ALLAN J. GOTTLIEB, '67

Funny Money

After using Matthew Fountain's problem for N/D 1 below, we have just one computer and no chess problems remaining. As mentioned last issue, this may be the market speaking. If, however, you would like to see more chess and computer problems, please send them in.

Wilbur DeHart, while commenting on JUL 1, claims that an interesting version of bridge, which he calls "inverse bridge," can be played if the bidding proclaims the minimum number of tricks to be taken by the *defenders*. DeHart enjoys seeing the kings and queens fall on the lead of an ace!

I have long known that historians write more than mathematicians or computer scientists but still found the following observation from Matthew Fountain surprising:

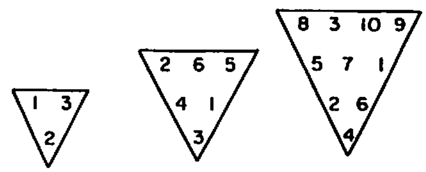
"As treasurer of our Historical Society I have inherited the papers and pictures of our most distinguished historian, who died at the age of 92. I have to inventory them for the executor of the estate. They were delivered to me in a pick-up truck as a car was too small."

We end with a remark from long-time contributer Mary Lindenberg, who ten years ago switched from teaching mathematics to high school students to teaching watercolor painting to adults. Ms. Lindenberg reports that, when her present students discover her previous vocation, they invariably ask if she started out "painting by the numbers"?

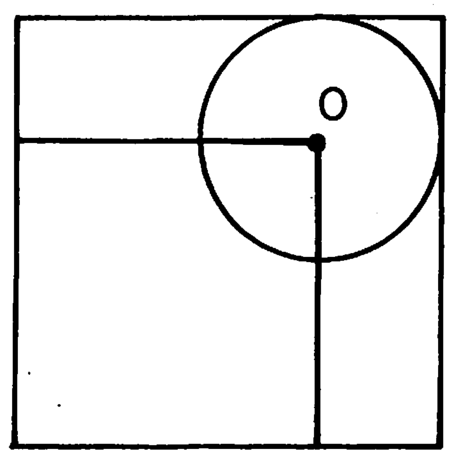
Problems

N/D 1. Matthew Fountain proposes, as a computer-oriented problem, finding the smallest nonprime integer, N > 1, that divides $2^{N-1} - 1$. As a noncomputer addition, explain why there are so "few" such Ns.

N/D 2. Nob. Yoshigahara wants you to find the rule for each of the following triangular configurations of numbers and to produce a configuration one size larger.



N/D 3: Arthur Lewbel wants you to find the area of the small square without using Pythagoras's theorem. The area of the large square is one. The circle has center O, is tangent to two sides of the large square, and passes through the center of the large square.



N/D 4. Phelps Meaker wants to know the radius of a sphere circumscribing a regular tetrahedron.

N/D 5. Our final regular problem is from Gordon Rice, who notes that, by successive flips of a coin, we may generate a random number between 0 and $2^N - 1$. Each flip determines one digit in the binary representation of the number; heads it's a one, tails it's a zero.

Suppose we have a biased coin, which comes up heads with probability P. What is the expected value of an N-bit number generated with such a coin?

Speed Department

SD 1. Greg Detefano wants to know by what date Puzzle Corner will find it necessary to make a policy change?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

SD 2. Jim Landau asks how many successful jumps must a parachutist make before s/he can graduate from jump school?

Solutions

JUL 1. Robert Bart offers a problem he attributes to Robert Darves in which South is to make seven spades with the assistance of all four players.

North
♠ 8 5
♥ Q 9 7 6 5 4 2
♦ 5
♣ 4 3 2

West	East
♠ K 10 7 4	♠ Q 9 6 3
♥ K J 6	♥ A 10
♦ J 10 3	♦ A K Q 4
♣ A Q 10	♣ K J 9

South
♠ A J 2
♥ 3
♦ 9 8 7 6 2
♣ 8 7 6 5

The following solution is from Brad Pines:
I am pleased to be able to finally submit an answer to one of the Puzzle Corner problems; better yet I am fairly certain that the answer is correct.

