ALLAN J. GOTTLIEB, '67

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

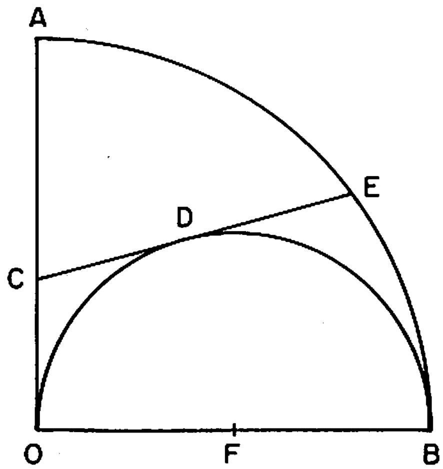
In each issue I present five regular problems (the first of which is chess, bridge, or computer-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later, one submitted solution is printed for each problem; I also list other readers whose solutions were successful. For example, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solutions to this issue's speed problems are given below. Only rarely are comments on speed problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

OCT 1. We should be proposing a new bridge problem this issue but, unfortunately, I mistakenly omitted a part of the M/J bridge problem. The corrected problem appears in the solution section below and will serve as the bridge problem for this issue. Sorry.

OCT 2. Matthew Fountain's figure shown below depicts a semicircle of radius 1 in a quartercircle of radius 2. What is the largest area that the curved region ACE may have, if CE is tangent to the semicircle?



OCT 3. Ron Raines sent us the following classic problem. It sounds familiar so I would not be surprised to hear that it appeared in "Puzzler Corner" 15-20 years ago! However, I think it is a lot of fun and worth the risk of being a repeat: On a train, Smith, Robinson, and Jones are the firemen, brakeman, and engineer, but NOT NECESSARILY respectively. Also aboard the train are three businessmen who also have the same names: a Mr. Smith, a Mr. Robinson, and a Mr. Jones.

(1) Mr. Robinson lives in Detroit.
(2) The brakeman lives exactly halfway between Chicago and Detroit.
(3) Mr. Jones earns exactly $20,000 per year.
(4) The brakeman's nearest neighbor, one of the passengers, earns exactly three times as much as the brakeman.
(5) Smith beats the fireman at billiards.
(6) The passenger whose name is the same as the brakeman's lives in Chicago.

Who is the engineer?

OCT 4. Richard Hess has a two part question that he calls "deduce your number":
Three of you in a room are told you each have a prime number written on your forehead and that they form the sides of a triangle with prime perimeter. Each person is asked in turn if he can deduce his number.

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

(a) You see a 5 and 7 and have heard "don't know" from the other two. What is your number?

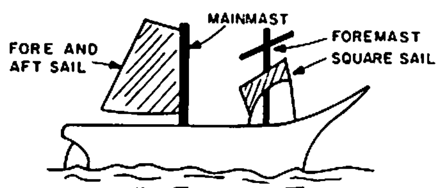
(b) You see a 5 and 11 and have heard "don't know" from the others on each of their first two turns. You have stated "don't know" on your first turn. It is now your second turn; what is your number?

OCT 5. For some unknown reason, Scott Berkenblit wants to know the largest integer that is less than

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{162{,}754}.$$

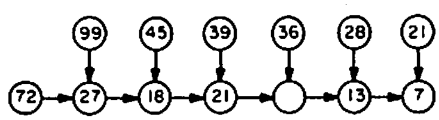
Speed Department

SD 1. Our first speed problem is from James Landau, who reports that he is temporarily living in Brigantine, NJ, and then notes that a "brigantine" is a 2-masted sailing ship with square sails on the foremast and a fore-and-aft sail on the mainmast.



Landau wants to know what, mathematically speaking, is the difference between a square sail and a fore-and-aft sail.

SD 2. Nob Yoshigahara wants you to fill in the empty circle. He warns you that the last 7 is NOT an 8.



Solutions

M/J 1. I inadvertently omitted part of the problem. Specifically, West leads the ten of spades, and East encourages with the seven. How do you play this hand? Hence, the problem is now re-opened and a solution will be given in February with the new problems presented this month.

