ALLAN J. GOTTLIEB, '67

# Whole Lotta Shakin' Goin' On

This being the first issue of another year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (1, 9, 9, and 0) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 1989 yearly problem is in the "Solutions" section.

### Problems

**Y1990.** Form as many as possible of the integers from 1 to 100 using the digits 1, 9, 9, and 0 exactly once each and the operators +, −, × (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1, 9, 9, and 0 are preferred. Parenthesis may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

**JAN 1.** Doug Van Patter offers us a real life bridge problem from the 1988 Lancaster Regional.

You (South) are declarer in a shaky 3 NT contract. West's opening lead of seven of clubs is taken by your eight. What line of play will give you a reasonable chance for nine tricks?

NORTH
♠ K 7 6 4 3
♥ A 7 6
♦ 9 3
♣ K 10 6

SOUTH
♠ J
♥ K Q J 10
♦ A K 6 2
♣ J 8 3 2

**JAN 2.** Walter Cluett wants us to solve Filene's 1985 Christmas problem.

Among the shoppers one snowy Saturday morning were members of "December 25," a Christmas shoppers' club of 12 married couples. A clerk waited on all 12 couples consecutively, as they bought a total of 8 each of the following items:

1. Gloves
2. Book
3. Perfume
4. Pearl Strands
5. Football Sweater
6. Handbag

Each husband and wife were waited on together. Each couple bought 4 items. No two couples bought the same combination of items, and none of the couples bought two or more of the same item. Using the following clues, can you determine:

1. The full name of each husband and wife
2. What order they were waited on
3. What items each couple bought

**The Clues**

Hint: One husband is Bob; one wife is Elizabeth and one surname is Stanton.

1. The Craigs, who bought a handbag, were waited on before the Murphys, who were not waited on last.
2. The Collins bought gloves, a sweater, a handbag, and perfume.
3. The couples waited on 8th and 10th bought a book.
4. These five couples were waited on consecutively: the Smiths, Gary and his wife, a couple who bought a book and a handbag, the Swains, and Bill and his wife.
5. Geraldine and her husband did not buy either a handbag or a sweater.
6. The couple who were waited on last did not buy pearls.
7. One of the items Tom and his wife bought was a book.
8. The Marshalls did not buy perfume or pearls.
9. Evelyn and her husband bought gloves but not perfume.
10. These five couples were waited on consecutively: Martha and her husband; Jack and his wife; the couple who bought gloves, perfume, a book, and a handbag; the couple who did not buy either pearls or a book; and Margaret and her husband.
11. The first five couples waited on all bought perfume.
12. Chuck and his wife did not buy gloves.
13. The couples waited on first, second and fourth did not buy a sweater.
14. Eleanor and her husband did not buy perfume.
15. Neither Allen and his wife, who did not buy a handbag, nor the Anthonys bought gloves.
16. Cheryl and her husband, who were not waited on 10th or 12th, and John and his wife are two couples who bought both a sweater and a handbag.
17. The Douglases, who did not buy gloves or a sweater, were waited on 9th.
18. Adam and his wife, who did not buy a handbag, were waited on immediately before the Days.
19. Steve and his wife bought pearls, a book, a sweater and one other item.
20. The last three couples waited on did not buy gloves.
21. The Joneses did not buy a sweater.
22. Susan and her husband bought pearls.
23. George and his wife bought a sweater.
24. The four couples who did not buy gloves are (in no particular order): Dorothy and her husband; the Craigs; Joe and his wife; and Rosalyn and her husband (who did not buy a sweater).
25. The O'Connors bought both perfume and a sweater.
26. Sandra and her husband, who did not buy a sweater, were waited on immediately before Cathleen and her husband.

**JAN 3.** Dave Mohr asks a discrete variant of 1988 M/J 2.

Two gamblers, you and Low Stakes, tire of dice and poker. You agree to wager on each of a number of plays of a game with the following set of rules: Each would write privately on a slip of paper three nonnegative integers whose sum must be 10. Zero is allowed (0, 2, 8 for example). Repeats are also allowed (3, 3, 4 for example). Then you compare amounts, largest against largest, smallest against smallest, and median against median. If any tie, the bet is a draw. The one with the larger amount in 2 of the 3 categories wins the wager. According to what strategy do you plan to select the numbers?

