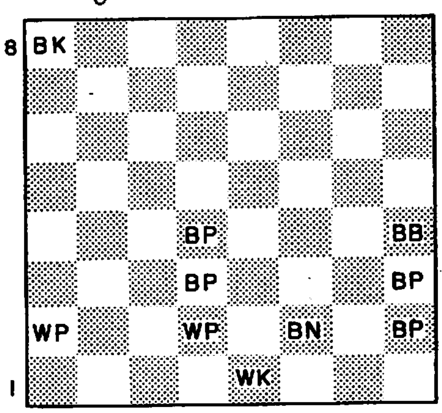
PUZZLE CORNER

ALLAN J. GOTTLIEB, '67

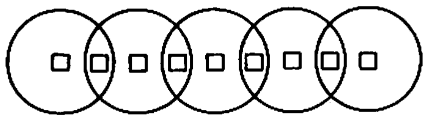
As I have mentioned previously, George Almasi and I recently wrote a book entitled *Highly Parallel Computing* published by Benjamin Cummings. George has asked me about the possibility of doing another book, this time based on "Puzzle Corner." The new book would mainly include selected problems and solutions but would also have a few of the more personal comments that have appeared from time to time. As some of you may remember, several times in the past I mentioned inquires about a possible *Best of Puzzle Corner* book, but none of them has ever developed. George, however, will make sure that a book appears if we decide to go ahead. What do you think? Go or No Go, and if the former any suggestions on content or format?

Problems

M/J 1. We begin with a two-part chess problem that appeared in *The Tech* during 1984. First, in the figure below, find a helpmate in 7, i.e., black moves first and cooperates with white so that black is mated on white's 7th move. Second, solve the same problem with the bishop on H4 gone.



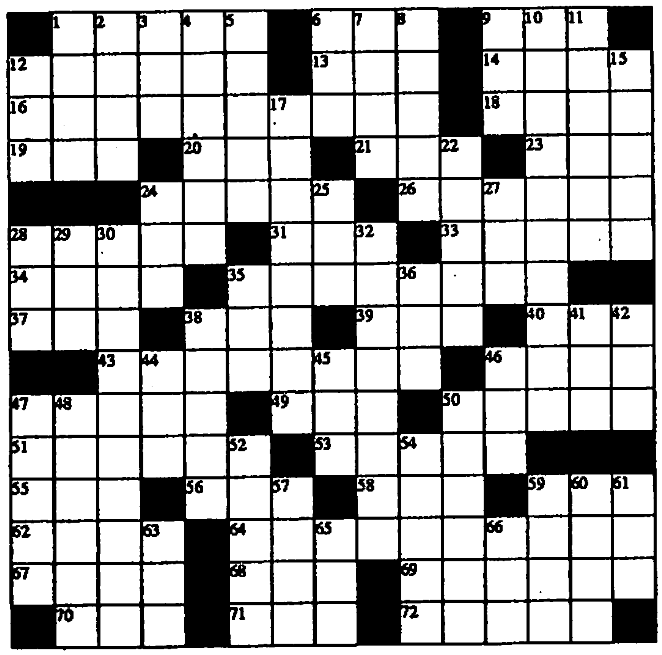
M/J 2. Nob. Yoshigahara wants you to put a unique digit from 1 to 9 in each of the nine boxes so that the sums in the circles are all equal.



M/J 3. Thomas Murley asks, for a random physical constant, what is the probability that its *second* digit is n?

M/J 4. Robert Bart extends a problem posed last year and asks for the smallest positive integer A_i such that $(A_i)^{1/i}$ begins with 10 distinct digits. Note that A_1 = 1023456789 (leading zeros are not permitted) and that we established A_2 = 1362 last year ($1362^{1/2} = 36.90528417\cdots$). Mr. Bart specifically asks for A_3 through A_{10}.

M/J 5. Our last regular problem is not that regular; it is a crossword puzzle from Andrew Greene published last year in *The Tech*. I have no objection to including these kinds of problems but want to know what *you* think.



