PUZZLE CORNER
ALLAN J. GOTTLIEB, '67

Canceled Checks

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

In each issue I present five regular problems (the first of which is chess, bridge, or computer-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later, one submitted solution is printed for each problem; I also list other readers whose solutions were successful. For example, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solutions to this issue's speed problems are given below. Only rarely are comments on speed problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

Problems

OCT 1. The market may be speaking. This month we are scheduled to begin with a chess problem but none have been submitted and we will begin instead with a bridge problem from Tom Harriman. If you would like to see chess problems, please submit them!

OCT 2. Gordon Rice supposes that some time in the (not too distant?) future, the art of pencil-and-paper arithmetic has been forgotten. Also, your computer is giving off smoke. With no way to add, subtract, multiply, or divide except an eight-digit calculator, can you evaluate the following expressions?

3 \cdot 180997^2 - 313496^2
3 \cdot 37467^2 - 64896^2

OCT 3. John Rule has a three-digit number that, when divided by the product of its digits, yields as quotient the hundredth digit. Rule wants you to find this number and show that it is unique.

OCT 4. David Evans notes that on an 8 x 8 checkerboard, if two squares of the same color are removed, it is impossible to cover the remaining 62 squares with 31 1 x 2 tiles (since each tile covers one white and one black square). Is the converse true, i.e., if you remove 2 squares of opposite colors, can the remaining 62 squares always be covered by 31 1 x 2 tiles?

OCT 5. Chuck Coltharp poses the following partitioning question. Let S be a finite set of size 4n and let P be a collection of partitions of S, each of which partitions S into two disjoint sets of size 2n. Let the ith partition be the two sets A_i and B_i. We require that, for i \neq j, A_i \cap B_j is of size n. The question is how large can P be, that is, for each n what is the largest number of partitions that can be found satisfying the above properties?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, 67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.
Speed Department

SD 1. Jim Landau wants two non-identical functions \( f(x) \) and \( g(x) \) such that \( \frac{df(x)}{dx} = g(x) \) and \( \frac{dg(x)}{dx} = f(x) \).

SD 2. Edward Wallner notes that both William Shakespeare and Miguel Cervantes died on April 23, 1616, and asks who died first?

Solutions

MfJ 1. We begin with a two-part chess problem that appeared in The Tech during 1984. First, in the figure below, find a helpmate in 7, i.e., black moves first and cooperates with white so that black is mated on white’s 7th move. Second, solve the same problem with the bishop on H1 gone.

![Chess Diagram](image)

According to Richard Hess, the key to this problem is not to promote all pawns to queens. Hess’s solution is:

**A: black white**

1. \( b7 \) \( p8 = a7 \)
2. \( n4 \) \( n2 \)
3. \( b4 \) \( p4x \)
4. \( n3 \) \( p6x \)
5. \( p8 = r \) \( p6 = r \)
6. \( r8 = q = q6 = q \)

B: black white

1. \( b7 \) \( p6 = q6 \)
2. \( c6 \) \( p5 = a5 \)
3. \( d5 \) \( p6 = a6 \)
4. \( e4 \) \( p7 = a7 \)
5. \( d3 \) \( p4 = q \)
6. \( g2 \) \( r1 \)
7. \( h1 = n = n5 = n \)

Also solved by Robert Bart, Matthew Fountain, Winslow Hartford, and James Walker.

MfJ 2. Nob. Yoshigahara wants you to put a unique digit from 1 to 9 in each of the nine boxes so that the sums in the circles are all equal.

Please paste in figure from MfJ

Gordon Rice was able to solve this without a computer-assisted search; his solution follows:

We require that the nine numbers A, B, C, D, E, F, G, H, and I, representing a permutation of 1, 2, ..., 9, satisfy, for a given sum S, the equations:

\[
\begin{align*}
A + B &= S \\
B + C + D &= S \\
D + E + F &= S \\
F + G + H &= S \\
H + I &= S \\
\end{align*}
\]

We need that

\[
A + 2B + C + 2D + E + 2F + G + 2H + I &= 55
\]

Let us call a set of values for B, D, F, and H (ignoring the order of assignment) an "overlap set". There are 26 possible overlap sets:

\[
S_{11} = S_{12} = S_{13} = S_{14} = S_{15} = \ldots
\]

The 14 overlap sets marked with an "x" may be eliminated by the rule that if two members of the overlap set add up to 5, no solution is possible. We establish this rule as follows.