**JAN 4.** The following problem is from the book *The Puzzling Adventures of Dr. Ecco*, written by my NYU colleague Dennis Shasha.

"Receiving a telegram these days is most unusual," Ecco said as he tore open the envelope. After reading the message, he said, "The contents are even more so. What do you make of it, Professor?"

The telegram read: DR ECCO NEED YOUR HELP ON RIDDLE STOP BELIEVE IT FROM GREEK MYTHOLOGY STOP WILL CALL ON YOU TOMORROW AFTERNOON END.

"You are something of an amateur scholar of Greek mythology, aren't you?" I asked, pointing to a row of books.

"Quite amateur, indeed," Ecco responded modestly, "but considering the tone of this telegram, possibly more knowledgeable than our client, who may even now be pressing the doorbell."

The young man at the door looked very athletic with his polo shirt and tanned face. After introductions, he explained his problem.

"The woman I love is a graduate student in Greek literature," he said. "Her

*SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.*

latest eccentric demand is that I solve a riddle adapted from her researches. She will marry me if I can answer three questions. Will you listen?"

"By all means," said Ecco. "Please proceed."

"There is a party," said the young man, launching into the riddle. "Everybody at the party has shaken hands with three of the other people, except for one person, who has shaken hands with only one of the other people.

"That's all the information you get, Dr. Ecco." Then he stated the three questions.

1. What is the smallest number of people who could be at such a party?
2. Could there be 21 people at such a party?
3. Is there a general pattern of how many people could be at such a party?

## Speed Department

**SD 1.** Speedy Jim Landau wants to know how to drop an egg 4 feet without breaking it.

**SD 2.** Frank Model wants you to give a baseball team with all nine players major leaguers (present or past) having the last name Johnson. The three outfield positions may be combined, i.e., three outfielders are required but not all three positions need be represented (e.g., all three players could be centerfielders).

## Solutions

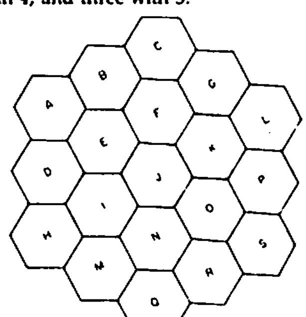
**Y1989.** Form as many as possible of the integers from 1 to 100 using the digits 1, 9, 8, and 9 exactly once each and the operators +, −, × (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1, 9, 8, and 9 are preferred. Parenthesis may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

The following solution is from John Drumheller, who extended our criteria for breaking ties. Drumheller ranks potential solutions via:
1) Minimum number of operators
2) 4 digits in order
3) 3 digits in order of 2 pairs in order
4) 2 digits in order
5) The simplest operators (ordered +, −, ×, /, exponential)

1 $1^{989}$
2 91 − 89
3 (19 + 8)/9
4 8/((9/9) + 1)
5 −
6 8 − ((9/9) + 1)
7 98 − 91
8 9 − $1^{98}$
9 $1^{98}$ × 9
10 $1^{98}$ + 9
11 (1 + 98)/9
12 −
13 −
14 −
15 −
16 (8 + 9) − $1^9$
17 18 − (9/9)
18 99 − 81
19 91 − (9 × 8)
20 19 − 8 + 9
21 189/9
22 198/9
23 −
24 −
25 9 + 8 + 9 − 1
26 (1 × 9) + 8 + 9
27 1 + 9 + 8 + 9
28 −
29 −
30 −
31 −
32 −
33 −
34 −
35 −
36 19 + 8 + 9
37 −
38 −
39 −
40 −
41 −
42 −
43 −
44 −
45 −
46 −
47 −
48 −
49 −
50 −
51 —
52 —
53 (8 × 9) − 19
54 ((8 − 1) × 9) − 9
55 (8 × (9 − 1)) − 9
56 —
57 —
58 —
59 —
60 —
61 —
62 (9 × 8) − 9 − 1
63 81 − 9 + 9
64 1 − 9 + (8 × 9)
65 —
66 —
67 —
68 —
69 —
70 89 − 19
71 ((1 + 9) × 8) − 9
72 $1^9$ × 8 × 9
73 ((1/9) + 8) × 9
74 91 − 9 + 8
75 —
76 —
77 —
78 —
79 98 − 19
80 (89 × 1) − 9
81 99 − 18
82 81 + (9/9)
83 —
84 —
85 —
86 —
87 —
88 89 − $1^9$
89 (1 × 98) − 9
90 1 + 98 − 9
91 819/9
92 91 + 9 − 8
93 —
94 —
95 —
96 —
97 89 − 1 + 9
98 (1 × 9) + 89
99 891/9
100 $1^8$ + 99

