ALLAN J. GOTTLIEB, '67

Can You Keep the Peace Among 12 Golfers?

It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Let me do so now.

I have close to a year's supply of regular, chess, bridge, and speed problems but have run out of computer-related problems, one of which would otherwise appear as the first problem this month. This may well be a case of the market speaking. If no computer-related problems arrive when their turn comes up again in three issues, I will drop that class and return to alternating bridge and chess problems for problem 1.

Problems

APR 1. Our first problem, from Nob. Yoshigahara, involves multiplying a "time expression," i.e., one involving hours, minutes, and seconds, by a scalar to obtain another time expression. Another requirement is that all ten digits are to be used once each.
ab:cd × e = f:gh:ij.

APR 2. William Pulver knows 12 golfers who play weekly in 3 foursomes, 2 players as a team in each foursome competing against the other pair in that foursome. The problem is to arrange a schedule so that each golfer plays with each of the other eleven the same number of times and against each of the eleven the same number of times.

APR 3. Matt Stenzel wants you to show that for $p = 2, 3, 4, \ldots$

$$\frac{1}{\sum_{n=1}^{\infty} \frac{n}{p^{n-1}}}$$

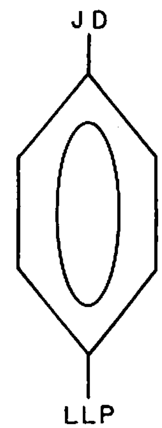
is a perfect square.

APR 4. Gordon Rice would like you to find non-equilateral triangles containing a 60° angle, all of whose sides are integers. How about 30°?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

Speed Department

SD 1. Jim Landau wonders what the following chemical formula represents.



SD 2. Warren Himmelberger is interested in wives of U.S. presidents and wants you to identify Anne I, Claudia I, Thelma I, Sarah II, and Dorothea I.

Solutions

N/D 1. Gordon Rice reminds us that a number in a particular number base is *palindromic* if the digits (leading zeros excluded) read the same right-to-left as left-to-right. For each integer $N > 2$ let $P(N)$ be the least integer exceeding $2N$ that is palindromic both in base N and in base 2. What is the smallest N such that $P(N) > P(3)$?

David Simen writes that "this is a good problem for the C programming language, which handles numerical computations and string manipulation with equal ease." Mr. Simon attached a C program, which is available from the editor, to solve the problem. The program reveals that P(3) = 6643, which is not exceeded until P(41) = 7671.

Also solved by Richard Hess, Matthew Fountain, Jerry Grossman, and the proposer.

N/D 2. You encounter three people who know each other. One always tells the truth; one lies all the time; and one gives random answers. How can you tell, by asking only three questions directed to only one person at a time, which is which?

Rather surprisingly, Andrew Marshall appears to be able to solve this problem asking only two questions (chosen from a set of three questions). His solution follows:
If the second question is based on the answer to the first question, then the solution can be arrived at with just two questions. First, label the individuals A, B, and C. For the first question, ask A, "Would B say that 1 = 1?" (or any obviously true statement). If the answer is "No," then the second question (addressed to B) would be, "Would C say that you (B) would say that 1 = 1?." If, on the other hand, the answer to question 1 is anything else, then the alternative second question (addressed to C) would be, "Would A say that 1 = 1?" It is essential that

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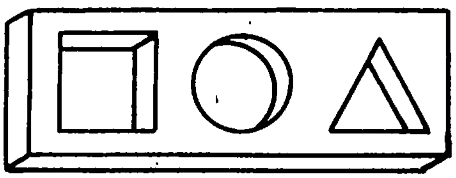
there are four possible answers to each question—Yes (= Y), No (= N), I don't know (= ?), and I know (= !), which is the falsification of "I don't know." The following table gives the meaning of each pair of answers.

A	B	C	Q1	Q2	Q2A
True	False	Random	N	!	
False	True	Random	N	?	
Random	True	False	N	N	
Random	False	True	N	Y	
True	Random	False	?		N
False	Random	True	!		N
Random	True	False	Y or ? or !		!
Random	False	True	Y or ? or !		?

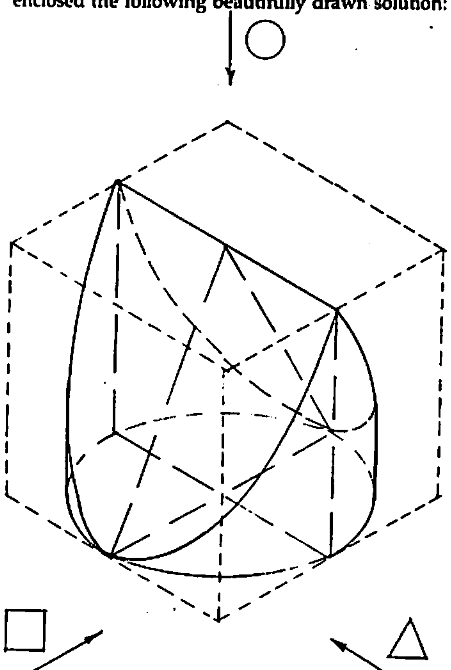
Any answer not noted is impossible. It may appear that question 2 is more complex than necessary, but the simpler question to B, "Would C say that 1 = 1?" would be unable to distinguish between *Random True False*, and *Random False True*.