ACROSS

1 Follicle outgrowths
6 Mal de ____
9 760 mmHg
12 Portion of
13 *Much ____ About Nothing*
14 Anti-fly team?
16 Part of M.I.T.
18 "____, I don't think we're in Kansas anymore"
19 Draft org.
20 Compass dir.
21 NCC-1701-D crewmember Tasha ____
23 Dine
24 Bloodsucker
26 Strata
28 "...____ him now or wait 'til you get home"–Bugs Bunny
31 Golf item
33 ____ Jones' locker
34 First video game
35 Massachusetts____
37 Ripen
38 Preposition
39 B'way abbr.
40 Type of gate
43 Kind of hash
46 Po follower
47 *Gewitter*, ____ (Movement from Beethoven's sixth symphony)
49 Red or Black, e.g.
50 Sanctuaries
51 Wrote
53 *Dynasty* star Linda
55 Finish
56 Coke cops
58 British record co.
59 Bee chaser
62 *Cheers* cheer
64 M.I.T. sports, e.g.
67 ____ Scott case
68 Teleflora competitor
69 Falcons
70 "Star Wars"
71 Affirmative
72 *Hamlet* word

DOWN

1 Shades
2 Fundamentals
3 Man from Tel Aviv
4 Stomach lining
5 British weight
6 Chinese ruler
7 Nervous
8 Kingly
9 Computer supply co.
10 Contest on Nov. 24, 1987
11 Ripen
12 Pt. of Course XXI
15 Turvy's partner
17 LSC events
22 No soap
24 Journal
25 Pronoun
27 Gab
28 Health resort
29 Swine
30 Integer often following 26
32 One who captures and sells people
35 Mazel ____!
36 Be nosy
38 Well-known
41 OPEC concern
42 Co. between NBC and GE
44 Bird
45 Born as
46 Curve on a road
47 Use money
48 Some Logarythms
50 Beast
52 Worship as a god
54 Collect
57 Poker start
59 Indian tribe
60 Wyatt ____
61 Subways that aren't subways
63 MMMII halved
65 6 pts. in football
66 Fiddle's cousin

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

Speed department

SD 1. We once again begin this section with an offering from Jim ("Sir Speedy") Landau who wants to know in what year *The Pirates of Penzance* takes place.

SD 2. Frank Model writes that he and Mike Bertin have been concocting what they call "theme teams". The idea is to assemble a high-quality baseball team using past or present players all of whose surnames meet a given condition. For this speed problem the condition is that the surname is Robinson. Players must play their natural position but all three outfields are considered interchangeable.

Solutions

JAN 1. Robert Bart wants to know the shortest chess game that ends in a true smothered mate, i.e., only the square the king is on is under attack, all the adjacent squares are blocked by "friendly" forces.

Two variations on the same theme from Andrew Kolbert:

e3	Nc6		e3	Nc6
Ne2	Ne5	and	Ne2	Ne5
g3	Nf3 mate		c3	Nd3 mate

Also solved by Gordon Rice, Thomas Harriman, Bob High, Timothy Allen, Joe Kesselman, Philip Dangel, Lyndon Tracy, Greg Spradlin, Stephen Callaghan, Nancy Burstein, and the proposer.

JAN 2. A non-Satanic pentagram (Figure A) is formed by intersecting five circular arcs evenly spaced around a circle. These arcs are tangent to a radius 1 circle concentric to the first and having half its area. Ken Rosato wants to know the radius of the arcs.

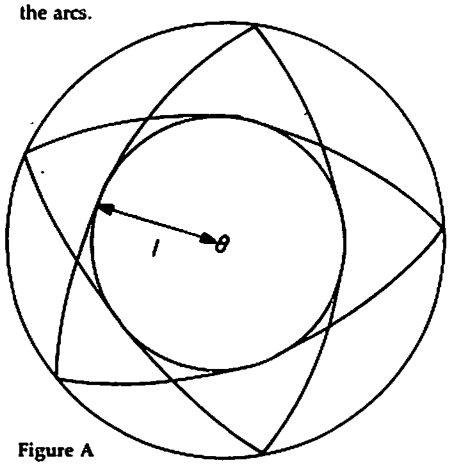


Figure A

Timothy Allen sent us the following solution.