Also solved by Allen Tracht, Bob High, Joe Feil, Daniel Mullins, Stephen Callaghan, Greg Spradlin, Steve Feldman, Charles Dale, Robbie Smith, Albert Smith, and Avi Ornstein.

**A/S 1.** My plea for computer-related problems has inspired Warren Himmelberger. I still need more such problems. Mr. Himmelberger wants us to devise a computer program to use the numbers 1 through 19 once each to label the hexagons below so that each diagonal sums to the same value. Note that there are six diagonals with 3 hexagons each, six with 4, and three with 5.




Matthew Fountain reports that solving this problem taught him how to use sets in Pascal. He filled the cells in a somewhat odd order and relettered the cells to correspond to that order, which was

A H B
M G N I
F O R P C
L Q S J
E K D

Fountain's solution follows:

My program running on a 20MH 386 found the following solution in less than six seconds. It took 12.4 minutes to show that it was the only solution except for mirror images and rotations.

3 19 16
17 7 2 12
18 1 5 4 10
11 6 8 13
9 14 15

It is interesting that 19 + 1 + 4 + 14 = 12 + 7 + 8 + 11 = 17 + 6 + 2 + 13 = 38, an additional symmetry involving 38.

For a complete description of how Fountain arrived at his solution and the text of his program, please write to Tech Review Puzzle, W59-212, 201 Vassar St., Cambridge, MA 02139.

Also solved by Gordon Burns, Paul Herkart, Winslow Hartford, Steven Feldman, Al Danzis, Alan Schwartz, Robert Bart, Scott Maley, Harry Zaremba, Donald Savage, Gordon Rice, and the proposer.

**A/S 2.** A magic square problem from Ronald Martin, who wants you to arrange the numbers from 1 to 121 in an 11 × 11 grid so that all rows, columns, and diagonals sum to 671. Note that to form most diagonals you must imagine a copy of the square placed next to the original.

The following solution is from Victor Christensen: The specific solution, an 11 × 11 magic square, is shown below:

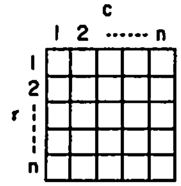
n = 11

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 68 | 81 | 94 | 107 | 120 | 1 | 14 | 27 | 40 | 53 | 66 | |
| 80 | 93 | 106 | 119 | 11 | 13 | 26 | 39 | 52 | 65 | 67 | |
| 92 | 105 | 118 | 10 | 12 | 25 | 38 | 51 | 64 | 77 | 79 | |
| 104 | 117 | 9 | 22 | 24 | 37 | 50 | 63 | 76 | 78 | 91 | |
| 116 | 8 | 21 | 23 | 36 | 49 | 62 | 75 | 88 | 90 | 103 | |
| 7 | 20 | 33 | 35 | 48 | 61 | 74 | 87 | 89 | 102 | 115 | 671 |
| 19 | 32 | 34 | 47 | 60 | 73 | 86 | 99 | 101 | 114 | 6 | |
| 31 | 44 | 46 | 59 | 72 | 85 | 98 | 100 | 113 | 5 | 18 | |
| 43 | 45 | 58 | 71 | 84 | 97 | 110 | 112 | 4 | 17 | 30 | |
| 55 | 57 | 70 | 83 | 96 | 109 | 111 | 3 | 16 | 29 | 42 | |
| 56 | 69 | 82 | 95 | 108 | 121 | 2 | 15 | 28 | 41 | 54 | |

