Funny Money

After using Matthew Fountain's problem for N/D 1 below, we have just one computer and no chess problems remaining. As mentioned last issue, this may be the market speaking. If, however, you would like to see more chess and computer problems, please send them in.

Wilbur DeHart, while commenting on JUL 1, claims that an interesting version of bridge, which he calls "inverse bridge," can be played if the bidding proclaims the minimum number of tricks to be taken by the defenders. DeHart enjoys seeing the kings and queens fall on the lead of an ace!

I have long known that historians write more than mathematicians or computer scientists but still found the following observation from Matthew Fountain surprising:

"As treasurer of our Historical Society I have inherited the papers and pictures of our most distinguished historian, who died at the age of 92. I have to inventory them for the executor of the estate. They were delivered to me in a pick-up truck as a car was too small."

We end with a remark from long-time contributor Mary Lindenberg, who ten years ago switched from teaching mathematics to high school students to teaching watercolor painting to adults. Ms. Lindenberg reports that, when her present students discover her previous vocation, they invariably ask if she started out "painting by the numbers"?

Problems

N/D 1. Matthew Fountain proposes, as a computer-oriented problem, finding the smallest nonprime integer, N > 1, that divides 2\(^{N-1} - 1\). As a noncomputer addition, explain why there are so "few" such Ns.

N/D 2. Nob. Yoshigahara wants you to find the rule for each of the following triangular configurations of numbers and to produce a configuration one size larger.

N/D 3. Arthur Lewbel wants you to find the area of the small square without using Pythagoras's theorem. The area of the large square is one. The circle has center O, is tangent to two sides of the large square, and passes through the center of the large square.

N/D 4. Phelps Meaker wants to know the radius of a sphere circumscribing a regular tetrahedron.

N/D 5. Our final regular problem is from Gordon Rice, who notes that, by successive flips of a coin, we may generate a random number between 0 and 2\(^n\) - 1. Each flip determines one digit in the binary representation of the number; heads it's a one, tails it's a zero. Suppose we have a biased coin, which comes up heads with probability P. What is the expected value of an N-bit number generated with such a coin?

Speed Department

SD 1. Greg Detefano wants to know by what date Puzzle Corner will find it necessary to make a policy change?
SD 2. Jim Landau asks how many successful jumps must a parachutist make before s/he can graduate from jump school?

Solutions

JUL 1. Robert Bart offers a problem he attributes to Robert Darves in which South is to make seven spades with the assistance of all four players.

North

\[ \spadesuit 8 \] 
\[ \heartsuit 9 \] 
\[ \clubsuit 7 \] 
\[ \diamondsuit 6 \]

West

\[ \spadesuit K 10 7 4 \] 
\[ \heartsuit J K 6 \] 
\[ \clubsuit J 10 3 \] 
\[ \diamondsuit A Q 10 \]

JUL 2. As noted below, problem FM 4 was misprinted (sorry). Here is the correct version. A offers to run three laps while B does two but gets only 150 yards into his third lap when B wins. He then offers to run four laps to B’s three, and quickens his pace in the ratio of 43. B also quickens his pace, in the ratio of 98, but in the second lap falls to his original pace, and in the third he goes only 9 yards for the 10 he went in the first race. A wins the race by 180 yards. How long is each lap?

The following very generous letter is from first-time responder David Glass: As this is the first time I have sent answers to a puzzle column, I have to say that I am one of the "silent majority" who try puzzles but never write in about them. I have followed this practice because, after all, it’s no big trick to get the answers to a few puzzles, right? Unfortunately a recent bad experience with a couple of Colomo’s puzzles (I found proofs for both of my answers, and they both turned out wrong) convinced me that it’s not sport to avoid risking ridicule for wrong answers, even though the risk might be tiny (ha ha). Another thing I should say is that DOING PUZZLES IS FUN, and thank you very much, even for the ones I didn’t answer, and all the ones I never wrote about.

