Cleaning Out the Problem Attic

In reviewing the May/June issue in preparation for the solution section below, I noticed that the following two personal contributions from the readers had inexplicably not appeared. Sorry for the delay. One of Mary Lindberg’s water colors appears in the 1990 American Press Calendar. She has kindly send a copy of the calendar to me; all three pictures, which depict various New England settings, are lovely. Congratulations.

Matthew Fountain writes that a house across the street has been purchased by a young engineer who, with the part time help of his three brothers and seven sisters, improved the foundation, planted trees, and put in a driveway and retaining wall. Knowing the demands put on the parents of just two boys, my heart goes out to the engineer’s mother and father.

As we reported in July, “Puzzle Corner” is being phased out due to increased pressure for space in the Alumni section of Technology Review and the current issue contains the last installment of this column. As a result there are no new problems this month. I doubt that there are very many people with considerable experience in gracefully terminatating a column that has been running continuously for 23 years; certainly I am not one. Let me conclude by thanking my faithful readers whose contributions have been the heart and soul of “Puzzle Corner” for all these years and also thank everyone who has written to me (and to the editors) after hearing that the column is to be discontinued. Thank you all.

Solutions

MJ 1. Doug Van Patter reports that most decliners in the 1989 Cherry Hill Regional failed to make six hearts on the deal shown below. West leads the jack of diamonds (his highest card). Is there a chance of making 12 tricks?

SEND COMMENTS TO ALLAN J. GOTTLIEB, 67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012. OR TO: gottlieb@nyu.edu

The following solution is from Jonathan Hardis: Win the opening lead with the K of diamonds in dummy. Draw trump in 3 rounds, and play an additional round of hearts for a total of four. Take the three quick-tricks in clubs, the ace of spades, and then play the low spades. After this point each player has five cards left, and East is marked for the queen of diamonds.

If East takes this spade trick, you win. He is out of hearts, a club or spade return allows you to rough and to discard the diamond loser. A diamond return is a free finesse, since he is marked for the queen. If West takes the spade trick, you also win on a club or spade return. However, a diamond return requires some luck. Play the 10. If East lets it ride, you win. If East covers with the queen, take the trick with the ace. The remaining 7 of diamonds may or may not be a winner.

Also solved by Doug McMahon, Nike Agman, Richard Hess, Winlow Hartford, Eric Lund, Steven Feldman, Frederick Furland, Matthew Fountain, John Chandler, Daniel Loeb, Bill Huntington, Eugene Bick, and the proposer.

MJ 2. Randall Whitman proposes the following generalization of 1989 F/M 2. For each positive integer n, consider writing the integers from 1 to n inclusive and let f(n) be the number of times the digit 1 was used. For what values of n does f(n) = n?