For the general case, use the following variables:
n = positive integer representing the number of cells along one edge of the square
c = current column number
r = current row number

In general, to create a magic square using each of the integers from 1 to $n^2$ (where n is *odd*) only once, the following method may be used:

Think of an empty square as a grid of empty cells. The position of a cell is identified using column and row coordinates (c and r, respectively), as shown below:




Place the integers in the cells using the following algorithm:

1) Place the integer 1 in the cell at coordinates $\left(c = \frac{n+1}{2}, r = 1\right)$; this is the middle cell of the top row.
2) To determine the next cell to fill (call it the "target cell"), attempt to find an empty cell at coordinates (c = c + 1, r = r − 1); this cell will be upper-right diagonally adjacent to the current cell. In this attempt, one of four possibilities will result:
   a) The target cell would be beyond the upper edge of the square (r < 1); in this case, place the next successive integer in the bottom cell (row r = n) of column c = c + 1 and continue to the next target cell.
   b) The target cell is empty; in this case, place the next successive integer in the target cell and continue to the next target cell.
   c) The target cell would be beyond the right edge of the square (c > n); in this case, place the next successive integer in the far left cell (column c = 1) of row r = r − 1 and continue to the next target cell.
   d) The target cell is within legal bounds (1 ≤ c ≤ n, 1 ≤ r ≤ n), but is not empty; in this case, place the next successive integer in the cell directly below the current cell, at coordinates (c, r = r + 1) and continue to the next target cell.
3) Recurse through step 2 until all cells are filled; the magic square is now complete.

To determine what the "magic sum" (S) will be even before a square is filled or started, divide the sum of the integers in the square (that is, the sum of the integers from 1 to $n^2$) by the number of columns (or rows, since they're equal) in the square.

From our math experience, we know that the sum of the integers from 1 to n is

$$\frac{n(n + 1)}{2}.$$

Therefore, the sum of the integers from 1 to $n^2$ is

$$\frac{n^2(n^2 + 1)}{2}.$$

Dividing this quantity by the number of columns/rows, n, gives us

$$\frac{n(n^2 + 1)}{2}.$$

Therefore, the magic sum is represented as:

$$S = \frac{n(n^2 + 1)}{2}$$

With the 11 × 11 square, we have

$$\frac{11(121 + 1)}{2} = 671,$$

as desired.

Another, more subtle, use of the above formula may be found after the square is filled (at least to the point where the center cell is filled). The center cell of a square formed using the above algorithm will always equal

$$\frac{n^2 + 1}{2};$$

in the case of the 11 × 11 square, we have

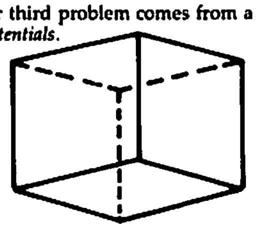
$$\frac{121 + 1}{2} = 61,$$

which is in the center cell. Multiplying this number by n (in this case, n = 11) gives us the desired 671 as the magic sum. Symbolically, it results in the same formula for S as shown above.

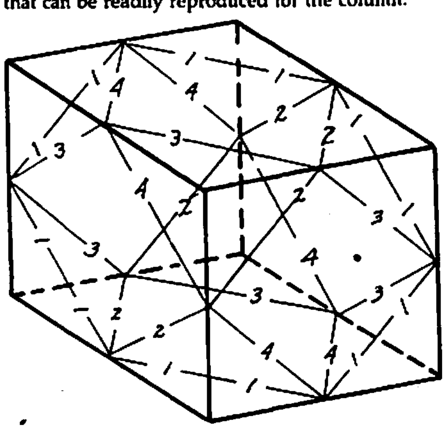
I'm sure there are more intriguing things to be discovered about magic squares, but at least now you have a basic idea of them.

