ALLAN J. GOTTLIEB

Making Mittens Inside Outside

I am in the very last stages of producing, with George Almasi from IBM, a book entitled *Highly Parallel Computing*. Since we are producing camera-ready copy and work on computer systems with incompatible text formatters (Script and Troff), we have managed to learn more than we might wish about many formatting details. It gives me more appreciation for the effort involved in publishing a book.

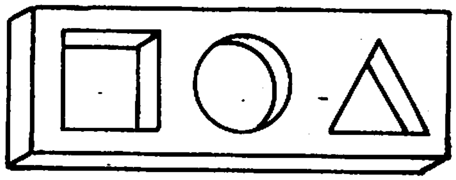
Our supply of computer-oriented problems is very low, so submissions of such problems are encouraged. A full listing of our backlogs is given each April.

Problems

N/D 1. Our first problem this month is computer-oriented. Gordon Rice reminds us that a number in a particular number base is *palindromic* if the digits (leading zeros excluded) read the same right-to-left as left-to-right. For each integer $N > 2$, let $P(N)$ be the least integer exceeding $2N$ that is palindromic both in base N and in base 2. What is the smallest N such that $P(N) > P(3)$?

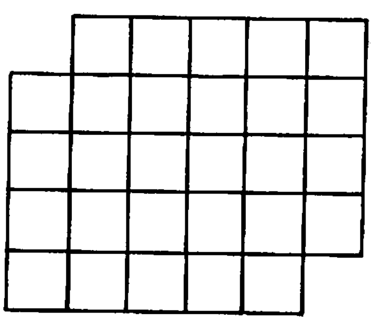
N/D 2. The following problem appeared in the "Gamesman" column of the October 1986 issue of *IEEE Potentials*, where it was attributed to Rajiv Gupta. You encounter three people who know each other. One always tells the truth; one lies all the time; and one gives random answers. How can you tell, by asking only three questions directed to only one person at a time, which is which?

N/D 3. Gary Schmidt and Joe Horton want you to find a single shape that will fit snugly through all three holes in the board shown below. Each edge of the square is the same length as the diameter of the circle and as the bottom edge of the triangle. You are free to specify the other two edges of the triangle as part of your solution.



N/D 4. Frank Rubin wants to know in how many ways the integers from 1 to N can be permuted so that the result consists of a strictly ascending sequence followed by a strictly descending sequence. For example, with $N = 9$ we could have, 1,4,5,7,9,8,6,3,2. [These sequences are sometimes called *bitonic*—ed.]

N/D 5. The following problem, from Solomon Golomb's column entitled "Golomb's Gambits," appeared in the October 1987 issue of *Johns Hopkins Magazine*. Find four ways to divide the figure below into 4 congruent pieces.



Speed Department

SD 1. Jim Landau forgot the last line of the following poem:

Of the skin he made him mittens
Made them with the fur side inside
Made them with the skin side outside
He, to get the warm side inside,
Put the inside skin side outside
He, to get the cold side outside
Put the warm side fur side inside
That's why he put the fur side inside
Why he put the skin side outside
. . .

SD 2. Mark Astolfi had one plate appearance during an inning of baseball

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

and walked. What is the largest number of bases he could have stolen during that inning?

Solutions

JUL 1. Write an efficient computer program for searching for positive integer solutions to the equation
$x^3 + y^3 + c = z^3$
and list all values of c between 0 and 100 for which you find solutions. You will need to bound x and y (thereby bounding z). Efficiently searching all values less than a few thousand is a modest (but not trivial) computation.

The most complete solution is from the Richard Hess, who writes:
A small program that loops on x and y finds
$z = [(x^3 + y^3)^{1/3}] + 1$,
and determines whether
$C = z^3 - x^3 - y^3 \leq 100$,
was run in double precision on my PC for a few hours to check $x, y \leq 5000$. The values of c for which there were solutions are 1, 2, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 25, 27, 28, 29, 34, 35, 36, 37, 43, 44, 46, 47, 48, 53, 54, 55, 56, 57, 60, 62, 63, 64, 65, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 87, 88, 90, 91, 92, 93, 97, 98, 99, and 100.

Also solved by Winslow Hartford, Harry Zaremba, and the proposer, Matthew Fountain.

JUL 2. Solve the following Naymandidge from Neil Macdonald's Puzzle column in the September-October issue of *Computers and People*:
In a naymandidge, an array of random or psuedorandom digits ("produced by *nature*"), such as that at the bottom of this column, has been subjected to a "definite systematic operation" ("chosen by *nature*"). The problem ("which *man* is faced with") is to figure out what was *nature's* operation. A "definite systematic operation" meets the following requirements: the operation must be performed on all the digits of a definite class which can be designated; the result must display some kind of evident, systematic, rational order and completely remove some kind of randomness; and the operation must be expressible in not more than four English words. (But *man* can use more words to express the solution and still win.)

