Can You Keep the Peace Among 12 Golfers?

I t has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Let me do so now.

I have close to a year's supply of regular, chess, bridge, and speed problems but have run out of computer-related problems, one of which would otherwise appear as the first problem this month. This may well be a case of the market speaking. If no computer-related problems arrive when our turn comes up again in three issues, I will drop that class and return to alternating bridge and chess problems for problem 1.

Problems

APR 1. Our first problem, from Nob. Yoshigahara, involves multiplying a "time expression," i.e., one involving hours, minutes, and seconds, by a scalar to obtain another time expression. Another requirement is that all ten digits are to be used once each.

\[
\begin{align*}
&\text{abcd} \times e = fghij.
\end{align*}
\]

APR 2. William Pulver knows 12 golfers who play weekly in 3 foursomes, 2 players as a team in each foursome competing against the other pair in that foursome. The problem is to arrange a schedule so that each golfer plays with each of the other eleven the same number of times and against each of the eleven the same number of times.

APR 3. Matt Stenzel wants you to show that for \( p = 2, 3, 4, \ldots \)

\[
\sum_{i=1}^{p-1} \frac{1}{i} \text{ is a perfect square.}
\]

APR 4. Gordon Rice would like you to find non-equilateral triangles containing a 60° angle, all of whose sides are integers. How about 30°?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 351 MERCER ST., NEW YORK, N.Y. 10012.
there are four possible answers to each question—
Yes (= Y), No (= N), I don’t know (= ?), and I know
(= b), which is the falsification of “I don’t know.”
The following table gives the meaning of each pair of answers.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Q1</th>
<th>Q2</th>
<th>Q2A</th>
</tr>
</thead>
<tbody>
<tr>
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<td>False</td>
<td>Random</td>
<td>N</td>
<td></td>
<td>!</td>
</tr>
<tr>
<td>False</td>
<td>True</td>
<td>Random</td>
<td>N</td>
<td></td>
<td>?</td>
</tr>
<tr>
<td>Random</td>
<td>True</td>
<td>False</td>
<td>N</td>
<td>N</td>
<td></td>
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<tr>
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<td>False</td>
<td>True</td>
<td>Y</td>
<td>?</td>
<td>!</td>
</tr>
</tbody>
</table>

Any answer not noted is impossible. It may appear that question 2 is more complex than necessary, but the simpler question to B, “Would C say that 1 = 1?” would be unable to distinguish between Random True False, and Random False True.

Also solved by Larry Bell, Robert Bart, Gordon Rice, and Matthew Fountain.

N/D 3. Find a single shape that will fit snugly through all three holes in the board shown below. Each edge of the square is the same length as the diameter of the circle and as the bottom edge of the triangle. You are free to specify the other two edges of the triangle as part of your solution.

Walter Cluett notes that the answer is a shape, fitting within the outlines of a cube, that is a circle when viewed from the top, a square from the front, and a triangle from the side. The height of the triangle equals half of the circle. Mr. Cluett also enclosed the following beautifully drawn solution:

Nob. Yoshigahara, a frequent contributor of problems for this column, sent us his first solution:

Gordon Rice found several solutions where an individual piece was a non-connected region.

N/D 4. In how many ways can the integers from 1 to 9 be permuted so that the result consists of a strictly ascending sequence followed by a strictly descending sequence? For example, with N = 9 we could have 1, 4, 5, 7, 9, 8, 6, 3, 2. [These sequences are sometimes called bitonic—ed.]

Gordon Rice responds that the answer is 2^n-1 and adds that “this is really a question about subsets, not permutations. Once we choose a set of numbers which form the ascending part of the sequence, everything else is determined. The number of distinct subsets of N things is 2^N. The reason that our answer is half of this is that we don’t include N itself in the subset. The position of N is always between the ascending and descending parts of the sequence, and need not be thought of as part of either. If the monotonic sequences 1, 2, ..., N and N, ..., 1 are not accepted as degenerate cases of bitonic sequences, subtract 2 from the answer.”

Jerry Grossman reports that this problem “as well as thousands more” will appear in his undergraduate discrete mathematics text to be published by Macmillan next year.


N/D 5. Find four ways to divide the figure below into four congruent pieces.

Weingarten, Schurpin, Gagnebin & Hayes

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