Also solved by Maria Silva, Richard Bator, Don Berliner, Warren Himmelberger, Winslow Hartford, Harry Zaremba, Matthew Fountain, and the proposer.

**A/S 3.** Our third problem comes from a 1986 issue of *IEEE Potentials.*



Norman Wickstrand sent me a work of art for this one, a beautiful 4-color (plus black and white) drawing for me to enjoy and a black-and-white version that can be readily reproduced for the column.



If we connect the midpoint of each edge of the cube to the other nearest edges we will obtain the 24 light lines shown on the enclosed diagram. If we then select the lines as indicated by the numerals in the breaks in the lines we will have the required four regular hexagons. These lines are all of equal length—by construction. Opposite lines are all parallel. Hence the hexagons are all regular. The numerals 1, 2, 3, and 4 show that there are four regular hexagons. Each of these four hexagons are in planes that divide the cube into equal parts.

Also solved by Phelps Meaker, Leonard Nissim, Avi Ornstein, Mary Lindenberg, Harry Zaremba, Matthew Fountain, Robert Bart, Winslow Hartford, Ken Rosato, and Randall Whitman.

**A/S 4.** Richard Hess has taken craps to an extreme and writes: In Extreme Craps, four dice are thrown and the middle two dice are ignored. Otherwise, it is played the same as ordinary craps. What is the shooter's probability of winning Extreme Craps?

I perhaps should have saved this one for a computer problem. Michael Jung's BASIC program, reprinted below, shows that the winning probability is about 53% for extended craps. For regular craps the corresponding probability is about 49%.

```
200 REM ——————————
210 REM INITIALIZATION
220 REM ——————————
230 GOSUB 2430: REM INITIALIZE SCREEN
240 GOSUB 2330: REM DIMENSION ARRAYS
1000 REM ——————————
1010 REM MAIN SEQUENCE
1020 REM ——————————
1030 GOSUB 2030: REM COMPUTE SHOT
     PROBABILITIES
1040 GOSUB 2530: REM COMPUTE WIN
     PROBABILITY
1050 END
2000 REM ——————————————————
2010 REM COMPUTE SHOT PROBABILITIES
2020 REM ——————————————————
2030 PRINT "    COMPUTING SHOT
     PROBABILITIES"
2040 FOR I1% = 1 TO 6
2050 FOR I2% = 1 TO 6
2060 FOR I3% = 1 TO 6
2070 FOR I4% = 1 TO 6
2080 REM SELECT LARGEST
2090 IF I1% >= I2% AND I1% >= I3% AND
     I1% >= I4% THEN X = I1%: GOTO 2140
2100 REM COMPUTE SHOT PROBABILITIES
     (CONTINUED)
2110 IF I2% >= I1% AND I2% >= I3% AND
     I2% >= I4% THEN X = I2%: GOTO 2140
2120 IF I3% >= I1% AND I3% >= I2% AND
     I3% >= I4% THEN X = I3% ELSE X = I4%
2130 REM SELECT SMALLEST
2140 IF I1% <= I2% AND I1% <= I3% AND
     I1% <= I4% THEN X = X + I1%: GOTO
     2180
2150 IF I2% <= I1% AND I2% <= I3% AND
     I2% <= I4% THEN X = X + I2%: GOTO
     2180
2160 IF I3% <= I1% AND I3% <= I2% AND
     I3% <= I4% THEN X = X + I3% ELSE
     X = X + I4%
2170 REM RECORD EVENT
2180 PROB(X) = PROB(X) + 1
2190 NEXT I4%
2200 REM COMPUTE SHOT PROBABILITIES
     (CONTINUED)
2210 NEXT I3%
2220 NEXT I2%
2230 NEXT I1%
2240 FOR I = 1 TO 12
2250 PROB(I) = PROB(I)/1296
2260 NEXT I
2270 RETURN
2300 REM ————————————
2310 REM DIMENSION ARRAYS
2320 REM ————————————
2330 DIM PROB(12)
2340 RETURN
2400 REM ————————————
2410 REM INITIALIZE SCREEN
2420 REM ————————————
2430 SCREEN 0,0,0
2440 KEY OFF
2450 PEN OFF
2460 CLS
2470 RETURN
2500 REM ——————————————————
2510 REM COMPUTE WIN PROBABILITY
2520 REM ——————————————————
2530 PRINT
2540 PRINT "    COMPUTING WIN
     PROBABILITY"
2550 REM SEVEN ON COMEOUT ROLL
2560 PWIN = PROB(7)
2570 REM ELEVEN ON COMEOUT ROLL
2580 PWIN = PWIN + PROB(11)
2590 REM 4, 5, OR 6 ON COMEOUT ROLL
2600 REM COMPUTE WIN PROBABILITY
     (CONTINUED)
2610 FOR I = 4 TO 6
2620 PWIN = PWIN+PROB(I)*PROB(I)/(PROB(I)
     + PROB(7))
2630 NEXT I
2640 REM 8, 9, OR 10 ON COMEOUT ROLL
2650 FOR I = 8 TO 10
2660 PWIN = PWIN+PROB(I)*PROB(I)/(PROB(I)
     + PROB(7))
2670 NEXT I
2680 PRINT
2690 PRINT "    WIN PROBABILITY = "; PWIN
2700 REM COMPUTE WIN PROBABILITY
     (CONTINUED)
2710 RETURN
```