4	8	0	1	8	9	1	7	5	1	0	4	4	6	2	1	7	4	7	7
5	1	8	6	4	8	9	1	4	4	3	0	0	4	2	1	0	9	8	7
9	3	2	9	9	6	5	2	3	5	6	0	5	8	2	2	9	7	5	0
4	9	6	2	4	8	7	7	5	3	6	4	5	0	7	1	9	7	9	3
8	0	1	5	0	2	9	4	6	0	9	9	4	3	3	1	1	6	7	0
8	5	7	8	6	6	2	2	4	6	9	8	7	5	8	1	7	9	2	1
4	7	4	3	8	7	4	7	6	1	1	5	6	1	4	2	8	4	0	4
0	5	4	1	7	4	1	6	2	1	1	2	7	0	3	2	2	7	9	3
6	8	9	9	2	2	9	4	6	7	7	2	4	1	0	1	6	8	6	8
2	6	6	7	8	0	9	7	6	5	1	7	8	4	3	2	0	1	0	0

The only submission is from Matthew Fountain, who notes that the sixteenth column contains only 1 and 2 and thus conjectures that nature's operation was to replace the old value x in the sixteenth column by $(x \bmod 2) + 1$.

JUL 3. To a pure sinusoidal tone, add a second pure sinusoidal tone of identical amplitude and frequency but with a random phase shift. What is the probability that the resulting sound is lower in volume than the original tone?

The following solution is from John Prussing:
The function can be expressed as $f = \sin(t) + \sin(t + k)$. This is equivalent to $f = (1 + \cos k)\sin(t) + \sin(k)\cos(t)$, which has an amplitude A given by
$A^2 = (1 + \cos k)^2 + (\sin k)^2 = 2(1 + \cos k)$.
For this amplitude to be less than one requires that $\cos k < -1/2$, which is satisfied by $2\pi/3 < k < 4\pi/3$. Assuming a uniform probability density for the random variable k on the interval $0 < k < 2\pi$ results in a probability equal to 1/3.

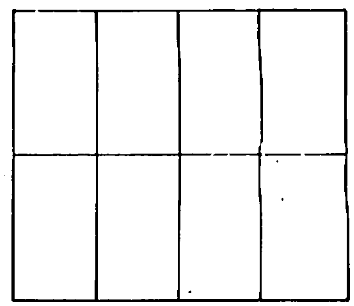
Also solved by Alan Taylor, Richard Hess, Matthew Fountain, Ken Rosato, Winslow Hartford, Harry Zaremba, Daniel Morgan, Steven Feldman, Brian McCue, and the proposer, Dave Mohr.

JUL 4. A doubly palindromic number would be a number N that is palindromic when expressed in each of two bases p and q, where gcd(p,q) = 1 and N exceeds the product pq. Are there any such N? If so, is there a largest?

The following solution is from Linda Klaver:
An example of a doubly palindromic number is 52. Let p = 3 and q = 5. Then (p,q) = 1, and 52 is written 1221 in base 3 and 202 in base 5. There is no largest number N which is doubly palindromic. To show this, let p be any integer greater than 3 and let q = p + 1. Then (p,q) = 1. Take $N = q^2 + pq + 1$. In the base q, N is written 1p1, which is palindromic. Expressed as a polynomial in p,
$N = (p + 1)^2 + p(p + 1) + 1$
$= p^2 + 2p + 1 + p^2 + p + 1$
$= 2p^2 + 3p + 2.$
In the base p, therefore, N is written 232, which is palindromic. Note that the general solution does not work for p = 3, because $3p = p^2$. In the general solution, if p = 3, then q = 4 and N = 29. In the base 3, N is written 1002, which is not palindromic.

Also solved by Avi Ornstein, Richard Hess, Matthew Fountain (who also considered numbers palindromic in three bases), Ken Rosato, and Gordon Rice.

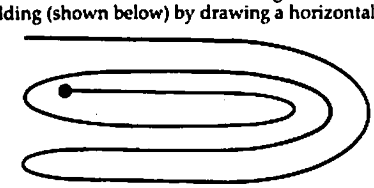
JUL 5. Take an 8½ × 11 inch piece of paper (preferably with writing on both sides) and fold it in half three times, as if you were about to store it into your shirt pocket. Now unfold it. If you are like most of us, the fold lines will separate the paper into two rows of four rectangles all of which are identical:



In how many unique ways can you completely fold up the paper, assuming that you only make flat folds along the fold lines? Now suppose you had a strip of paper containing one row of six rectangles. How many ways can you fold that? What are the odds that the large map inside your automobile glove compartment is correctly folded?

Our last solution is from Matthew Fountain:
There are 264 ways of folding the paper into eight rectangles. There are 140 ways of folding the strip into a six-rectangle stack. The probability my map is folded correctly is about 0.5 when new and very small when old.