The unfortunate typographical error that resulted in (2) 6H and (0) 8H luckily does not affect the solution. The cards can be in either location and the solution remains the same. I have included a specific solution that shows the play of all cards on every trick. There are many other solutions that follow the same line of play. Leading or discarding cards in a different order at certain points in the play will not effect the outcome. I will attempt to give a general solution at the end.

Contract 7S Opening Lead: J of Hearts

Trick	South	West	North	East
1	3H	JH	QH	10H
2	2S	KH	2H	AH
3	2D	3D	5D	4D
4	5C	6/8H	9H	QD
5	6C	10D	7H	KD
6	7C	JD	6/8H	AD
7	8C	10C	5H	9C
8	AS	KS	4H	QS
9	9D	QC	2C	JC
10	8D	AC	3C	KC
11	7D	7S	8S	6S
12	JS	10S	4C	9S
13	6D	4S	5S	3S

General Solution for this line of play.
For South: Clubs can be played in any order on tricks 4, 5, 6, 7. Diamonds can be led in any order on tricks 9, 10, 11, 13.
For West: Diamonds can be played in any order on tricks 5, 6. Clubs can be played in any order on tricks 7, 9, 10.
For North: Q Hearts must be played at trick 1, 9 Hearts must be led at trick 4, any other heart can be led at tricks 2, 5, 6, 7, 8. Clubs can be played in any order on tricks 9, 10, 12.
For East: Diamonds can be played in any order on tricks 4, 5, 6. Clubs can be played in any order on tricks 7, 9, 10.
The spades played by West, East, and South on tricks 8 and 12 can be switched, as long as A, K, Q and J, 10, 9 are grouped in the same trick. The spades played by West, East, and North on tricks 11 and 13 can be switched as long as 8, 7, 6 and 5, 4, 3 are grouped in the same trick.
With West-East NOT assisting and with good play (West leading trump) North-South can be limited to only (1) trick, the A of trump.

Also solved by Robert Bart, Richard Hess, Eric Raiten, Charlie Larson, John Stiehler, Doug McMahon, Sonney Taragin, Kelly Woods, Leonard Nissim, Wilbur DeHart, Dennis Loring, John Chandler, Winslow Hartford, Bill Huntington, Donald Boynton, Matthew Fountain, Doug Van Patter, Mark Foringer, Graham Woerner, and the proposer (who points out that the bridge problem is due to Robert Darvas).

JUL 2. As noted below, problem F/M 4 was misprinted (sorry). Here is the corrected version. *A* offers to run three laps while *B* does two but gets only 150 yards into his third lap when *B* wins. He then offers to run four laps to *B*'s three, and quickens his pace in the ratio of 4:3. *B* also quickens his pace, in the ratio of 9:8, but in the second lap falls off to his original pace, and in the third goes only 9 yards for the 10 he went in the first race. *A* wins the race by 180 yards. How long is each lap?

The following very generous letter is from first-time responder David Gluss:
As this is the first time I have sent answers to a puzzle column, I have to say that I am one of the "silent majority" who try puzzles but never write in about it. I have followed this practice because, after all, it's no big trick to get the answers to a few puzzles, right? Unfortunately a recent bad experience with a couple of Golomb's puzzles (I found proofs for both of my answers, and they both turned out wrong) convinced me that it's not sporting to avoid risking ridicule for wrong answers, even though the risk might be tiny (ha ha). Another thing I should say is that DOING PUZZLES IS FUN, and thank you very much, even for the ones I didn't answer, and all the ones I never wrote about.

Let L be the length of the track, A be the speed at which *A* runs, and B the speed at which *B* runs. We write the results of the two races thus:

$$\frac{2L + 150}{A} = \frac{2L}{B},$$

$$\frac{4L}{4/3\ A} = \frac{L}{9/8\ B} + \frac{L}{B} + \frac{L - 180}{9\ B}.$$

Solve each one for A/B, and eliminate the A/B:

$$\frac{L + 75}{L} = \frac{A}{B} = \frac{3\ L}{3\ L - 200}$$

We can now cross-multiply and solve for L.
$3\ L^2 = 3\ L^2 + 25\ L - 15000$, or
$L = 600$.