Also solved by Larry Bell, Robert Bart, Gordon Rice, and Matthew Fountain.

N/D 3. Find a single shape that will fit snugly through all three holes in the board shown below. Each edge of the square is the same length as the diameter of the circle and as the bottom edge of the triangle. You are free to specify the other two edges of the triangle as part of your solution.



Walter Cluett notes that the answer is a shape, fitting within the outlines of a cube, that is a circle when viewed from the top, a square from the front, and a triangle from the side. The height of the triangle equals the base of the triangle. Mr. Cluett also enclosed the following beautifully drawn solution:



Norman Spencer and Alan Stiehl sent in wood models they produced in their home shops. These two models, which are about the same size, now have honored locations atop my computer display. Joe Alfano found this problem mentioned in Martin Gardner's August 1958 column in *Scientific American*.

Also solved by Robert Bart, Richard Hess, Winslow Hartford, Larry Bell, Harry Zaremba, Ken Rosato, Matthew Fountain, Charles Sutton, and the proposers, Gary Schmidt and Joe Horton.

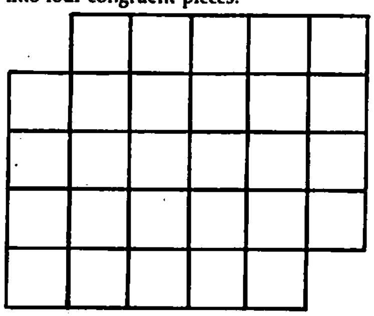
N/D 4. In how many ways can the integers from 1 to N be permuted so that the result consists of a strictly ascending sequence followed by a strictly descending sequence? For example, with $N = 9$ we could have, 1, 4, 5, 7, 9, 8, 6, 3, 2. [These sequences are sometimes called *bitonic*—ed.]

Gordon Rice responds that the answer is 2^{N-1} and adds that "this is really a question about subsets, not permutations. Once we choose a set of numbers which form the ascending part of the sequence, everything else is determined. The number of distinct subsets of N things is 2^N. The reason that our answer is half of this is that we don't include N itself in the subset. The position of N is always between the ascending and descending parts of the sequence, and need not be thought of as a part of either. If the monotonic sequences 1,2,. . .,N and N,. . .,2,1 are not accepted as degenerate cases of bitonic sequences, subtract 2 from the answer."

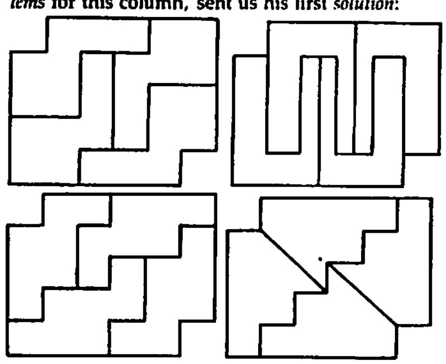
Jerry Grossman reports that this problem "as well as thousands more" will appear in his undergraduate discrete mathematics text to be published by Macmillan next year.

Also solved by Joe Alfano, Robert Bart, Jerry Grossman, Richard Hess, Winslow Hartford, Harry Zaremba, Walter Nissen, Larry Bell, Steven Feldman, David Simen, Matthew Fountain, Charles Sutton, and the proposer, Frank Rubin.

N/D 5. Find four ways to divide the figure below into four congruent pieces.



Nob. Yoshigahara, a frequent contributor of *problems* for this column, sent us his first *solution*:



Gordon Rice found several solutions where an individual piece was a non-connected region.

Also solved by Matthew Fountain, Richard Hess, Harry Zaremba, and the proposer, Solomon Golomb, in whose *Johns Hopkins Magazine* column the problem first appeared.

Better Late Than Never

1988 A/S 3. Alan Prince sent us a more direct solution to this problem by writing
tan(80)·tan(40)·tan(20) = tan(60 + 20)·tan(60 − 20)·tan(20),
applying the double angle formula, and noticing that the result fits the formula for tan($3x$) with x = 20.

Proposers' Solutions to Speed Problems

SD 1. Paralegals.

SD 2. Nancy Reagan, Lady Bird Johnson, Pat Nixon, Jane Wyman (first wife of Ronald Reagan), and Dolley Madison.