North
♠ 4 3
♥ K 7 6
♦ A Q J 5 2
♣ K Q 5

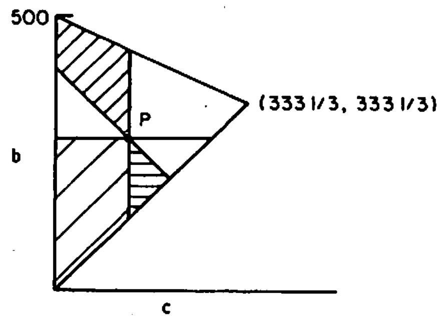
South
♠ K 6
♥ J 10 8 5 3 2
♦ K 7
♣ A 6 4

Bidding

N	E	S	W
1D	1S	2H	Pass
3H	Pass	4H	Pass
Pass	Pass		

M/J 2. Two gamblers, High Roller and Poker Face, love to gamble but not with each other, as High Roller always wins at dice and Poker Face always wins at cards. A mutual friend suggests a fair and nontrivial bet that they could make. Each could privately write three amounts adding to $1,000 on a slip of paper. Then they could compare their amounts, the largest against largest, the smallest against the smallest, and the median against the median. The one with the larger amount in two of these three comparisons would win the bet and take the $1,000. Can you help Poker Face decide what to write on his slip?

Robert Bart drew the following diagram.



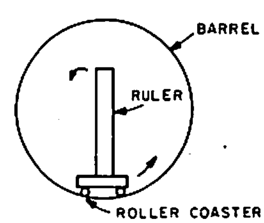
In this picture the horizontal axis is c, the smallest value, and the vertical axis is b, the median value. The space of feasible solutions is the triangle shown. The point P has coordinates (b, c) and a = 1000 − b − c is also determined by P. The first point to note is that the shaded area gives the feasible solutions that lose to P. Hence the goal is to maximize this area. Bart asserts that this occurs (approximately) when c = 231, b = 384, a = 385; which agrees with Robert Buegler's computer search.

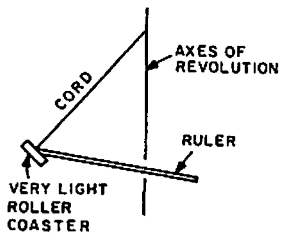
Also solved by Robert High, Winslow Hartford, Ken Rosato, and the proposer.

M/J 3. Imagine a roller coaster with a ruler, initially pointing straight up, balanced on it. Even ignoring air drag, the ruler's attitude is unstable, and it will eventually fall over. Is there a path, e.g. a series of hills and loops, for the coaster such that the ruler's attitude remains stable and it will not fall over?

The following solution is from Matthew Fountain: Two arrangements make it possible. The first diagram shows the path to be the inside surface of a barrel. The width of the ruler makes it stable against tipping in the direction of travel. The centrifugal force acting on one end of the ruler keeps it in contact with the roller coaster while the smaller, opposing centrifugal force on the other end keeps it from tipping over toward an end of the barrel.

The second diagram shows another far-fetched arrangement which will work even if the ruler is a thin rod normally unstable in all directions. A thin rod hanging from a light cord attached to one end and properly set in motion so that its upper end moves in one circle while its lower end moves in a smaller circle 180° behind the upper end will remain in stable rotation. With practice anyone can demonstrate this by starting with the hanging rod at rest and then swirling the upper end of the cord horizontally in a tightening circle. When spinning at constant speed, the center of gravity moves in a circle with constant radius. At constant speed any small departure of the center of gravity from this circle results in changes in the path of the upper end with the result that the tension in the cord varies during each revolution, being greatest when in position to oppose the departure.





Also solved by Robert Bart and the proposer.

M/J 4. Four suspects, each of different height, are in a house surrounded by a posse. The actual criminal is known to be the tallest. The sheriff is constrained to make only a single arrest. The suspects can be arrested only as they leave the building. They do so one at a time. Which suspect—the first to leave, the second, etc.—should the sheriff arrest?