Also solved by Robert Bart, Richard Hess, John Stiehler, Kelly Woods, John Chandler, Winslow Hartford, Bill Huntington, Donald Boynton, Matthew Fountain, David Simen, Ray Kinsley, Avi Ornstein, Phelps Meaker, Thomas Lewis, Gardner Perry, Steven Feldman, Ken Rosato, Harry Zaremba, Bob High, Randall Whitman, Linda Kalver (who obtained her solution to this revised problem by word processing her demonstration that the original problem was faulty), and Roger Spellman.

JUL 3. A Cryptoquiz from David Wagner:

PNUIU HC QGSL KIQFUO EUPNQO QY JCCHCPHGA PNU JOFJGRUEUGP QY KZIU CRHUGRU - PNJP QY KHRBHGA EUG QY AUGHZC, DJRBHGA PNUE NUJFHSL, JGO SUJFHGA PNUE PQ OHIURP PNUECUSFUC.

MJEUC DILJGP RQGJGP

H. William Wardle, with a little help from some home-brew software, had little trouble with this one. He writes:
Below is my solution to your Cryptoquiz puzzle, in the form of a screen dump, so that the original text with the solution written above appear together. Some 3 or 4 years ago I wrote a short computer program to assist in solving these crypto puzzles. Often, like yours, they are printed with insufficient room above to write the solution, so why not copy the coded text onto a computer screen? Then, to help in solving, one guess of a coded letter instantly makes all identical letters in the puzzle likewise change to the guessed letter, and so on. It's quick and fun. This puzzle took only 2½ minutes, so you might as well have made it a Speed Department puzzle. By the way, it looks like someone left out a word between the 3rd & 4th words of the coded text: perhaps "QGU" which translates to "ONE."

THERE IS ONLY PROVED METHOD OF ASSISTING
PNUIU HC QGSL KIQFUO EUPNQO QY JCCHCPHGA

THE ADVANCEMENT OF PURE SCIENCE - THAT
PNU JOFJGRUEUGP QY KZIU CRHUGRU - PNJP

OF PICKING MEN OF GENIUS, BACKING THEM
QY KHRBHGA EUG QY AUGHZC, DJRBHGA PNUE

HEAVILY, AND LEAVING THEM TO DIRECT
NUJFHSL, JGO SUJFHGA PNUE PQ OHIURP

THEMSELVES. — JAMES BRYANT CONANT
PNUECUSFUC. — MJEUC DILJGP RQGJGP

Also solved by Robert Bart, Richard Hess, John Chandler, Winslow Hartford, Donald Boynton, Matthew Fountain, David Simen, Ray Kinsley, Peter Silverberg, Bob Rosin, Avi Ornstein, Phelps Meaker (who reports that he found this problem rather easy, despite being in his 89th year), Alan Sopelak, Bob Sutton, Fred Furland, Steven Feldman, Walter Caldwell, Edward Cohen, Peter Weuder, Ken Rosato, Harry Zaremba, Jeffrey Harris, Marcia Spellman, David Gluss, and Alan Taylor.

JUL 4. Matthew Fountain has been looking at "constellations," a subject previously studied by Euler. Fountain wants us to find eight distinct positive integers such that

$A + B + C + D = E + F + G + H$

$A^2 + B^2 + C^2 + D^2 = E^2 + F^2 + G^2 + H^2$

and

$A^3 + B^3 + C^3 + D^3 = E^3 + F^3 + G^3 + H^3$

First time responder Richard Steuer considered this problem a very pleasant exercise, particularly since he has lots of time available in retirement. He writes:

Given $A + B + C + D = E + F + G + H$, let $Q = (D - H) - (E - A) = (G - C) - (B - F)$, $L = E = A$, and $R = B - F$. Then $D - H = L + Q$ and $G - C = R + Q$. Given

$A^2 + B^2 + C^2 + D^2 = E^2 + F^2 + G^2 + H^2$,

then

$(D^2 - H^2) - (E^2 - A^2) = (G^2 - C^2) - (B^2 - F^2)$.

