ALLAN J. GOTTLIEB, '67

Transferring Liquid Assets

Sid Shapiro thinks that I should have had a poll for "best problem of the decade." Well, better late than never: the polls are open! Mr. Shapiro's favorite was the short conversation between a Mr. P, who knew the product of two integers, and Mr. S, who knew their sum, from which one could deduce the two numbers.

Due to space limitations this issue, we will have only four problems.

Problems

M/J 1. Doug Van Patter reports that most declarers in the 1989 Cherry Hill Regional failed to make six hearts on the deal shown below. West leads the jack of diamonds (his highest card). Is there a chance of making 12 tricks?

North
♠ A 8
♥ Q J 10 4 3
♦ K 10 6
♣ A K 10

South
♠ 5 3
♥ A K 7 5 2
♦ A 7 4
♣ Q J 5

M/J 2. Randall Whitman proposes the following generalization of **1989 F/M 2.** For each positive integer n, consider writing the integers from 1 to n inclusive and let $f(n)$ be the number of times the digit 1 was used. For what values of n does $f(n) = n$?

M/J 3. Richard Hess has a drinking problem he wants us to help him solve. Consider three containers that hold 15 pints, 10 pints, and 6 pints. The 15-pint container is full and other two are empty (15,0,0). Through transferring liquid among the containers you are to measure out exactly 2 pints to drink, drink the 2 pints, and continue transferring liquid to end up with 8 pints in the 10-pint container and 5 pints in the 6-pint container (0,8,5).

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

M/J 4. Gordon Rice wants you to extend the following sequence of Pythagorean triangles at least four more steps.

3	4	5
20	21	29
119	120	169
696	697	985
4059	4060	5741
23660	23661	33461

Speed Department

SD 1. Edward Wallner wants to know how Archbishop Whitgift could have died on 29 Feb 1603?

SD 2. Another calendar quicky. This one, from Alex Okun and Gene Lieberman, was transmitted by Speedy Jim Landau. The "October Revolution" in Russia began on 24 October 1917. What day of the week was that?

Solutions

JAN 1. Doug Van Patter offers us a real life bridge problem from the 1988 Lancaster Regional.

You (South) are declarer in a shaky 3 NT contract. West's opening lead of seven of clubs is taken by your eight. What line of play will give you a reasonable chance for nine tricks?

North
♠ K 7 6 4 3
♥ A 7 6
♦ 9 3
♣ K 10 6

South
♠ J
♥ K Q J 10
♦ A K 6 2
♣ J 8 3 2

The following solution is from John Chandler: Having taken the first trick, declarer needs only eight more, including four sure hearts and two sure diamonds. It comes down to promoting one more club and one spade. West clearly starts out with at least ace, queen, 9, and 7 of clubs, so the lead of the 7 is a conventional "fourth from the longest and strongest suit." Thus, it would be reasonable to suppose that East has the ace or queen of spades, unless West has only a few spades. I think the best play is to lead the jack of spades and let it ride. If East takes the trick with the ace, that sets up Dummy's king; if East takes it with the queen, we haven't lost much, particularly if East also has the ace. Indeed, if West has both ace and queen, that either lets the jack win or (more likely) clears the ace to set up Dummy's king. In any case, that makes it about 50-50 on setting up a spade trick right off, and the aftermath still holds the possibility of the defenders cashing the ace of spades for want of the 10, 9, or 8. It's easy enough to collect another club trick by leading a small one from South—that essentially forces West to play the ace, making Dummy's king of clubs good.

Also solved by Robert Bart, Amy Lowenstein, Matthew Fountain, Gordon Rice, and the proposer.

JAN 2. Walter Cluett wants us to solve Filene's 1985 Christmas problem.

Among the shoppers one snowy Saturday morning were members of "December 25," a Christmas shoppers' club of 12 married couples. A clerk waited on all 12 couples consecutively, as they bought a total of 8 each of the following items:

1. Gloves
2. Book
3. Perfume
4. Pearl Strands
5. Football Sweater
6. Handbag

Each husband and wife were waited on together. Each couple bought 4 items. No two couples bought the same combination of items, and none of the couples bought two or more of the same item. Using the following clues, can you determine:

1. The full name of each husband and wife
2. What order they were waited on
3. What items each couple bought

The Clues

Hint: One husband is Bob; one wife is Elizabeth and one surname is Stanton.