Let L be the length of the track. A is the speed at which A runs, and B the speed at which B runs. We write the results of the two races thus: 

- For the first race: 
  \[ 2L + 150 = 43L \]
  \[ A = \frac{B}{43} \]

- For the second race: 
  \[ 2L + 180 = 98L \]
  \[ 4/9A = 98B - 9B \]

Solve each one for A/B, and eliminate the A/B:

\[ L + 75 = 3L \]
\[ 4L = L + L + L - 180 \]
\[ L = 100 \]

Also solved by Robert Bart; Richard Hess, John Steihler, Kelly Woods, John Chandler, Winslow Hartford, Donald Boynton, Matthew Fountain, David Simen, Ray Kinsey, Avi Ornstein, Phelps Meaker, Thomas Lewis, Gardner Perry, Steven Feldman, Ken Rosato, Harry Zarbo, Bob High, Randall Whitman, Linda Kalver (who obtained her solution by means of a revised method of puzzle by word processing her demonstration that the original problem was faulty), and Roger Spellman.

JUL 3. A Cryptoquiz from David Wagner:

**PUUJU HC QGSL IQFUQU EUPNQO QY JCCHPFHPA PNUJ FOJGRUEUIG QY KZIU CRHUGU - PNJP QY KHRBHGQ EUG QY AUGHCZC**

H. William Wardle, with a little help from some home-brew software, had little trouble with this one. He writes:

Below is my solution to your Cryptos puzzle, in the form of a screen dump, so that the original text with the solution written above appear together. Some 3 or 4 years ago I wrote a short computer program to assist in solving these crypto puzzles. Often, like yours, they are printed with insufficient room above to write the solution, so why not copy the coded text onto a computer screen? Then, to help in solving, one guess of a coded letter instantly makes all identical letters in the puzzle likewise change to the guessed letter, and so on. It’s quick and fun. This puzzle took only 20 minutes, so you might as well have made it a Speed Department puzzle. By the way, it looks like someone left out a word between the 3rd & 4th words of the coded text: perhaps "OUP" which translates to "ONE". THERE IS ONLY PROVED METHOD OF ASSISTING PUUJU HC QGSL IQFUQU EUPNQO QY JCCHPFHPA THE ADVANCEMENT OF PURE SCIENCE - THAT PNUJ FOJGRUEUIG QY KZIU CRHUGU. PNJP OF PICKING MEN OF GENIUS, BACKING THEM QY KHRBHGQ EUG QY AUGHCZC. DJRBHGA PNUE NUJFHL SLO SJUHFLA PNUE PQ OHIUUP PNUE CUSUC.

**MEJUC DILJGP RQGJGP**

NAME

STREET

CITY STATE ZIP

MAIL TO: MIT ALUMNI CAREER SERVICES, 8M 12-170, CAMBRIDGE, MA 02139

617-253-4733

DJOJOHDA PNUE NUHFL SLO SJUHFLA PNUE PQ OHIUUP PNUE CUSUC.**

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rather easy, despite being in his 89th year). Alan Sopelsak, Bob Sutton, Fred Furland, Steven Feldman, Walter Caldwell, Edward Cohen, Peter Weidner, Ken Rosato, Harry Zaremba, Jeffrey Harris, Marcia Spellman, David Guss, and Alan Taylor.

JUL 4. Matthew Fountain has been looking at "constellations," a subject previously studied by Euler. Fountain wants us to find eight distinct positive integers such that
\[ A + B + C + D = E + F + G + H \]
\[ A^1 + B^2 + C^3 = D^4 = E^5 + F^6 + G^7 + H^8 \]

First time responder Richard Steuer considered this problem a very pleasant exercise, particularly since he has lots of time available in retirement. He writes:

Given \( A + B + C + D = E + F + G + H \) let \( Q = (D - H) = (E - A) = (G - C) = (B - F), L = B - A = C - R. Then D - H = L + Q \)

and

\[ A^1 + B^2 + C^3 = D^4 = E^5 + F^6 + G^7 + H^8 \]

\[ (n,r) \text{ stands for } n/r \text{ which equals } \frac{n!}{r!(n-r)!} \]

\[ (n,r,s) \text{ stands for } n/r/s \text{ which equals } \frac{n!}{r!(n-r)!} \]

I view the deck of cards as ordered from top to bottom: If the Kings are randomly placed in the deck, a King can marry up if a Queen is immediately above it, or can marry down if a Queen is immediately below it. Thus for every King, there are nominally two marriage slots or eight in the deck. The probability of at least one marriage is one minus the probability of no marriages. No marriages occur if all four Queens land in the remaining unoccupied 40 positions. This can occur in 40(40) unique ways, versus the 48! distinct ways the Queens can be distributed in the spaces not occupied by Kings. Thus, nominally the probability of at least one marriage is 1 - 91,390,194,580 = 53.03%.