As noted by the proposer and others, it is important to state what assumptions are being made. Here we assume that the suspects come out at random and that the goal is to maximize the probability of getting the correct suspect (but that it is no worse to choose the wrong suspect than not to choose any suspect). With these assumptions Bob High shows that the best strategy is to let the first suspect go by and then arrest the next suspect who is taller than all preceding suspects. Indeed, High generalizes the problem to n suspects and shows that one should then let n/e go by and arrest the next one taller than all preceding. For large n, the chances of catching the criminal are about $1/e$. High's analysis is available from the editor.

Also solved by Winslow Hartford, Matthew Fountain, Robert Bart, Gordon Rice, R.L. Loesch, Steve Feldman, Bob Metcalfe, Norman Spencer, and the proposer.

M/J 5. Given a point, a line, a compass, and a straight edge, construct the perpendicular to the line through the point using the compass only once and the straight edge as many times as necessary.

The following beautiful (and beautifully drawn) solution below is from Edward Dawson:

The given straight line is AB, and C is the given point. With C as a center draw a circle intersecting line AB at points D and E. Using the straight edge draw diameters DG and EF, and chords DF and EG. From an arbitrarily located point H on line AB draw line CH which intersects line EG at point J. Draw line DJ which intersects diameter FCE at point K. Draw line HK extended to intersect diameter DCG at point L.

Point L is harmonic to point G with respect to points D and C, so that DL:DG = CL:CG. Denote radius CG by r, then:

$$\frac{DL}{2r} = \frac{CL}{r} \text{ and } CL = \frac{DL}{2} = \frac{r}{3} = \frac{CD}{3}.$$

Therefore, as radius CF is equal to radius CE, line FLR is a median of triangle FDE, and DR = RE. Hence, CR is the required perpendicular to line AB.

Also solved by Matthew Fountain, Robert Bart, Winslow Hartford, Ken Rosato, Phelps Meaker, Wilbur DeHart, Mary Lindenberg, Avi Ornstein, and the proposer.

Better Late Than Never

1987 N/D 5. Thomas Harriman, Stephen Kanter, and Jack Bogdanski have responded.

1988 JAN 1. James Poitras, Robert Keston, and Frank Model note that if the Diamonds split 4-2 and the Spade finesse fails, the opponents are likely to cash the A and K of Diamonds and then ruff a third round. Hence it is better to play for the drop rather than the finesse in Spades. To quote Mr. Poitras, "Otherwise, you are playing the suit, not the hand."

F/M 2. The proposer, Frank Rubin, writes that the problem was designed to be open-ended and suggests the following avenues for solutions.

(1) For $x = y = z = 2$, the well-known formula $(p^2 - q^2)^2 + (2pq)^2 = (p^2 + q^2)^2$ gives a family of solutions. This can be iterated, by setting $p = r^2 - s^2$ and $q = 2rs$ to give solutions of the form $a^2 + b^2 = c^{2^n}$, for all n.

(2) From $2^n + 2^n = 2^{n+1}$ we can factor n and $n + 1$ to get many solutions, like $8^5 + 32^3 = 16^4$.

(3) From $1 + 2^3 = 3^2$, we can multiply by 3^{3n} to get $(3^3)^n + (2 \cdot 3^n)^3 = 3^{3n+2}$.

(4) We can multiply any $a^x + b^y = c$ by $c^{lcm(x,y)}$. Similarly, we can multiply any $a^x + b = c^z$ by $b^{lcm(x,z)}$.

F/M 3. Robert Buegler has responded.

APR 2. Morton Hecht and Donald Savage have responded.

APR 4. Michael Jung has responded.

M/J SD2. Robert Keston believes that one should consider the effect of giving the cube to the opponent.

Proposers' Solutions To Speed Problems

SD 1. The answer is geometric but has little to do with the shape of the sails. A square sail can present either vertical edge to the wind, but always presents the same side. A fore-and-aft sail can present either side to the wind, but always presents the same edge.

SD 2. 12.