1. The Craigs, who bought a handbag, were waited on before the Murphys, who were not waited on last.
2. The Collins bought gloves, a sweater, a handbag, and perfume.
3. The couples waited on 8th and 10th bought a book.
4. These five couples were waited on consecutively: the Smiths, Gary and his wife, a couple who bought a book and a handbag, the Swains, and Bill and his wife.
5. Geraldine and her husband did not buy either a handbag or a sweater.
6. The couple who were waited on last did not buy pearls.
7. One of the items Tom and his wife bought was a book.
8. The Marshalls did not buy perfume or pearls.
9. Evelyn and her husband bought gloves but not perfume.
10. These five couples were waited on consecutively: Martha and her husband; Jack and his wife; the couple who bought gloves, perfume, a book, and a handbag; the couple who did not buy either pearls or a book; and Margaret and her husband.
11. The first five couples waited on all bought perfume.
12. Chuck and his wife did not buy gloves.
13. The couples waited on first, second and fourth did not buy a sweater.
14. Eleanor and her husband did not buy perfume.
15. Neither Allen and his wife, who did not buy a handbag, nor the Anthonys bought gloves.
16. Cheryl and her husband, who were not waited on 10th or 12th, and John and his wife are two couples who bought both a sweater and a handbag.
17. The Douglases, who did not buy gloves or a sweater, were waited on 9th.
18. Adam and his wife, who did not buy a handbag, were waited on immediately before the Days.
19. Steve and his wife bought pearls, a book, a sweater and one other item.
20. The last three couples waited on did not buy gloves.

21. The Joneses did not buy a sweater.
22. Susan and her husband bought pearls.
23. George and his wife bought a sweater.
24. The four couples who did not buy gloves are (in no particular order): Dorothy and her husband; the Craigs; Joe and his wife; and Rosalyn and her husband (who did not buy a sweater).
25. The O'Connors bought both perfume and a sweater.
26. Sandra and her husband, who did not buy a sweater, were waited on immediately before Cathleen and her husband.

This problem was popular in two senses: Many readers solved it and many said it was enjoyable. However, describing the solution technique was not easy. Several readers gave detailed accounts of their reasoning but the results seem too long to print. Also available, thanks to our proposer Walter Cluett, is a solution prepared by the original contest organizers. Although this solution is nicely typed and of appropriate length (but still a little long), I prefer to use one of our own solutions. I am giving Michael Baumann's, which was a good example of the summaries that several other solvers furnished.

Baumann's solution follows:

Please see chart (at bottom of page). Detailing how the solution was obtained would be quite laborious. Suffice it to say that I started by determining the surnames, what order they paid for their purchases, and to some extent, what they purchased. Once the couples were ordered correctly, it was fairly easy to go back and assign first names and fill out what was purchased.

Also solved by Robert Bart, Mary Lindenberg, Allen Wiegner, Sander Liehsten, Frank Binns, Raisa Deber, Susan Levitin, Samuel Levitin, Reba Hite, Tom Lydon, Donald Eckhardt, Amy Lowenstein, Randy Koloch, John Chandler, Steve Feldman, Bob High, Charlie Masison, Larry Bell, Laura Fricke, Loren Bonderson, Ray Gamino, Matthew Fountain, Libbie Merrow, Michele Rosen, Harry Zaremba, Julie Sussman, Jim Klucar, and the proposer.

JAN 3. Dave Mohr asks a discrete variant of 1988 M/J 2.

Two gamblers, you and Low Stakes, tire of dice and poker. You agree to wager on each of a number of plays of a game with the following set of rules: Each would write privately on a slip of paper three nonnegative integers whose sum must be 10. Zero is allowed (0, 2, 8 for example). Repeats are also allowed (3, 3, 4 for example). Then you compare amounts, largest against largest, smallest against smallest, and median against median. If any tie, the bet is a draw. The one with the larger amount in 2 of the 3 categories wins the wager. According to what strategy do you plan to select the numbers?