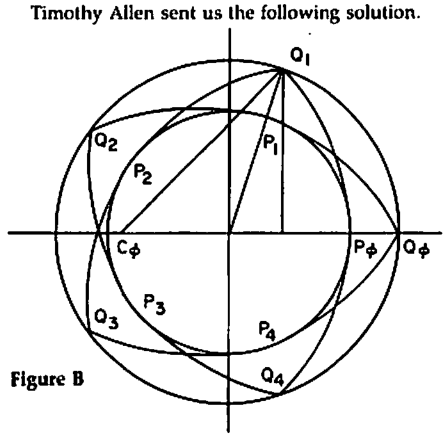


Figure B

All five tangent arcs have equal length radii. That length, r, has the following value and is given by the following formula:

$r = 1.888124745$
$r = (1.5 - c)/(1.0 - c)$
where
$\sqrt{2}\cos 2\pi/5$.

First, note the five-fold symmetry of Figure A. Without loss of generality, we can impose the coordinates (shown in Figure B). Both circles are centered at the origin. The inner circle has radius 1. The outer circle, which has twice the area of the inner circle, has radius $\sqrt{2}$. By symmetry, the origin, the tangent points on the inner circle, and the corresponding intersection points on the outer circle are collinear. Also by symmetry, the five tangent points on the inner circle, P_K, have coordinates (when written in complex number form) equal to the five fifth (ie, 1/5) roots of unity; and the five intersection points on the outer circle, Q_K, are scalar multiples, as follows:

$P_K = cis(2\pi K/5)$
$Q_K = \sqrt{2}\, P_K$
where:
K ranges from 0 to 4,
$cis(x) = \cos(x) + i \sin(x)$, in complex form,
$cis(x) = (\cos(x), \sin(x))$, in coordinate form.

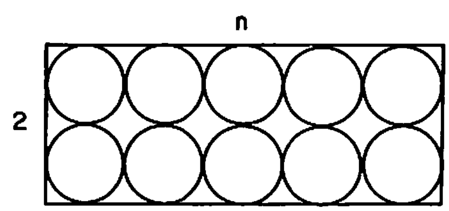
Consider the arc that is tangent to the inner circle at the unity point, $P_o = cis\,(O)$. Its center, C_o, is located on the negative side of the x-axis. The distance from the center, C_o, to the tangent point, P_o, which is the radius, r, equals the distance from the center, C_o, to the first intersection point on the outer circle, Q_1, which is also the radius, r. Also, $Q_1 = \sqrt{2}\ cis\ (2\pi/5)$. This yields the following equations:

$C_o = (1 - r, O)$
Distance (C_o to P_o) = distance (C_o to P_1)
$r^2 = [(1 - r) - \sqrt{2}\cos(2\pi/5)]^2 + \sqrt{2}\sin(2\pi/5)]^2$

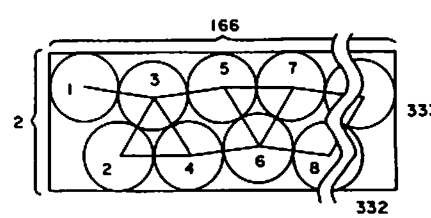
By simplifying this, we get the value and formula given above.

Also solved by Thomas Harriman, Bob High, Harry Zaremba, Gordon Rice, Robert Buegler, Winslow Hartford, Scott Berkenbilt, Eugene Sard, Sidney Williams, Kelly Woods, Andy Schwartz, Edward Dawson, Mary Lindenberg, Fred Furland, and the proposer.

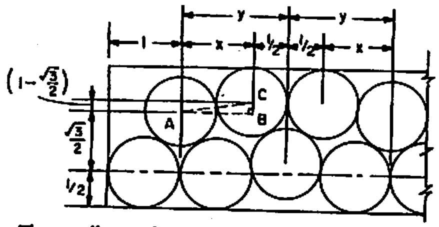
JAN 3. As illustrated below it is easy to put $2n$ unit diameter circles inside a $2 \times n$ rectangle. Nob. Yoshigahara and J. Akiyama want to know the smallest value of n for which you can fit $2n + 1$ circles.