This can be reduced to

$$L(H - A) + Q(H + L) = R(C - F) + Q(C + R). \quad (1)$$

Similarly,

$(D^3 - H^3) - (E^3 - A^3) = (G^3 - C^3) - (B^3 - F^3)$

can be reduced to:

$$L(H - A)(H + A + L) + Q(H + L)(H + L + Q) = R(C - F)(C + F + R) + Q(C + R)(C + R + Q). \quad (2)$$

Consider the case of $Q = 0$; (1) and (2) can then be reduced to:

$L(H - A) = R(C - F)$

and

$H + A + L = C + F + R$

These can be combined to yield:

$C = A + 1/2[(1 + L/R)(H - A) + L - R]$

and

$F = C - L/R(H - A)$

It can be shown that if a constant is added to each term of a solution, it will yield another solution; therefore we can arbitrarily set $A = 1$. If L and R are chosen as positive integers with $L < R$, then a family of solutions can be obtained, where:

A = 1
B = 1 + R + KX
C = 1 + L + KY
D = 1 + R + L + KZ
E = 1 + L
F = B − R
G = C + R
H = D − L

K is any positive integer greater than L/X, and also greater than (R − L)/(Y − X). X, Y and Z depend upon whether (L + R) is odd or even:

L + R:	ODD	EVEN
X =	(R − L)	1/2 (R − L)
Y =	(R + L)	1/2 (R + L)
Z =	2R	R

The condition on the size of K insures that the eight terms of the solution will all be different integers. The lowest solution appears to be obtained in the case of L = 1, R = 2 and K = 2, yielding:

1 + 5 + 8 + 12 = 2 + 3 + 10 + 11.

In this case X = 1, Y = 3 and Z = 4, so the next larger solution in this family (for K = 3) is

1 + 6 + 11 + 16 = 2 + 4 + 13 + 15.

Varying L, R and K yields an infinite number of solutions for the case of Q = 0. There may also be other solutions for non-zero values of Q.

Also solved by Robert Bart, Richard Hess, Donald Boynton, Harry Zaremba, and the proposer.

JUL 5. Walter W. Hill and Walter L. Hill (no kidding) want to know the probability that at least one marriage occurs in a normal deck of 52 cards. A marriage is said to occur if a King and Queen are consecutive, e.g., the King of clubs is the 13th card and the Queen of hearts is the 14th.

Mark Lively sent us a very thorough solution, which uses the following two abbreviations for combinations:

(n:r) stands for $\binom{n}{r}$, which equals $\frac{n!}{r!(n-r)!}$

(n:r,s) stands for $\binom{n}{r\ s}$, which equals $\frac{n!}{r!s!(n-r-s)!}$

I view the deck of cards as ordered from top to bottom. If the Kings are randomly placed in the deck, a King can marry up if a Queen is immediately above it, or can marry down if a Queen is immediately below it. Thus for every King, there are nominally two marriage slots or eight in the deck. The probability of at least one marriage is one minus the probability of no marriages. No marriages occur if all four Queens land in the remaining, unspecified 40 positions. This can occur in (40:4) distinct ways, versus the (48:4) distinct ways the Queens can be distributed in the spaces not occupied by Kings. Thus, nominally the probability of at least one marriage is 1-91,390/194,580 = 53.03%.

CONTESTED SLOTS

But sometimes Kings are placed such that there are fewer than eight marriage slots. First, there is the possibility that a King is on top of the deck and cannot improve itself by marriage. (My feminist wife sees this as an unlikely event.) Second, there is the possibility that a King is on the bottom of the deck and cannot marry down. (Vicki sees this as a very likely event.) Third, a marriage slot can be contested by a superior King and an inferior King, giving the Queen a choice of marrying up or marrying down. (This is much like having a coed sit between two male students in class.) Fourth, two Kings may be adjacent, causing the elimination of two marriage slots. One King can't marry up, the other can't marry down. (Much the situation in most campus living groups.) Finally, the above situations can occur in combination.