John Chandler found them all and writes: Obviously, the first is n = 1. After that, they get scarce, and the next is n = 199,981. To see that it is useful to count the 1’s in a column separately, so that we can make use of the “clumping.” For example, all the ten’s digit 1’s from 1-100 lie in 10-19, and there are exactly 10 of them. Thus, we see that (9) = 1, (99) = 10 x (9) + 10 x 2, (999) = 10 x (99) + 100 = 300, and so on. It is easy to see that the ratio f(n)/n reaches a local minimum each time n reaches 10^(m-1) - 1 (or 10^m - 1 modulo 10^m) and then grows faster than average until n reaches 2 x 10^(m-1) - 1 (or modulo). From the values shown above, it is clear that the minimum is x/10 (plus a little) and that there will be no solutions for n > 10^(m-1). Also, the local maximum at 2 x 10^(m-1) - 1 is (5 + x)/10 plus a little, so the first solution after n = 1 must be about 2 x 10^m. We get f(199,999) = 200,000, so we must back off past the last number ending in “1” to get f(n) = n, and then back off again to get numbers with f(n) < n. Thus, all numbers from 199,981 through 199,990 are solutions, as well as 200,000 and 200,001. From there to 300,000, it is clear that f(n) = 200,000 + f(n-200,000), and so on up to 10^m, at which point, the ratio climbs again from the low of 0.6. Since (999,999) = 600,000, and gains by 50,000 per 100,000, we find that (1,999,999) = 1,600,000, and we pick up ten more solutions: 1,599,981 through 1,599,990. The ratio then continues to climb until n = 1,999,999.
Falmouth, Mass.
Andrew T. Regan, '33; May 15, 1990; Kingsport, Tenn.
Simeon I. Rosenthal, '33; December 28, 1989; Boynton Beach, Fla.
Stanley H. Walters, '33; November 8, 1989; East Sullivan, Me.
Vito F. Battista, '34; May 24, 1990; Brooklyn, N.Y.
Julian A. Dorr, '34; March 24, 1990; Punta Gorda, Fla.
Joseph A. Serrallach, '34; December, 1989
John R. Whitney, '35; February 1, 1989; Pompano Beach, Fla.
Sydney J. Karofsky, '37; June 14, 1990; Weston, Mass.
David J. Whitney, '37; May 28, 1989; Bristol, N.H.
Donald W. Waterman, '39; October 10, 1989; Easton, Conn.
Walter M. Foster, '40; 1985; Annapolis, Md.
Harold L. Leibnian, '40; May 15, 1990; Stanford, Conn.
Barton L. Weller, '40; May 25, 1990; Easton, Conn.
Knut J. Johnsen, '41; June 6, 1990; Newburgh, N,Y.
John E. Demoss, '42; April 19, 1990; Chester, Mass.
Charles R. Stempf, '42; August, 1988; Newport, Australia
John O'Meara, '43; April 26, 1990; St. Louis, Mo.
John Farley, '44; June 2, 1990; Westfield, N.J.
Robert J. Horn, '44; June 8, 1990; Concord, Mass.
Arthur F. Peterson, Jr., '44; January 3, 1989; Star Lake, N.Y.
Robert J. Reilly, '44; April 19, 1990; Riviera Beach, Fla.
John Upton, Jr., '44; May 24, 1990; Pittsburgh, Pa.
Margaret E. Knutzen, '47; 1976; Bronxville, N.Y.
Jerry C. Kuczynski, '47; May 16, 1990; South Bend, Ind.
Donald Marshall, '48; April 22, 1990; La Jolla, Calif.
Joseph V. Yancey, '48; December 7, 1989; Alexandria, Va.
Stanley A. Murray, '49; April 11, 1990; Kingsport, Tenn.
Syed M.S. Ali, '50; December, 1987; Woodland Hills, Calif.
Francis L. Fleming, Jr., '50; March 13, 1989; Manhattan Beach, Calif.
Richard S. Paull, '50; 1987
Fred J. Rayfield, Jr., '50; May 24, 1990; Jericho, Vt.
Federico G. Baptista, '51; March 26, 1981; Caracas, Venezuela.
Jorgen Elkan, '51; May 26, 1990; West Newton, Mass.
Richard R. Fidler, '51; June 13, 1990; Andover, N.H.
Robert J. Greeney, '51; May 17, 1990; Rockville, Md.
Sergio F. Valdes, '52; November 24, 1989; Canyon Country, Calif.
Robert N. Noyce, '53; June 3, 1990; Austin, Tex.
David W. Dennew, '54; June 21, 1990; Cambridge, Mass.
William F. Stuart, Jr., '55; September 14, 1989; Alpharetta, Ga.
John D. Crowley, '57; April 21, 1990; New London, Conn.
Donald L. Jarrell, '57; December 3, 1989; McLean, Va.
Don W. Smith, '57; June 10, 1990; New York, N.Y.
William H. Moore, '58; 1989; Princeton, N.J.
James F. Hurley, III, '59; September 15, 1989; Chula Vista, Calif.
David I. Weissblat, '59; April 21, 1990; Galesburg, Mich.
Alfred J. Diefenderfer, '61; November 16, 1989; Fullerton, Calif.
George Brydon, '64; October 27, 1989; Don Mills, Ontario, Canada
Nils G. Wahlstron, '69; 1990; Lidingo, Sweden
Cyrus Behain, '72; September 2, 1989; San Clemente, Calif.
Joyce D. Waye, '75; September 25, 1989; Boston, Mass.
and then falls back so that \( (2,599,999) = 2,600,000, \) and we pick up two more: 2,600,000 and 2,600,001. Similarly, the ratio hits another minimum at 10\(^5\) and climbs up again to give a solution of 13,199,998. On the subsequent descent, we cross the break-even point at 35,000,000 and then repeat the initial pattern: 35,000,001, 35,199,981 through 35,199,998, 352,000,000, 352,000,001, 353,199,981 through 353,199,998, 501,000,000, 502,000,001, 513,199,981, 535,000,000, 535,000,001, 535,199,981 through 535,199,998, 552,000,000, and 553,000,001. The next ascent gives a solution of 1,111,111,110—and that's the last one, since the following ascent reaches the minimum at 10\(^6\) without hitting a solution. All solved by Jonathan Hadjis, Keith Price, Richard Hess, Daniel Lobel, Winslow Hartford, Ken Rosato, Jim McNamara, Matthew Fountain, Bill Huntington, Steven Feldman, Michael Baumann, Harry Zaremba, Bob High, and Nob Yosihigaha (who has a different generalization), and the proposer.