Pretty tricky! Many respondents believed that no solution was possible. Bob High found a solution for n = 167 and one of the proposers, Nob. Yoshigahara, included the following figure to illustrate a solution for n = 166 and notes that the clearance in the long direction is less than 0.0007.



The minimal solution for which I received the complete construction is the following $n = 166.5$ submission from Harry Zaremba:



The smallest n for which $(2n + 1)$ unit diameter circles can fit into a $2 \times n$ rectangle equals 166.5. An arrangement where this is possible is indicated in the figure. The circles are arranged in groups of three with the circles in each group tangent to each other, and alternate groups have two of their circles tangent to opposite sides of the rectangle. From the geometry,

$x = \overline{AB} = (\overline{AC}^2 - \overline{BC}^2)^{1/2} = [1 - (1 - \sqrt{3}/2)^{1/2}]^{1/2} = (4\sqrt{3} - 3)^{1/2}/2$

and,

$y = x + 1/2 = 1.4909848.$

It is noted that, after the first circle is positioned at the left end of the rectangle, a uniform increment of three circles is added whereby each increment increases the rectangle's length by a constant y units and its area by 2y square units. Hence, for each addition of three circles, the increase in circles exceeds the increase in area by $(3 - 2y)$ or 0.0180304. The number of y increments that must be added to make up the one-circle deficit at the beginning, and to accrue a one-circle excess at the end, is given by

$k = 2/(3 - 2y) = 110.92$ or 111.

It follows that the smallest length of the rectangle that can contain $(2n + 1)$ circles is

$n = 1 + ky = 166.5.$

The number of circles in the rectangle is $(1 + 111 \times 3) = 334$, and the rectangle's area is $2n = 333$.

Also solved by Thomas Harriman, Winslow Hartford, Ken Rosato, Norman Wickstrand, Joe Kesselman, Scott Berkenbilt, and Eugene Sard.

JAN 4. John Rule is interested in perfect squares that when written (in base 10) use all ten digits once each. What is the smallest such number? What is the largest?

Edward Wallner solved a more general problem to illustrate that Rule was wise in specifying *squares*:

To generalize slightly the "Diophantine" equation $K = M^N$ has the following numbers of solutions in which K uses each integer including zero once and only once:

N	Number of solutions
0	0
1	3628800
2	117
3 . . . ∞	0

This assumes that a leading zero is allowed. If not the number of solutions for $N = 1$ is $10! - 9! = 3265920$ and for $N = 2$ is 87. For $N = 2$ the solutions are included here for both assumptions.

A number of factors reduce the space to be searched for solutions. First, it is quicker to search among the $\approx \sqrt{9876543210} \approx 10^5$ possible roots than among the $10! \approx 4 \times 10^6$ permutations of the ten digits. Second, since the digital root of K is zero, the digital root of M, for $N>1$, must be 0, 3, or 6. Since the digital root is divisible by 3 so is M, which reduces the search by a factor of 3. A program search from $M \geq \sqrt{0123456789}$ to $M \leq \sqrt{9876543210}$ by 3's gave the printed results. Similar searches gave no solutions for N from 3 to 7 at which point the possible values of M are 27 or less. The multiples of 3 up to 27 were then checked for values of N giving K's between 0123456789 and 9876543210. None of these few values used each of the digits once only.

Also solved by Thomas Harriman, Peter Groot, Alan Taylor, Bob High, Harry Zaremba, Gordon Rice, Steven Feldman, Robert Buegler, and Winslow Hartford.

Better Late Than Never

1988 A/S 5. John Maynard's *on-time* solution just turned up, having been misplaced in my files. I apologize.

ND/ 1. Alan Taylor has responded.

N/D 3. Peter Groot, Thomas Harriman, and Fred Furland have responded.

N/D 4. Peter Groot and Thomas Harriman have responded.

N/D 3. Thomas Harriman has responded.

Proposers' Solutions To Speed Problems

SD 1. 1877. Frederic becomes 21 years old at the time of the play, yet his 21st *birthday* is February 29, 1940.

SD 2. Pitcher Don, Catcher Wilbert, 1B Eddie, 2B Jackie, 3B Brooks, SS Craig, and outfielders Fresh, Floyd, and Bill. Four hall-of-famers are included.