POSSIBLE COMBINATIONS

The relevant combinations include:

ID	FORM	KINGS	LOSSES	PERMS
A	?K?	1	0	1
B	K?	1	1	2
C	?K?K?	2	1	1
D	K?K?	2	2	2
E	?KK?	2	2	1
F	KK?	2	3	2
G	?K?K?K?	3	2	1
H	K?K?K?	3	3	2
I	?KK?K?	3	3	2
J	KK?K?	3	4	4
K	?KKK?	3	4	1
L	KKK?	3	5	2
M	?K?K?K?K?	4	3	1
N	K?K?K?K?	4	4	2
O	?KK?K?K?	4	4	3
P	KK?K?K?	4	5	6

Q	?KKK?K?	4	5	3
R	KKK?K?	4	6	6
S	?KKKK?	4	6	1
T	KKKK?	4	7	2

WHERE:

- ? indicates any card except K
- An initial K indicates a K on top or bottom
- Losses are the potential marriage slots forgone by this combination
- Perms are the valid ways to arrange the cards within the combination.

EIGHT SLOTS

The nominal case with 8 slots occurs when there are 4A and 40?. These can be combined (44:4) or 135,751 ways. In comparison, there (52:4) or 270,725 ways to combine 4K and 48?. Thus, the nominal case has a probability of 135,751/270,725, or 50.14%. The joint probability of the nominal case and no marriages is 50.14% × 53.03% = 26.59%.

SEVEN SLOTS

Seven slots can occur in two different forms:

FORM	COMBOS	PERMS	WAYS
1B3A41?	(44:3)	2	26,488
1C2A41?	(44:1,2)	1	39,732

or a total of 66,220 ways, with a probability of 66,220/270,725 = 24.46%. If there are seven marriage slots, the four Queens can avoid marriage by being placed in the remaining 41 slots (41:4) or 101,270 ways. The probability of at least one marriage is thus 1-101,270/194,580 = 47.95%. The joint probability of seven marriage slots and at least one marriage is 24.46% × 47.95% = 11.73%.

SIX SLOTS

Six marriage slots can occur in six different forms:

FORM	COMBOS	PERMS	WAYS
1E2A42?	(45:1,2)	1	42,570
1D2A42?	(44:2)	2	1,892
1G1A42?	(44:1,1)	1	1,892
2B2A42?	(44:2)	1	946
1B1C1A42?	(44:1,1)	2	3,784
2C42?	(44:2)	1	946

or a total of 52,030 ways, with a probability of 19.21%. If there are six marriage slots, the four Queens can avoid marriage by being placed in the remaining 42 slots (42:4) or 111,930 ways. The probability of at least one marriage is thus 42.48%. The joint probability is 8.16%.

FIVE SLOTS

Five marriage slots can occur in ten different forms:

FORM	COMBOS	PERMS	WAYS
1F2A43?	(45:2)	2	1,980
1I1A43?	(45:1,1)	2	3,960
1H1A43?	(44:1)	2	88
1M43?	(44:1)	1	44
1E1B1A43?	(45:1,1)	2	3,960
1E1C43?	(45:1,1)	1	1,980
1D1B1A43?	(44:1)	2	88
1D1C43?	(44:1)	2	88
1G1B43?	(44:1)	2	88
2B1C43?	(44:1)	1	44

or a total of 12,320 ways, or a probability of 4.55%. If there are five marriage slots, the four Queens can avoid marriage by being placed in the remaining 43 slots (43:4) or 123,410 ways. The probability of at least one marriage is 36.58%. The joint probability is 1.66%.

FOUR SLOTS

Four marriage slots can occur in 12 different forms:

FORM	COMBOS	PERMS	WAYS
1K1A44?	(46:1,1)	1	2,070
1K1A44?	(45:1)	4	180
1O44?	(45:1)	3	135
1N44?	(44:0)	2	2
1F1B1A44?	(45:1)	2	90
1F1C44?	(45:1)	2	90
1I1B44?	(45:1)	4	180
1H1B44?	(44:0)	2	2
2E44?	(46:2)	1	1,035
2D2E44?	(45:1)	2	90
2D44?	(44:0)	1	1
1E2B44?	(45:1)	1	45

or a total of 3,920 ways, or a probability of 1.45%. If there are four marriage slots, the four Queens can avoid marriage by being placed in the remaining 44 slots (44:4) or 135,751 ways. The probability of at least one marriage is 30.23%. The joint probability is 0.44%.

