ALLAN J. GOTTLIEB, '67

439,792 Ways to Seat Ten Men

Mary Lindenberg wrote (on 29 March) that April 4 would be the 70th birthday of her husband, Martin, and in celebration she sent in a solution to APR 4. She also mentioned that the published solution to N/D 3 gives a hint of a possible attack on Fermat's Last Theorem.

In the December issue, Jim Landau conjectured that the radius of an n-dimensional regular tetrahedron is 1/n the radius of the circumscribing sphere. Dennis White reports validating the conjecture.

Phelps Meaker reports on several very simple looking rational functions for which the solution is x = .61803398875 . . .

Finally, William Eaton directs us to an interesting article on string figures—that is, taking a loop of string, placing your hands inside, and manipulating the loop to form interesting patterns. The article appeared in the April 25th issue of *Insight*.

Problems

A/S 1. David Evans has placed white knights on a1, b1, and c1 and black knights on a4, b4, and c4. He wants you to find the minimum number of moves needed to interchange the positions of the knights disregarding possible captures. Only the first four ranks and the first three files are to be used.

A/S 2. Matthew Fountain reports that a computer expert wanted to find the average length obtained for the largest part of a line of unit length when the line is randomly divided into four parts. The expert wrote a program that summed four random numbers between zero and one and divided the largest of these four random numbers by their sum. Is the average result he obtained from his program the length he sought?

A/S 3. Scott Berkenblit poses a challenge he saw in a Russian book of math problems:
Find the exact value of the product
tan(80) × tan(40) × tan(20),
where all angles are expressed in degrees.

A/S 4. Ken Rosato's rocket accelerates from 0 velocity to C (the velocity of light, 186,000 miles per second) with a constant acceleration (relative to a stationary observer) of 1g = 32 feet per second². It carries a clock synchronized to an identical clock at rest with the stationary observer. When the velocity of the rocket reaches that of light, how far behind the stationary clock will the clock on the rocket be?

A/S 5. Our last regular problem comes from the February 1986 issue of *IEEE Potentials*, where it was attributed to Bruce Layman:
An IEEE student member entered the north end of a tunnel of length L. After walking the distance L/4 into the tunnel, he noticed a car approaching the north entrance at 40 miles per hour. The student knew his own speed and calculated that no matter which end of the tunnel he ran to, he would arrive there at the same time as the car. What is his top speed? Hint: he might do better as a professional athlete than as an engineer.

Speed Department

SD 1. Jim Landau wants to know the value of

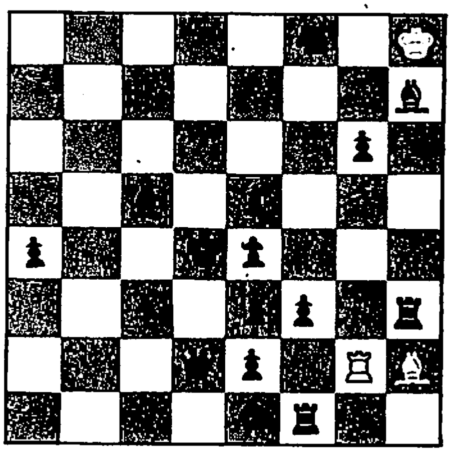
$$\frac{\begin{vmatrix} 12 & 8 \\ 3 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 8 \\ 3 & 4 \end{vmatrix}}$$

SD 2. Alex Okun recalls when Archimedes was working on a math problem in the sand and a passerby asked him, "How long until I get to Syracuse?" Archimedes said only, "Go!" The passerby started walking on and Archimedes called after him, "About two hours." Why?

Solutions

APR 1. Black moves first, and White and Black are to cooperate so that Black is mated on White's fourth move.

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.



Karim Roshd sent us the following solution:

	White	Black
1.		Q-d3
2.	R-f2	PxR
3.	B-g3	K-e3
4.	B-h4	B-d4
5.	B-g5	mate

Also solved by Robert Bart (who feels the problem should be called "one step at a time"), Matthew Fountain, Charles Larson, and Richard Hess.

APR 2. Find two positive integers differing by five such that the sum of their squares is a perfect cube, and show that the solution is unique.

Dennis White found that there are at least two solutions, so no uniqueness proof is possible. He found
$5^2 + 10^2 = 5^3$ and $47^2 + 52^2 = 17^3$.
White also showed that the number being cubed must itself be a sum of two squares, but like everyone else was unable to show uniqueness.

