ALLAN J. GOTTLIEB, '67

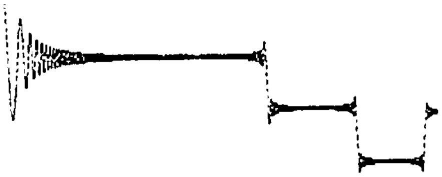
Auld Lang Sine

Readers interested in Rubik (and other) puzzles might wish to contact Peter Beck at Just Puzzles, 54 Richwood Place, Denville, N.J. 07834. Robert High suggests that those interested in the game of Go contact the American Go Association, P.O. Box 397, Old Chelsea Station, New York, N.Y. 10113.

As we reported last month, "Puzzle Corner" is being phased out due to increased pressure for space in the alumni section of *Technology Review* and the next issue will contain the last installment of this column. As a result there are no new problems this month.

Solutions

APR 1. We begin with a computer-oriented problem from Jim Landau who wants you to write a program that generates (a reasonable facsimile of) the following graphic output.



Richard Hess observed that the graph looks like a partial Fourier expansion of a step function

$$\sum_{i=1}^{n} \frac{1}{2k+1} \sin(2k+1)\pi x.$$

Winslow Hartford's function was

$$ke^{-g(x)x}\cos(f(x)\cdot x + \varphi),$$

where k, g, f, and φ are determined by curve fitting. Ken Rosato's function was

$$A(x) + B(x)\cdot \cos(C(x)),$$

where again A, B, and C are parameters. The proposer's solution, which appears below, is rather different and includes some "Puzzle Corner history:

This program was written in an unsuccessful attempt to solve a Puzzle Corner problem (I don't have the number, but it appeared in 1986 ± 1) which asked for the curve generated by the integral from 0 to x of sin (t to the nth), where n > 1. The correct answer is a sine-like curve with decaying amplitude and wavelength.

This program attempts to find integral of sin (t to the n) by evaluating t to the n at evenly-spaced intervals and summing the values. This technique works fine if the intervals are small relative to the period of the sine wave.

APR 1

```
5 INPUT "power of t", N          rem  input the shape parameter N
10 CLS                           rem  clear the screen
20 PI = 3.14159265#              rem  set up constants
30 INC = PI / 360                rem  INC is ½ degree in radians
40 PI8 = PI * 8
50 F = 0
60 FOR T = 0 TO PI8 STEP INC     rem  loop for t from 0 to 8 pi
70      SINTN = SIN (T^N)        rem  sin (T to the Nth)
80      F = F + SINTN * INC      rem  add to accumulator
90      XP = T * 25              rem  convert to PC pixel coordinates
100     YP = F * 200
110     PSET (XP, YP)            rem  turn on pixel at (XP, YP)
120 NEXT T
```

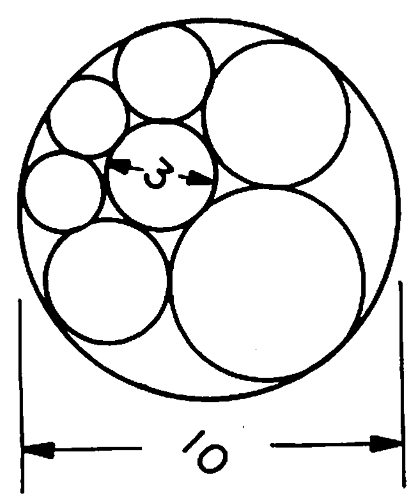
HOWEVER, the sine wave is decaying in wavelength, and eventually the wavelength becomes about as small as the sampling interval, which means the program is now sampling once per wave at roughly the same spot on each wave. Hence the accumulator F, instead of oscillating as does sin (t to the n), becomes monotonic and remains so for a short while, until the wavelength decays even more and becomes less than the sampling interval. At this point the program is sampling from every second wave and is generating reasonable results again for a while, until the wavelength decays to the point where the sampling occurs at almost the same spot on each wave, whereupon F becomes monotonic once again. And so *ad infinitum*.

The program's output is therefore incorrect but when plotted the results are rather pretty and it takes a while to realize that it is preposterous for such a simple function to have such an elaborate graph.

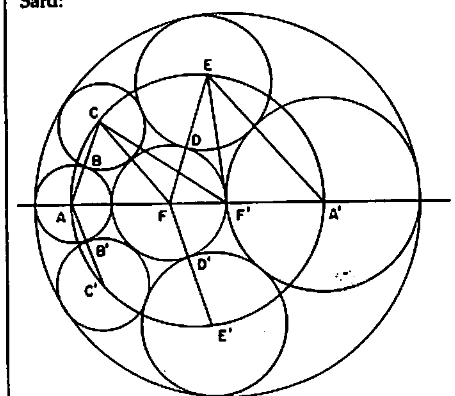
APR 2. Frank Rubin wants the largest prime having a digit that can be replaced by any of the nine other digits with the resulting number still prime.

Matthew Fountain, Robert Bart, Robert High, Avi Ornstein, Richard Hess, Donald Zalkin, and the proposer all observed that no such prime can exist since at least one of the nine results would be divisible by 3. This is so since a number is divisible by 3 if the sum of its digits is divisible by three.

APR 3. Matthew Fountain knows only that the inner and outer circles in the figure below have diameters of 3 and 10 but still wants you to determine the distance between their centers.