THREE SLOTS

Three marriage slots can occur in seven different forms:

FORM	COMBOS	PERMS	WAYS
1L1A45?	(46:1)	2	92
1Q45?	(46:1)	3	138
1P45?	(45:0)	6	6
1K1B45?	(46:1)	2	92
1J1B45?	(45:0)	4	4
1F1E45?	(46:1)	2	92
1F1D45?	(45:0)	2	2

or a total of 426 ways, or a probability of 0.16%. If there are three marriage slots, the four Queens can avoid marriage by being placed in the remaining 45 slots (45:4) or 148,995 ways. The probability of at least one marriage is 23.43%. The joint probability is 0.04%.

TWO SLOTS

Two marriage slots can occur in four different forms:

FORM	COMBOS	PERMS	WAYS
1S46?	(47:1)	1	47
1R46?	(46:0)	6	6
2F46?	(46:0)	1	1
1L1B46?	(46:0)	2	2

or a total of 56 ways, or a probability of 0.02%. If there are two marriage slots, the four Queens can avoid marriage by being placed in the remaining 46 slots (46:4) or 163,185 ways. The probability of at least one marriage is 16.13%. The joint probability is 0.003%.

ONE SLOT

One marriage slot can occur in only one form:

FORM	COMBOS	PERMS	WAYS
1T47?	(47:0)	2	2

or only 2 ways, or a probability of 0.007%. If there is only one marriage slot, the four Queens can avoid marriage by being placed in the remaining 47 slots (47:4) or 178,365 ways. The probability of at least one marriage is 8.33%. The joint probability is 0.00006%.

SUMMARY

Four Kings can be placed in a deck of 52 cards in (52:4) or 270,725 distinct ways. Most of these combinations result in eight marriage slots. The other combinations result in as few as one marriage slot. Four Queens can be placed in the remaining 48 cards in (48:4) or 194,580 distinct ways. The following table lists the possible number of marriage slots, the possible number of combinations that produce those marriage slots (and the associated probability versus 270,725), the possible ways for the Queens to avoid any marriages out of the 194,580 positions, and the resulting joint probability of at least one marriage.

SLOTS	KINGS	(%)	QUEENS	(%)	JOINT
8	135,751	50.14	91,390	46.96	26.5922
7	66,220	24.46	101,270	52.05	11.7298
6	52,030	19.22	111,930	57.52	8.1634
5	12,320	4.55	123,410	63.42	1.6645
4	3,920	1.45	135,751	69.77	0.4378
3	426	0.16	148,995	76.57	0.0369
2	56	0.02	163,185	83.87	0.0033
1	2	0.00	178,365	91.67	0.0001
SUM	270,725	100.00			48.6279

Thus, the probability is just under half. I tested this result using a Monte Carlo simulation with 1,000 deals and found 482 deals with at least one marriage.

Also solved by Richard Hess, Donald Boynton, Matthew Fountain, David Simen, Ken Rosato, Harry Zaremba, Bob High, David Smith, David Gluss, Guy Jacobson, Randall Whitman, and the proposer.

Better Late Than Never

JAN 2. Naomi Markovitz has responded.

APR 2. Thomas Harriman and Ken Rosato have responded.

APR 3. Naomi Markovitz and Thos Harriman have responded.

APR 4. Thos Harriman and Ken Rosato have responded.

M/J 1. Sol Vidor has responded.

M/J 2. Naomi Markovitz, Ken Rosato and Sol Vidor have responded.

M/J 3. Sol Vidor and Ken Rosato have responded.

M/J 4. Sol Vidor has responded.

M/J 5. Ken Rosato and Sol Vidor have responded.

JUL SD2. Wilbur DeHart can move just one match and be left with no triangles—he uses the match to burn all the others!

Proposers' Solutions To Speed Problems

SD 1. By January 2000, since the yearly problem (generating values from 1-100 using the four digits of the year) will have only three solutions [Don't count on it!—ed.].

SD 2. All of them.