MJ 3. Richard Hess has a drinking problem he wants us to help him solve. Consider three containers that hold 15 pints, 10 pints, and 6 pints. The 15-pint container is full and the other two are empty (15,0,0). Through transferring liquid among the containers you are to measure exactly 2 pints to drink, drink the 2 pints, and continue transferring liquid to end up with 8 pints in the 10-pint container and 5 pints in the 6-pint container (6,5). David Webster sends his answer, adding “We had fun solving this one, using beer as the liquid!”


MJ 4. Gordon Rice wants you to extend the following sequence of Pythagorean triangles at least four more steps.

<table>
<thead>
<tr>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>21</td>
<td>29</td>
</tr>
<tr>
<td>169</td>
<td>168</td>
<td>257</td>
</tr>
<tr>
<td>696</td>
<td>697</td>
<td>985</td>
</tr>
<tr>
<td>4059</td>
<td>4060</td>
<td>5741</td>
</tr>
<tr>
<td>23660</td>
<td>23661</td>
<td>33461</td>
</tr>
</tbody>
</table>

I guess it is fitting that the very last problem gave rise to a large number of exceptionally fine solutions. There were several fine analytic solutions that space considerations preclude printing (but I will send copies if Robert Oliver, George Ropes, Mark Lively, Scott Maley, James Wilcox, Richard Hess, Winslow Hartford, Eric Lund, Steven Feldman, Frederick W. Furland, Gerald Lebowitz, N. F. Trang, John Granlund, Frank Carbin, Charles Piper, Jim Landau, David Wagger, Matthew Fountain, John Chandler, Bill Huntington, Roy Sinclair, Mary Lindenberg, Avi Ornstein, Angel Silva, Walter Nissen, Harry Zaremba, Robin Pitcher, David Glass, Richard Boyd, Jim Landau, Bob High, and the proposer.

Better Late than Never

AJ S. 2. Victor Christiansen believes that “all diagonals” in a square means only the main and anti-diagonal. The better late than never remark given in May/June included diagonals that “wrap around” the square. For example in a 3 by 3 square one such diagonal would be \((1,2,3),(2,3,1)\).

APR 2. Jim Landau and Robert Hart have responded.

APR 3. David Glass and Robert Hart have responded.

APR 4. David Wagger and Robert Hart have responded.