CONTESTED SLOTS

But sometimes Kings are placed such that there are fewer than eight marriage slots. First, there is the possibility that a King is on top of the deck and cannot improve itself by marriage. (My feminist wife sees this as an unlikely event.) Second, there is the possibility that a King is on the bottom of the deck and cannot marry down. (Vicki sees this as a very likely event.) Third, a marriage slot can be contested by a superior King and an inferior King, giving the Queen a choice of marrying up or marrying down. (This is much like having a coed sit between two male students in class.) Fourth, two Kings may be adjacent, causing the elimination of two marriage slots. One King can't marry up, the other can't marry down. (Much the situation in most campus living groups.) Finally, the above situations can occur in combination.

POSSIBLE COMBINATIONS

The relevant combinations include:

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<th>FORM</th>
<th>KINGS</th>
<th>LOSSES</th>
<th>PERMS</th>
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<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>B</td>
<td>K7</td>
<td>1</td>
<td>1</td>
<td>2</td>
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<tr>
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<td>K7K</td>
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<td>1</td>
<td>2</td>
</tr>
<tr>
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<td>K7K7</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
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<tr>
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<td>2</td>
</tr>
<tr>
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<td>2</td>
</tr>
<tr>
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<td>3</td>
<td>2</td>
</tr>
<tr>
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<td>3</td>
<td>3</td>
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</tr>
<tr>
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<tr>
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<td>KKKK</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

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WHERE:
* ? indicates any card except K
* An initial K indicates a K on top or bottom

**Losses are the potential marriage slots forgone by this combination**

**Perm s are the valid ways to arrange the cards within the combination.**

EIGHT SLOTS

The nominal case with 8 slots occurs when there are 4A and 4J. These can be combined (44) or 135,751 ways. In comparison, there are 524 or 270,725 ways to combine 4K and 487. Thus, the nominal case has a probability of 135,751/270,725, or 50.14%. The joint probability of the nominal case and no marriages is 0.5014 × 0.50323 = 0.2599.

SEVEN SLOTS

Seven slots can occur in two different forms:

**FORM COMBOS PERMS WAYS**

1B4A47 (44:3) 2 26,498
1C2A47 (44:1,2) 1 39,732

or a total of 66,220 ways, with a probability of 66,220/270,725 = 0.2446. If there are seven marriage slots, the four Kings can avoid marriage by being placed in the remaining 41 slots (41:4) or 101,270 ways. The probability of at least one marriage is thus 101,270/194,500 = 0.5255. The joint probability of seven marriage slots and at least one marriage is 24.46% × 0.5255 = 0.1273.

SIX SLOTS

Six marriage slots can occur in six different forms:

**FORM COMBOS PERMS WAYS**

1E2A47 (45:1,2) 1 42,570
1D2A47 (44:2) 2 1,892
1G1A47 (44:1,1) 1 1,892
2B2A47 (44:2) 1 946
1B1C1A47 (44:1.1) 2 3,784
2C27 (44:2) 1 946

or a total of 52,030 ways, with a probability of 19.21%. If there are six marriage slots, the four Queens can avoid marriage by being placed in the remaining 42 slots (42:4) or 111,930 ways. The probability of at least one marriage is thus 42.48%. The joint probability is 8.16%.

FIVE SLOTS

Five marriage slots can occur in ten different forms:

**FORM COMBOS PERMS WAYS**

1J2A47 (45:2) 2 1,980
1JH47 (45:1,1) 2 3,960
1H1A47 (44:1) 2 88
1H47 (44:1) 1 44
1E1B1A47 (45:1,1) 2 3,960
1E1C1G47 (45:1,1) 1 980
1D1B1A47 (44:1) 2 88
1D1C1G47 (44:1) 2 88
1G1B47 (44:1) 2 88
2B1C1G47 (44:1) 1 44

or a total of 12,320 ways, or a probability of 4.55%. If there are five marriage slots, the four Queens can avoid marriage by being placed in the remaining 43 slots (43:4) or 122,410 ways. The probability of at least one marriage is 36.58%. The joint probability is 1.66%.