Also solved by Richard Hess, James Walker, David Smith, Jonathan Aronson, Ken Rosato, Charles Larson, Matthew Fountain, Winslow Hartford, Frederick Furland, Dennis White, Matthew Fountain, Avi Ornstein, Ken Rosato, Norman Wickstrand, Robert Bart, Edward Dawson, Harry Zaremba, Charles Rivers, and George Adams.

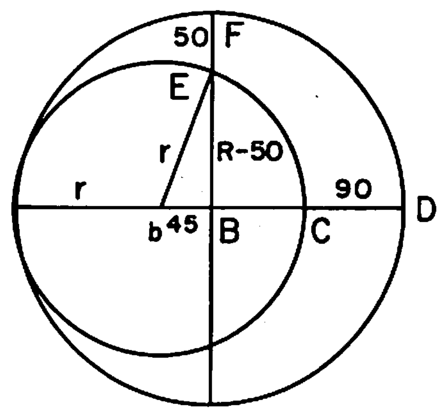
APR 3. As with the standard Towers of Hanoi problem, there are three pegs; initially there are *n* rings on one of the pegs, graded in size from largest at the bottom to smallest on top. The problem is to find the minimum number of moves to transfer all of the rings to another peg, one at a time, never having a larger ring on top of a smaller one. The variation is that only clockwise moves are permitted. For example, if the pegs are labeled A,B,C, then the only moves permitted are A to B, B to C, and C to A. There are two cases of solution, one for moving all the rings to the nearest peg in the clockwise direction, and one for moving to the third peg.

The following solution is from William Messner. Let f(n) be the number of moves to move a stack of n rings (an n-stack) one peg (all motions are clockwise), and let g(n) be the number of moves needed to move an n-stack two pegs. To move an n-stack one peg, one must first move an (n−1)-stack two pegs, then move the remaining ring one peg, and finally move the (n−1)-stack two pegs. Hence
$f(n) = 2g(n-1) + 1$.
To move an n-stack two pegs, one must first move an (n−1)-stack two pegs and the remaining ring one peg. Next move the (n−1)-stack one peg, and the remaining ring one peg. Finally, the (n−1)-stack moves two pegs. Hence
$g(n) = 2g(n-1) + f(n-1) + 2$.
Substituting for f(n−1) gives the nonhomogeneous second order linear finite difference equation
$g(n) = 2g(n-1) + 2g(n-2) + 3$.
A particular solution is $g_p(n) = -1$. The homogeneous solutions are found by assuming $g_h(n) = \lambda^n$, which leads to $\lambda = 1 \mp \sqrt{3}$ and hence
$g_h(n) = A(1+\sqrt{3})^n + B(1-\sqrt{3})^n$. Since g is the sum of the homogeneous and particular solutions, we have
$g(n) = A(1+\sqrt{3})^n + B(1-\sqrt{3})^n - 1$.
Plugging in g(0) = 0 and g(1) = 2 gives the values for A and B.

Also solved by Winslow Hartford, Scott Berkenblit, Dennis White, George Adams, Jonathan Aronson, Matthew Fountain, Richard Hess, Robert Bart, and the proposer, Peter Gottlieb.

APR 4. Two circles shown in the diagram touch at A. The larger circle has its center at B. The width of the crescent between points C and D is 90 mm. and between points E and F the distance is 50 mm. What are the diameters of the two circles?

Edgar Rose sent us the following solution:
Let point b designate the center of the small circle, and R and r the radii of the large and small circles respectively. All dimensions will be in mm.



bEB forms a right triangle in which:
$bE = r$
$EB = R - 50 = r - 5$ (since $r = R - 45$)
$Bb = 45$
Therefore
$r^2 = (r \times 5)^2 + (45)^2$
$r^2 = r^2 - 10r + 25 + 2025$
$r = 205$
Thus, the two diameters are 410 and 500 mm.

Also solved by David Smith, Frederick Furland, Harry Zaremba, Winslow Hartford, Kelly Woods, Norman Wickstrand, David Wagger, Howard Stern, Ken Rosato, Ray Ellis, Michael Riezenman, Greg Spradlin, Matt Stenzel, Jim Czajka, Norman Spencer, John Cushnie, David Margulies, Stephen Callaghan, David Deleeuw, George Parks, Harry Garber, Tomo and Shirley Hasegawa, Art Harris, Steven Feldman, L.J. Upton, William Messner, Robert Bart, Stephen Berkenblit, Jonathan Aronson, Matthew Fountain, Richard Hess, Dennis White, Mary Lindenberg, Phelps Meaker, George Adams, R.L. Loesch, Charles Rivers, H.J. de Garcia, R.L. Loesch, and an anonymous reader from Irving TX.