Also solved by Winslow Hartford, Robert Bart, Matthew Fountain, Peter Silverberg, and the proposer.

**A/S 5.** Frank Rubin tells us about Milo Mindbender, a student at Drudgery High. After every test, Milo figures out his cumulative average, which he always rounds to the nearest whole percent. Today he had two tests. First he got 75 in French, which dropped his average by 1 point. Then he got 83 in History, which lowered his average another two points. What is his average now?

Pretty tricky! Although a few readers suggested that this problem must be a misprint, it is in fact correctly printed as the following solution from Gordon Rice illustrates:

Suppose Milo's unrounded average before the last two tests was S/N, where N is the number of tests and S is their cumulative point total. After the French test his average became $(S + 75)/(N + 1)$, and after the History test it is $(S + 75 + 83)/(N + 2)$.

We can assert

$$S/N - (S + 75)/(N + 1) < 2,$$

otherwise his rounded average must have fallen by more than 1 after the French test. We have also

$$(S + 75)/(N + 1) - (S + 158)/(N + 2) > 1,$$

otherwise his rounded average would have fallen by less than 2 after the History test. Putting them both together with a bit of algebraic manipulation, we get

$$N^2 + 86N + 10 < S < 2N^2 + 77N.$$

For N < 11, the upper bound does not exceed the lower bound. For N > 13, S/N must exceed 100, which doesn't happen with tests. So the possibilities are:

| | |
|---|---|
| N = 11 | 1077 < S < 1089 |
| N = 12 | 1186 < S < 1200 |
| N = 13 | 1297 < S < 1300 |

The rest is numerical experimentation. A guiding principle is that we expect the fractional part of S/N to be just less than 1/2, so that it will round down but have as much room as possible to decrease without changing the rounded value. The solution is

S/N = 1083/11 = 98.45, rounding to 98
after French, 1158/12 = 96.50, rounding to 97
after History, 1241/13 = 95.46, rounding to 95.

Also solved by Steven Feldman, Peter Tzanetos, Winslow Hartford, Robert Bart, Matthew Fountain, Ken Rosato, Harry Zaremba, Mary Lindenberg, and the proposer.

**Proposers' Solutions To Speed Problems**

**SD 1.** Drop it 5 feet above the floor (and consider the top 4 feet).

**SD 2.**

| | | |
|---|---|---|
| 1B Deron | OF Lou | SS Howard |
| 2B Dave | OF Alex | C Daryl |
| 3B Billy | OF Vance | P Walter |