I explain the second part of the problem first, as the case of all folds along parallel axes is less complicated. I classified the distinct ways of folding the six rectangles into a six-layer stack according to the direction of the creases observed after the strip was unfolded. For example, there are seven foldings that leave the crease pattern RRRLL. To find these I marked the upper face of the first rectangle of the strip and numbered the creases from this point on from one to five. Next I drew a rough sketch of one folding (shown below) by drawing a horizontal line



from left to right to represent an edge view of the first rectangle. I then reversed the continuation of this line with a right, or R, turn and drew a second horizontal line below the first from right to left to represent the edge view of the second rectangle. I again reversed the direction of the line with another R turn and drew the edge view of the third rectangle. I chose to draw this above the first and second rectangles, but noted my next sketch would place the line between them. I again reversed the direction of the line with a third R turn and drew the fourth rectangle. The fifth and sixth rectangles were drawn following left turns so that the series of turns for a traverse of the line was RRRLL. A second sketch was made by retracing the first up to the point that I had noted and then proceeding on between the first and second rectangle. With a little attention one can discover all the possible foldings that yield a given pattern of turns or creases. Of the $2^5 = 32$ possible patterns it is necessary to only determine the number of folds corresponding to 10 patterns as patterns with mirror symmetry have the same number of folds. For example, RRRLL, LLLRR, RRLLL, and LLRRR all have 7 distinct ways of folding. The tabulation below shows that the 16 patterns starting with an R have a total of 70 folds. The 16 patterns starting with L are mirror images of the R patterns. Therefore there are 140 distinct ways of folding six rectangles into a stack.

Pattern	No. folds	Pattern	No. folds
RRRRR	5	RLRRR	5
RRRRL	5	RLRRL	3
RRRLR	5	RLRLR	1
RRRLL	7	RLRLL	3
RRLRR	5	RLLRR	4
RRLRL	3	RLLRL	3
RRLLR	4	RLLLR	5
RRLLL	7	RLLLL	5

The paper folded into eight rectangles by three parallel creases and one at right angles is more complicated. It is wise to check by actually folding a paper. Certain folds require a tuck fold of the sort commonly used to fold newspapers to keep them closed. I tabulated these separately as maps are not usually so folded. I divided the pattern of folds up as follows:

(a) First fold is lengthwise, the rest crosswise. After the lengthwise fold there are 16 continuations without a tuck and 8 ending with a tuck. As the first fold can be made in two ways there are a total of 32 regular foldings plus 16 tuck foldings.

(b) One end of the paper is folded over followed by a lengthwise fold that places the folded end inward. There are 6 continuations without a tuck and 4 with a tuck. As there are two ends and two directions of the first fold, there are 24 regular foldings plus 16 tuck foldings.

(c) Same as (b) but the lengthwise fold is in the opposite direction. There are 6 continuations without a tuck and 8 with a tuck. As there are two ends and two directions of the first fold, there are 24 regular foldings plus 32 with a tuck.

(d) First fold is across the middle of the paper and the second is the lengthwise fold. Only one more fold is made. As each of the three folds may be made in two ways, there are 8 foldings, all without a tuck.

(e) Both ends of the paper are folded over to the same side and the third fold is lengthwise with folded ends inward. There are two continuations. There are a total of 4 foldings; all without a tuck.

(f) Both ends of the paper are folded over to the same side and the third fold is lengthwise with folded ends outward. There are two continuations without a tuck and four with a tuck. There are a total of 4 foldings without a tuck and 8 with a tuck.

(g) Both ends of the end paper are folded over to opposite sides and the third fold is lengthwise. There are two continuations without a tuck and 2 with a tuck. There are 8 foldings without a tuck and 8 foldings with a tuck.

(h) One end is folded over, the middle fold is made, and then the lengthwise fold is made with folded end inward. There are two continuations. There are a total of 16 foldings without a tuck.

(i) One end is folded over, the middle fold is made, and then the lengthwise fold is made with the folded end outward. There are two continuations without a tuck and two with a tuck. There are a total of 16 foldings without a tuck and 16 foldings with a tuck.

(j) The lengthwise fold is made last. There are 16 foldings before the last fold which can be made in two ways. There are a total of 32 foldings without a tuck.

There are in all 32 + 24 + 24 + 8 + 4 + 4 + 8 + 16 + 16 + 32 = 168 regular foldings and 16 + 16 + 32 + 8 + 8 + 16 = 96 tuck foldings for a total of 264 foldings.

Also solved by Winslow Hartford, Ken Rosato, and the proposer, Robert Johnson.

Better Late Than Never

F/M 1. Naomi Markovitz has responded.