APR 5. Arrange the integers from 1 to 25 in a 5 x 5 grid so that each column and row sums to 65. Moreover, each of the 10 diagonals is to sum to 65. These 10 diagonals are formed by starting at each of the 5 elements in the first row and proceeding southwest and southeast, identifying the left and right edges of the grid. For example, one diagonal includes the elements located at (1,4), (2,3), (3,2), (4,1), (5,5).

The following solution is from Jonathan Aronson: Another way of looking at this problem is to use the numbers 0 to 24 and have all rows, columns, and diagonals sum to 60. One can view 0 to 24 as all the two-digit numbers base 5 and thus break up the original problem into two smaller problems, making magic squares for the ones and fives places. These are really the same problem since the latter can be obtained by multiplying each entry in the former by 5, which has the effect of adding a zero to each entry (base 5). Here is one solution obtained by placing 0 to 4 in the first row and then rotating:

0	1	2	3	4
3	4	0	1	2
1	2	3	4	0
4	0	1	2	3
2	3	4	0	1

As mentioned, multiplying by 5 gives the second square. Before adding the two squares, we rotate the second about its main diagonal (to prevent duplicate entries). Finally, add 1 to each entry so the numbers go from 1 to 25 as specified. This yields the final solution:

1	17	8	24	15
9	25	11	2	18
12	3	19	10	21
20	6	22	13	4
23	14	5	16	7

Also solved by L.J. Upton, Winslow Hartford, Harry Zaremba, Matthew Fountain, Richard Hess, Dennis White, and the proposer, Ronald Martin.

Better Late Than Never

1985 A/S 3. Dennis White agrees with the proposer that the four-port electronic device produces an arc of a circle, thus disagreeing with the published solution. Mr. White's argument, which would be hard to typeset, is available from the editor.

1987 N/D 3. Karim Roshd enjoyed the solution.

N/D 4. Mark Lively noticed that it is not hard to transform the figure printed into Pascal's triangle, giving a simpler method for calculating the entries. Gordon Rice also found a simpler solution.

N/D 5. Mark Lively, Richard Hess, Gordon Rice, Jorgen Hornse, and Matthew Fountain have objected to the solution printed. (The published answer was $(n-1)^{(n-2)}$.) Mr. Lively writes:
The first factor anchored an arbitrary woman and randomly placed the other $(n-1)$ women. As a minor point, isn't having someone to my right the same as having the same person to my left? Thus, shouldn't the first factor be $(n-1)/2$? The second factor allowed any man a choice of the $(n-2)$ empty seats not next to his date. That selection determined the selection for every other man. The men sit in the same order as their dates but all rotated around the table by 3 seats, or by 5 seats, or by 7 seats, . . . However, the selection process for the men need not be that rigid once we get past 4 couples. Assume 5 couples and that the women are seated. Refer to them as ABCDE in order around the table. Assuming the first position for men is between A and B and the last is between E and A, the men can sit

caeba	daebc	eabcd*
cdeab*	daecb	edabc
ceabd	deabc*	edacb
cebad	deacb	edbac
	debac	

The asterisks indicate the $(n-2=3)$ combinations that are in the same order as the women but with the men rotated 3, 5, or 7 seats from their dates.

Once n women are seated, how many ways *can* n men be seated? n! But many of these ways are illegal in that a man sits next to his date. a can't sit on either side of A, which rules out $2(n-1)!$ combinations. Similarly for b, c, d, etc. Once a has been seated, many of the $2(n-1)!$ illegal ways for b have already been eliminated. If b is in chair AB and a is in chair XA, there are $(n-2)!$ ways to arrange the other men. These combinations were eliminated in the placement of a. If b is in chair BC, a can be in XA or AB for additional double eliminations. Thus, once a has eliminated $2(n-1)!$ combinations, the number of combinations b can eliminate are
$2(n-1)! - 3(n-2)!$
Then the number of combinations c can eliminate are
$2(n-1)! - 7(n-2)! + 4(n-3)!$
Then the number of combinations d can eliminate are
$2(n-1)! - 11(n-2)! + 16(n-3)! - 5(n-4)!$

Due to space constraints, Lively's development of the coefficients of the factorials has been omitted but is available from the editor. The coefficients are the sums of the $(2n-1)$ and $(2n)$ terms. Once the 10 women are seated the 10 men *may* be seated 439,792 ways.

1988 FEB 1, FEB 2. Caroline Richardson, David Simen, and Jonathan Aronson have responded.

FEB 3. Jonathan Aronson has responded.

Proposers' Solutions to Speed Problems

SD 1. Indeterminant.

SD 2. Archimedes needed to know how fast the passerby walked.