The numbers chosen for B and H must be in the overlap set; the numbers chosen for A and I must not. If two numbers in the overlap set add up to 5, then they are both unavailable for B or H. That leaves two remaining. But at least one is too small (i.e., less than 5 - 9). The exception is S = 15, but there both pairs of the overlap set add up to 5.

That leaves 12 to try. We avoid symmetries by requiring that A be less than 1. "x" marks the point at which a trial fails.

**S set**

A B C D E F G H I

1 2 3 4 5 6 7 8 9

1. 1, 2, 3, 4, 5, 6, 7, 8, 9

2. 1, 2, 3, 4, 5, 6, 7, 8, 9

3. 1, 2, 3, 4, 5, 6, 7, 8, 9

4. 1, 2, 3, 4, 5, 6, 7, 8, 9

5. 1, 2, 3, 4, 5, 6, 7, 8, 9

6. 1, 2, 3, 4, 5, 6, 7, 8, 9

7. 1, 2, 3, 4, 5, 6, 7, 8, 9

8. 1, 2, 3, 4, 5, 6, 7, 8, 9

9. 1, 2, 3, 4, 5, 6, 7, 8, 9

10. 1, 2, 3, 4, 5, 6, 7, 8, 9

11. 1, 2, 3, 4, 5, 6, 7, 8, 9

12. 1, 2, 3, 4, 5, 6, 7, 8, 9

13. 1, 2, 3, 4, 5, 6, 7, 8, 9

14. 1, 2, 3, 4, 5, 6, 7, 8, 9

15. 1, 2, 3, 4, 5, 6, 7, 8, 9

16. 1, 2, 3, 4, 5, 6, 7, 8, 9

17. 1, 2, 3, 4, 5, 6, 7, 8, 9

18. 1, 2, 3, 4, 5, 6, 7, 8, 9

19. 1, 2, 3, 4, 5, 6, 7, 8, 9

20. 1, 2, 3, 4, 5, 6, 7, 8, 9

21. 1, 2, 3, 4, 5, 6, 7, 8, 9

22. 1, 2, 3, 4, 5, 6, 7, 8, 9

23. 1, 2, 3, 4, 5, 6, 7, 8, 9

24. 1, 2, 3, 4, 5, 6, 7, 8, 9

25. 1, 2, 3, 4, 5, 6, 7, 8, 9

26. 1, 2, 3, 4, 5, 6, 7, 8, 9

**S set**

A B C D E F G H I

1 2 3 4 5 6 7 8 9

1. 1, 2, 3, 4, 5, 6, 7, 8, 9

2. 1, 2, 3, 4, 5, 6, 7, 8, 9

3. 1, 2, 3, 4, 5, 6, 7, 8, 9

4. 1, 2, 3, 4, 5, 6, 7, 8, 9

5. 1, 2, 3, 4, 5, 6, 7, 8, 9

6. 1, 2, 3, 4, 5, 6, 7, 8, 9

7. 1, 2, 3, 4, 5, 6, 7, 8, 9

8. 1, 2, 3, 4, 5, 6, 7, 8, 9

9. 1, 2, 3, 4, 5, 6, 7, 8, 9

10. 1, 2, 3, 4, 5, 6, 7, 8, 9

11. 1, 2, 3, 4, 5, 6, 7, 8, 9

12. 1, 2, 3, 4, 5, 6, 7, 8, 9

13. 1, 2, 3, 4, 5, 6, 7, 8, 9

14. 1, 2, 3, 4, 5, 6, 7, 8, 9

15. 1, 2, 3, 4, 5, 6, 7, 8, 9

16. 1, 2, 3, 4, 5, 6, 7, 8, 9

17. 1, 2, 3, 4, 5, 6, 7, 8, 9

18. 1, 2, 3, 4, 5, 6, 7, 8, 9

19. 1, 2, 3, 4, 5, 6, 7, 8, 9

20. 1, 2, 3, 4, 5, 6, 7, 8, 9

21. 1, 2, 3, 4, 5, 6, 7, 8, 9

22. 1, 2, 3, 4, 5, 6, 7, 8, 9

23. 1, 2, 3, 4, 5, 6, 7, 8, 9

24. 1, 2, 3, 4, 5, 6, 7, 8, 9

25. 1, 2, 3, 4, 5, 6, 7, 8, 9

26. 1, 2, 3, 4, 5, 6, 7, 8, 9
The following solution is from John Chandler:

The probability density function of the second digit of a random constant is biased in the same way that the first, only not so strongly. I assert that the phrase "random physical constant" means just that the log of the constant has a uniform distribution, and I believe that assertion must be considered an axiom (which might not be true—remember, Dirac's "large number hypothesis" holds that large physical constants are all the same). Thus, just as the probability that the first digit is N is given by log N + 1/N, the probability that the second is N is given by

\[ P(N) = \log \left( \frac{11N}{9N+1} \right) + \log \left( \frac{21N}{20N+1} \right) + \log \left( \frac{31N}{30N+1} \right) + \cdots + \log \left( \frac{91N}{90N+1} \right) \]

In other words, the probability that the second digit is N, is, say, 2, is the sum of the probabilities that the first two digits are 12, 22, 32, 42, 52, 62, 72, 82, and 92. It is easy to show that Benford's law gives the proper normalization for the probability density, since the sum of all the individual terms of p(0) through p(9) is just log 10(10).

The following is a table of p(N).

<table>
<thead>
<tr>
<th>N</th>
<th>p(N)</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0.1199727</td>
</tr>
<tr>
<td>1</td>
<td>0.1138901</td>
</tr>
<tr>
<td>2</td>
<td>0.1088150</td>
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<tr>
<td>3</td>
<td>0.1042556</td>
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<tr>
<td>4</td>
<td>0.09980820</td>
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<td>5</td>
<td>0.09667724</td>
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<td>6</td>
<td>0.09337474</td>
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<tr>
<td>7</td>
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</tr>
<tr>
<td>8</td>
<td>0.08757005</td>
</tr>
<tr>
<td>9</td>
<td>0.08499735</td>
</tr>
</tbody>
</table>

Also solved by Jonathan Aronson, Matthew Fountain, Thomas Harriman, Winslow Hartford, Richard Hess, and Harry Zaremba.

MJ 5. Our last regular problem is not that regular; it is a crossword puzzle from Andrew Greene published last year in 'The Tech.' I have no objection to including these kinds of problems but want to know what you think. Our last solution is from Larry Bell:

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H A I L S M E N K W L U
I T T E R S A D O S H W W T Y
E V O L U T E N O R T O O
H E V E N L K T R E C K E R S
A E T Y N E R F R A Y E S
S H O T T E E D A V E Y
T M A N S I T R E T E N K E K
N E R N E D Y E V A N S
T U R E M T E M S A Y L A
E R N E R M I J E S E P
E N G E N E R A L N E S S T A T I C
C H E M I S T R Y N E R C E S E N
G E N E R A L N E S S T A T I C
E N G E N E R A L N E S S T A T I C
T E N N I S M T S N E R C E S E N
G E N E R A L N E S S T A T I C
E N G E N E R A L N E S S T A T I C
T E N N I S M T S N E R C E S E N
G E N E R A L N E S S T A T I C
E N G E N E R A L N E S S T A T I C
T E N N I S M T S N E R C E S E N
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As for the feedback you requested, I must say that although I do not consider myself a true crossword aficionado, I really did enjoy doing this one. The best part about it is the clues that are directed towards the "tech-minded" individual (e.g., me). Appropriate topics for crossword clues also include (my opinion): campus trivia, current "high tech" events, Star Trek, items from prior Tech Review articles, programming language keywords or concepts, etc.


Better Late than Never

JAN 3. Harry Zaremba now believes that the length of the rectangle can be reduced to 165.5.

MJ 5D1. Robert Bishop, Stephen Rawlinson, and Lorenzo Sadun note that 1900 was not a leap year according to the Gregorian calendar. Rawlinson suggests that perhaps the Julian calendar was being used and Bishop favors the conjecture that Frederic and W.S. Gilbert shared the widespread ignorance about the special status of 1900.

Proposer's Solution to Speed Problems

SD 1. \( -e^{-t} \) and \( e^{-t} \)

SD 2. Cervantes, Spain was using the Gregorian calendar and England the Julian.

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