F/M 4. The following submission is from Daniel Sleator:

I was amused by your note on palindromes. Guy Jacobson—using the dictionary representation described in his article in the May issue of the *Communications of the ACM*—took a crack at creating these. Here are the best ones he found (one word only, no proper nouns). (The comments are Guy's.)

Animal was I ere I saw lamina
  (the calming effects of layered materials)
Avid was I ere I saw diva
  (a disappointing opera)
Civic was I ere I saw Civic.
  (Hondas encourage rude behavior)
Cod was I ere I saw doc.
  (my delusion is cured)
Dog was I ere I saw god.
  (born again)
Drab was I ere I saw bard.
  (Shakespeare leads a whore to culture)
Drawer was I ere I saw reward.
Emir was I ere I saw rime.
  (of the Ancient Mariner, perhaps?)
Gelder was I ere I saw redleg.
Live was I ere I saw evil
Ogre was I ere I saw ergo
  (something Scherlis might say)
Pacer was I ere I saw recap
Redrawer was I ere I saw rewarder
Reified was I ere I saw deifier
Reined was I ere I saw denier
Repaid was I ere I saw diaper
  (a baby's revenge)
Reviled was I ere I saw deliver
Rewarder was I ere I saw redrawer
Smart was I ere I saw trams
Smug was I ere I saw gums
  (the heartbreak of gingivitis)
Stinker was I ere I saw reknits
Stressed was I ere I saw desserts
  (sugar lowers blood pressure)
Tubed was I ere I saw debut

A different game was played a few years ago in the computer science department here at Carnegie Mellon. Jim Saxe noticed that "a man, a plan, a canal—Panama" could be augmented to form "a man, a plan, a cat, a canal—Panama." Subsequently Dan Hoey created the following monstrosity:

A man, a plan, a caret, a ban, a myriad, a sum, a lac, a liar, a hoop, a pint, a catalpa, a gas, an oil, a bird, a yell, a vat, a caw, a pax, a wag, a tax, a nay, a ram, a cap, a yam, a gay, a tsar, a wall, a car, a luger, a ward, a bin, a woman, a vassal, a wolf, a tuna, a nit, a pall, a fret, a watt, a bay, a daub, a tan, a cab, a datum, a gall, a hat, a fag, a zap, a say, a jaw, a lay, a wet, a gallop, a tug, a trot, a trap, a tram, a torr, a caper, a top, a tonk, a toll, a ball, a fair, a sax, a minim, a tenor, a bass, a passer, a capital, a rut, an amen, a ted, a cabal, a tang, a sun, an ass, a maw, a sag, a jam, a dam, a sub, a salt, an axon, a sail, an ad, a wadi, a radian, a room, a rood, a rip, a tad, a pariah, a revel, a reel, a reed, a pool, a plug, a pin, a peek, a parabola, a dog, a pat, a cud, a nu, a fan, a pal, a rum, a nod, an eta, a lag, an eel, a batik, a mug, a mot, a nap, a maxim, a mood, a leek, a grub, a gob, a gel, a drab, a citadel, a total, a cedar, a tap, a gag, a rat, a manor, a bar, a gal, a cola, a pap, a yaw, a tab, a raj, a gab, a nag, a pagan, a bag, a jar, a bat, a way, a papa, a local, a gar, a baron, a mat, a rag, a gap, a tar, a decal, a tot, a led, a tic, a bard, a leg, a bog, a burg, a keel, a doom, a mix, a map, an atom, a gum, a kit, a baleen, a gala, a ten, a don, a mural, a pan, a faun, a ducat, a pagoda, a lob, a rap, a keep, a nip, a gulp, a loop, a deer, a leer, a lever, a hair, a pad, a tapir, a door, a moor, an aid, a raid, a wad, an alias, an ox, an atlas, a bus, a madam, a jag, a saw, a mass, an anus, a gnat, a lab, a cadet, an em, a natural, a tip, a caress, a pass, a baronet, a minimax, a sari, a fall, a ballot, a knot, a pot, a rep, a carrot, a mart, a part, a tort, a gut, a poll, a gateway, a law, a jay, a sap, a zag, a fat, a hall, a gamut, a dab, a can, a tabu, a day, a batt, a waterfall, a patina, a nut, a flow, a lass, a van, a mow, a nib, a draw, a regular, a call, a war, a stay, a gam, a yap, a cam, a ray, an ax, a tag, a wax, a paw, a cat, a valley, a drib, a lion, a saga, a plat, a catnip, a pooh, a rail, a calamus, a dairyman, a bater, a canal—Panama.

APR 4. Naomi Markovitz has responded.

M/J 1, M/J 2, M/J 4. Thomas Harriman has responded.

Proposers' Solutions to Speed Problems

SD 1. Why he turned them inside outside.
[George A. Strong, *The Song of Milkanwatha*]

SD 2. 6. He entered the game as a pinch runner at first base and stole three bases. The team batted around, he came up, was walked and stole three more.