FOUR SLOTS

Four marriage slots can occur in 12 different forms:

**FORM COMBOS PERMS WAYS**

1K1A41 (46:1,1) 1 2,070
1K1A47 (45:1) 4 180
1L4A41 (45:1) 3 135
1N4A47 (44:0) 2 2
1F1B1A47 (45:1) 2 90
1F1C47 (45:1) 2 90
1H1B47 (45:1) 4 180
1H1B47 (44:1) 2 2
2E47 (46:2) 1 1,035
2D2E47 (45:1) 2 90
2D47 (44:0) 1 1
1E2B47 (45:1) 1 45

or a total of 3,920 ways, or a probability of 1.45%. If there are four marriage slots, the four Queens can avoid marriage by being placed in the remaining 44 slots (44:4) or 135,751 ways. The probability of at least one marriage is 30.23%. The joint probability is 0.44%.

THREE SLOTS

Three marriage slots can occur in seven different forms:

**FORM COMBOS PERMS WAYS**

1L1A47 (46:1) 2 92
1Q47 (46:1) 3 138
1P47 (46:0) 6 6
1K1B47 (46:1) 2 92
1J1B47 (45:0) 4 4
1F1E47 (46:1) 2 92
1F1D47 (46:0) 2 92

or a total of 426 ways, or a probability of 0.16%. If there are three marriage slots, the four Queens can avoid marriage by being placed in the remaining 45 slots (45:3) or 148,995 ways. The probability of at least one marriage is 23.43%. The joint probability is 0.04%.

TWO SLOTS

Two marriage slots can occur in four different forms:

**FORM COMBOS PERMS WAYS**

1S467 (47:1) 1 47
1R467 (46:0) 6 6
2F467 (46:0) 2 2
1L1B46 (46:0) 2 2

or a total of 56 ways, or a probability of 0.02%. If there are two marriage slots, the four Queens can avoid marriage by being placed in the remaining 46 slots (46:4) or 163,185 ways. The probability of at least one marriage is 16.13%. The joint probability is 0.0005%.

ONE SLOT

One marriage slot can occur in only one form:

**FORM COMBOS PERMS WAYS**

1T478 (47:0) 2 2

or only 2 ways, or a probability of 0.007%. If there is only one marriage slot, the four Queens can avoid marriage by being placed in the remaining 47 slots (47:4) or 178,365 ways. The probability of at least one marriage is 8.33%. The joint probability is 0.00006%.

SUMMARY

Four Kings can be placed in a deck of 52 cards in (524) or 270,725 distinct ways. Most of these combinations result in eight marriage slots. The other combinations result in as few as one marriage slot. Four Queens can be placed in the remaining 48 cards in (484) or 194,580 distinct ways. The following table lists the possible number of marriage slots, the possible number of combinations that produce those marriage slots (and the associated probability versus 270,725), the possible ways for the Queens to avoid any marriages out of the 194,580 positions, and the resulting joint probability of at least one marriage.

**SLOTS KINGS (%)**

`| 270,725 | 100.00 |
`| 260,772 | 77.17 |
`| 245,340 | 89.93 |
`| 224,992 | 93.51 |
`| 205,638 | 96.20 |
`| 189,541 | 98.89 |
`| 174,329 | 100.00 |

The joint probability is 1.24%.

Thus, the probability is just under half. I tested this result using a Monte Carlo simulation with 1,000 deals and found 482 deals with at least one marriage.

Better Late Than Never

JAN 2. Naomi Markowitz has responded.

APR 2. Thomas Harriman and Ken Rosato have responded.

APR 3. Naomi Markowitz and Thos Harriman have responded.

APR 4. Thos Harriman and Ken Rosato have responded.

MAY 1. Sol Vidor has responded.

MAY 2. Naomi Markowitz, Ken Rosato and Sol Vidor have responded.

MAY 3. Sol Vidor and Ken Rosato have responded.

MAY 4. Sol Vidor has responded.

MAY 5. Ken Rosato and Sol Vidor have responded.

JUL 12. Wilbur DeHart can move just one match and be left with no triangles—he uses the match to burn all the others!

Proposers' Solutions To Speed Problems

SD 1. By January 2000, since the yearly problem generating values from 1-100 using the four digits of the year 2000 have only three solutions [Don't count on it—ed.].

SD 2. All of them.