The following solution technique was employed independently by Edward Dawson and Eugene Sard:



The center of the diameter 3 circle is at F, the center of the diameter 10 circle is at F', and the distance between these centers, FF', is 1.5. The centers of the six circles between the diameter 3 circle and the diameter 10 circle are on the ellipse ACEA'E'C' which has foci at F and F'. One of these circles, of diameter 2, has its center at A, and one of diameter 5 has its center at A'. The length of the major diameter of the ellipse, AA', is 6.5. The radius, BC, of the circle with its center at C can be obtained from triangles CAF and CAF'. In triangle CAF:

$$AC = BC + 1,\ AF = 2.5,\ CF = BC + 1.5,$$

$$\cos \tfrac{1}{2} \angle CAF = \sqrt{\frac{BC + 2.5}{2.5(BC + 1)}}.$$

In triangle CAF':

$$AC = BC + 1,\ AF' = 4,\ CF' = 5 - BC,$$

$$\cos \tfrac{1}{2} \angle CAF' = \sqrt{\frac{5(BC)}{4(BC + 1)}}.$$

Therefore, as

$$\angle CAF' = \angle CAF,\ \frac{5(BC)}{4} = \frac{BC + 2.5}{2.5},$$

$$\text{and } BC = \frac{20}{17} \cong 1.17647.$$

Likewise, from triangles EA'F and EA'F':

$$\frac{5(DE)}{2.5} = \frac{2.5(DE + 4)}{4},$$

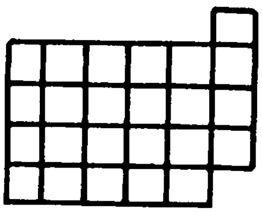
$$\text{and radius } DE = \frac{20}{11} \cong 1.81818.$$

As the ellipse is symmetric in the line AFF'A', radius B'C' = BC, and radius D'E' = DE.

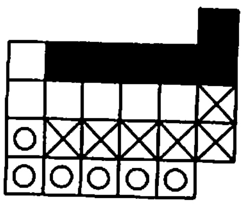
Also solved by Robert Bart, Robert High, Mary Lindenberg, Joe Feil, Harry Zaremba, Richard Hess, Winslow Hartford, Ken Rosato (who solved a more general problem involving *n* circles), Roy Sinclair, Norman Wickstrand, and the proposer.

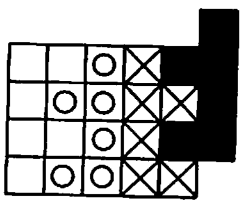
SEND SOLUTIONS AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012, OR TO: gottlieb@nyu.edu

APR 4. The following problem appeared in "Golomb's Gambits" edited by Solomon Golomb in the *Johns Hopkins Magazine*. You are to divide the figure below into four congruent pieces. There are two solutions.



The following solution is from Mary Lindenberg:





Also solved by Matthew Fountain, Robert Bart, Robert High, James Walker, Avi Ornstein, Joe Feil, Harry Zaremba, Richard Hess, Winslow Hartford, Donald Zalkin, Ken Rosato, Angel Silva, Alan Taylor, Bill Huntington, and Dan Garcia.

APR 5. Robert Bart's hypertension medication comes in 5 mg. tablets that can be divided in half to give morning and evening doses of 2.5 mg. each. At each dose, Bart selects a pill at random from the bottle. If it is 2.5 mg., it is used; if it is 5 mg., half is used and half returned to the bottle. Ignore size effects, i.e., the chance of getting a large pill is simply the percentage of pills that are large. Originally, Bart has N 5 mg. tablets and he wants to know what is likely to be the number of 5 mg. tablets remaining after i days. More specifically, he wants you to determine $P(n,N,i)$, the probability that n 5 mg. tablets remain after i days of use starting from N 5 mg. tablets and then determine the expected value of n as a function of N and i, i.e.,

$$E_n(N,i) = \sum_n nP(n,N,i)$$

In particular, what is $P(1,N,N-1)$, the probability that the *last* two doses will be a single 5 mg. tablet rather than two 2.5 mg. pieces. If a closed form is not possible, how about numerical solutions for $P(50,100,50)$, $E_n(50,20)$, $E_n(50,40)$, $P(10,20,10)$, and $E_n(10,5)$.

No one was able to obtain a closed form solution. Richard Hess and Steve Feldman each sent in the following numerical solutions based on computer calculations:

$$P(50,100,50) = 4.01368379 \times 10^{-20}$$
$$E_n(50,20) = 20.742281231$$
$$E_n(50,40) = 4.398673692$$
$$P(10,20,10) = 1.6093242 \times 10^{-4}$$
$$E_n(10,5) = 3.02588505$$

Bill Huntington notes that a similar problem occurs in his (dog) house since he has a large and a small dog who need a whole and half pill respectively each day.

Also solved by Matthew Fountain, Robert High, and Winslow Hartford.

Better Late Than Never

1988 F/M 4. Warren Himmelberger notes that Samuel Butler's novel was *Erewhon* and not *Erehwon* as printed. Hence one of the palindromes given was wrong.

1989 N/D 1. Howard Helman notes that in printing his solution various spacings were changed, a few of which resulted in syntactically invalid C programs. Mr. Helman's original solution is available from the editor and I apologize for not catching the errors in the galleys.

N/D 5. Carl Jockusch notes that the independence of the bits is not necessary since the expected value of the sum of random variables is always the sum of the expected values.

1990 JAN 2. Ronald Yoo and James Walker have responded.

JAN 4. Richard Hess has responded.

F/M 3. James Walker has responded.

F/M 5. James Walker has responded.

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