Several readers pointed out that considerations of bluffing and estimating the ability of your opponent are non-mathematical considerations that also apply. Gordon Rice included the following comment with his solution:

This problem is no more discrete than 1988 M/J 2, but it is more manageable. The earlier problem used integers up to 1,000, so it wasn't practical to build the payoff matrix. The result was that we got sucked off on a tangential issue. Like others, I approximated the finite solution by a continuous one, and tried to solve the calculus problem of finding which combination was the winningest, assuming that my opponent was idiot enough to choose his play at random from all possibilities. I almost got it, but there was a little area in the upper left where I couldn't complete the proof. I was disappointed to find that the published solution didn't help me with the difficulty.

Bob High noticed that for $n < 10$ there is an optimal solution consisting of always writing the same triple on your piece of paper, a so-called pure strategy. High's solution for $n \geq 10$ follows:

I wrote a program to test for optimal mixed strategies for larger values of n. My program tells me that the solution for n = 10 is the strategy mixing (6,4,0), (6,2,2), and (4,4,2) in equal parts. So n = 10 is in fact the first interesting case (the first requiring a mixed strategy). The full game matrix for the n = 10 version of the game is given by: (see chart at top of facing page).

Since the sum of the entries in the 7th, 9th, and 13th rows (corresponding to the three pure strategies we want to mix) is always ≥ 0, it is clear that the proposed mixed strategy is optimal.

In fact, there are optimal mixed strategies made up of equal parts of exactly three pure strategies for every value of n from 10 to 21. Sometimes there are several; for example, for n = 12, both (8,2,2), (7,5,0), (5,4,3) and (8,2,2), (6,6,0), (4,4,4) work. Sometimes there is only a single such strategy; the solution for n = 10 is unique, and for n = 20, the only solution I found was (12,5,3), (10,9,1), (8,7,5). The structure of the solutions, as proportions of n, seems pretty stable, but for n = 22, there is no such simple mixed strategy with only three components. I don't know what all this says about possible solutions to the continuous version—I had hoped the discrete solutions might converge to a continuous solution, but I don't know where the discrete solutions go for higher values of n.

Also solved by Robert Bart, Allen Wiegner, Sander Liehsten, Tom Lydon, Michael Baumann, Ken Rosato, Ken Haruta, Matthew Fountain, Harry Zaremba, and the proposer.

JAN 4. The following problem is from the book *The Puzzling Adventures of Dr. Ecco* written by my NYU colleague Dennis Shasha.

"Receiving a telegram these days is most unusual," Ecco said as he tore open the envelope. After reading the message, he said, "The contents are even more so. What do you make of it, Professor?"

The telegram read: DR ECCO NEED YOUR HELP ON RIDDLE STOP BELIEVE IT FROM GREEK MYTHOLOGY STOP WILL CALL ON YOU TO-

JAN 2 FIGURE

Order	Husband	Wife	Surname	Handbag	Book	Sweater	Pearls	Gloves	Perfume
1	Adam	Geraldine	Jones		X		X	X	X
2	Bob	Martha	Day	X			X	X	X
3	Jack	Susan	O'Connor			X	X	X	X
4	Tom	Sandra	Smith	X	X			X	X
5	Gary	Cathleen	Collins	X		X		X	X
6	John	Margaret	Marshall	X	X	X		X	
7	George	Cheryl	Swain	X		X	X	X	
8	Bill	Evelyn	Stanton		X	X	X	X	
9	Chuck	Rosalyn	Douglas	X	X		X		X
10	Steve	Eleanor	Craig	X	X	X	X		
11	Allen	Dorothy	Murphy		X	X	X		X
12	Joe	Elizabeth	Anthony	X	X	X			X

(10,0,0)	0	0	0	−1	0	−1	0	−1	−1	0	−1	−1	−1	−1
(9,1,0)	0	0	0	0	0	−1	0	−1	−1	0	−1	−1	−1	−1
(8,2,0)	0	0	0	0	0	0	0	−1	0	0	−1	−1	−1	−1
(8,1,1)	1	0	0	0	1	0	1	0	−1	1	0	−1	−1	−1
(7,3,0)	0	0	0	−1	0	0	0	0	1	0	−1	0	−1	0
(7,2,1)	1	1	0	0	0	0	1	0	0	1	0	−1	−1	−1
(6,4,0)	0	0	0	−1	0	−1	0	0	0	0	0	1	0	1
(6,3,1)	1	1	1	0	0	0	0	0	0	1	0	0	−1	0
(6,2,2)	1	1	0	1	−1	0	0	0	0	1	1	0	0	−1
(5,5,0)	0	0	0	−1	0	−1	0	−1	−1	0	0	0	1	1
(5,4,1)	1	1	1	0	1	0	0	0	−1	0	0	0	0	1
(5,3,2)	1	1	1	1	0	1	−1	0	0	0	0	0	0	0
(4,4,2)	1	1	1	1	1	1	0	1	0	−1	0	0	0	0
(4,3,3)	1	1	1	1	0	1	−1	0	1	−1	−1	0	0	0

JAN 3 FIGURE

MORROW AFTERNOON END.

