eugene Biek, Michael Baumann, Roy Sinclair, Angel Silva, Harry Zaremba, Robin Pitcher, Thomas Jabine, David Gluss, Richard Boyd, Bob High, Marlon Weiss, Jeffrey Harris, and the proposer.

M/J 4. Gordon Rice wants you to extend the following sequence of Pythagorean triangles at least four more steps.

3	4	5
20	21	29
119	120	169
696	697	985
4059	4060	5741
23660	23661	33461

I guess it is fitting that the very last problem gave rise to a large number of exceptionally fine solutions. There were several fine analytic solutions that space considerations preclude printing (but I will send copies if requested) as well as heuristics and extensive computer searches. The following solution, from David Harris (an entering freshman), has a little of each:

Let the three legs be a, a + 1, and a + n. Then, you can solve for a + n:

$$(a)^2 + (a + 1)^2 = (a + n)^2$$

$$n^2 + 2an - (a + 1)^2 = 0$$

$$n = \frac{-2a \pm \sqrt{4a^2 + 4(a + 1)^2}}{2}$$

; n is chosen to be greater than zero. Therefore:

$$n = \sqrt{2a^2 + 2a + 1} - a, \text{ or}$$

$$a + n = \sqrt{2a^2 + 2a + 1}$$

Thus if a + n is an integer, the three lengths form a Pythagorean triplet. Next, I noted that after the first few terms the ratios between successive "a" values monotonically approached an asymptote near 5.82 from above. I wrote a program that uses the ratio between the prior two a's to guess the next term. The algorithm is extremely efficient: it can calculate new terms faster than it can write them to the screen on a Mac II. Below are listed the first 24 triplets. After 24 triplets, my 80-bit floating point numbers were not sufficiently precise.

Also solved by Keith Price, Mary Lindenberg, Robert Bart, Robert Oliver, George Ropes, Mark Lively, Scott Maley, James Wilcox, Richard Hess, Winslow Hartford, Eric Lund, Steven Feldman, Frederick W. Furland, Gerald Leibowitz, N. F. Tsang, John Granlund, Frank Carbin, Charles Piper, Jim Landau, David Wagger, Matthew Fountain, John Chandler, Bill Huntington, Roy Sinclair, Mary Lindenberg, Avi Ornstein, Angel Silva, Walter Nissen, Harry Zaremba, Robin Pitcher, David Gluss, Richard Boyd, Jim Landau, Bob High, and the proposer.

Better Late Than Never

A/S 2. Victor Christensen believes that "all diagonals" in a square means only the main and antidiagonal. The better late than never remark given in May/June included diagonals that "wrap around" the square. For example in a 3 by 3 square one such diagonal would be [(1,2),(2,3),(3,1)].

APR 2. Jim Landau and Robert Bart have responded.

APR 3. David Gluss and Robert Bart have responded.

APR 4. David Wagger and Robert Bart have responded.

Triplet #1:	3	4	5
Triplet #2:	20	21	29
Triplet #3:	119	120	169
Triplet #4:	696	697	985
Triplet #5:	4059	4060	5741
Triplet #6:	23660	23661	33461
Triplet #7:	137903	137904	195025
Triplet #8:	803760	803761	1136689
Triplet #9:	4684659	4684660	6625109
Triplet #10:	27304196	27304197	38613965
Triplet #11:	159140519	159140520	225058681
Triplet #12:	927538920	927538921	1311738121
Triplet #13:	5406093003	5406093004	7645370045
Triplet #14:	31509019100	31509019101	44560482149
Triplet #15:	183648021599	183648021600	259717522849
Triplet #16:	1070379110496	1070379110497	1513744654945
Triplet #17:	6238626641379	6238626641380	8822750406821
Triplet #18:	36361380737780	36361380737781	51422757785981
Triplet #19:	211929657785303	211929657785304	299713796309065
Triplet #20:	1235216565974040	1235216565974041	1746860020068409
Triplet #21:	7199369738058939	7199369738058940	10181446324101389
Triplet #22:	41961001862379596	41961001862379597	59341817924539925
Triplet #23:	244566641436218639	244566641436218640	345869461223138161
Triplet #24:	1425438846754932240	1425438846754932241	2015874949414289041