"You are something of an amateur scholar of Greek mythology, aren't you?" I asked, pointing to a row of books.

"Quite amateur, indeed," Ecco responded modestly, "but considering the tone of this telegram, possibly more knowledgeable than our client, who may even now be pressing the doorbell."

The young man at the door looked very athletic with his polo shirt and tanned face. After introductions, he explained his problem.

"The woman I love is a graduate student in Greek literature," he said. "Her latest eccentric demand is that I solve a riddle adapted from her researches. She will marry me if I can answer three questions. Will you listen?"

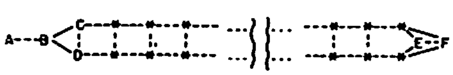
"By all means," said Ecco. "Please proceed."

"There is a party," said the young man, launching into the riddle. "Everybody at the party has shaken hands with three of the other people, except for one person, who has shaken hands with only one of the other people.

"That's all the information you get, Dr. Ecco." Then he stated the three questions.

1. What is the smallest number of people who could be at such a party?
2. Could there be 21 people at such a party?
3. Is there a general pattern of how many people could be at such a party?

As many readers pointed out, a very simple counting argument shows that the number of people must be even and an easy analysis shows that four people are not sufficient. The only real challenge was to find out which even numbers exceeding four are possible. Larry Bell showed that solutions exist for all such numbers. He drew the diagram below where A-F are always included (giving a six-person solution) and that pairs of stars can be added to give one more rung in the ladder (and a solution for the next even number).



Also solved by Avi Ornstein, Robert Bart, Donald Eckhardt, Mary Lindenberg, Amy Lowenstein, Peter Tzanetos, John Chandler, Michael Baumann, Allen Wass, Bob High, Ken Rosato, Matthew Fountain, Gordon Rice, Harry Zaremba, Frank Carbin, Susan Levitin, and Samuel Levitin.

Better Late Than Never

1989 JUL 1. Thomas Turnbull solved this problem on time. I mistakenly thought it was a new Bridge problem and misfiled it.

A/S 1. Juan Lavalle has responded.

A/S 2. Donald Eckhardt writes to say that the magic square shown is a de la Loubere magic square, named after Simon de la Loubere who, while in the service of the government of Louis XIV, learned the technique during a 1687 visit to Surat, India (at that time a major port just north of Bombay, which was then just a swampy little village). Eckhardt has written a paper on these magic squares that will soon appear in *Mathematics* Magazine.

Matthew Fountain and the proposer, Ronald Martin, note that the published solution does not meet the requirement that all the diagonals sum to 671. The following solution, from the proposer, does meet this requirement.

11	25	50	75	89	114	18	43	57	82	107
12	37	62	87	101	5	30	55	69	94	119
24	49	74	99	113	17	42	56	81	106	10
36	61	86	100	4	29	54	68	93	118	22
48	73	98	112	16	41	66	80	105	9	23
60	85	110	3	28	53	67	92	117	21	35
72	97	111	15	40	65	79	104	8	33	47
84	109	2	27	52	77	91	116	20	34	59
96	121	14	39	64	78	103	7	32	46	71
108	1	26	51	76	90	115	19	44	58	83
120	13	38	63	88	102	6	31	45	70	95

OCT 1. Joe Feil has responded.

N/D 2. Bruce Kulik has responded.

Proposers' Solutions to Speed Problems

SD 1. At that era in England years started on 25 March but the leap years still fell when the year starting 1 January was divisible by four [Learn something every day—ed.].

SD 2. That was 24 October in the Julian calendar, which after 1900 was lagging the current Gregorian calendar by 13 days. Hence the revolution began 6 November according to our calendar, which straightforward counting shows to be a Tuesday.