ALLAN J. GOTTLIEB

Palindromes and a Naymandidge

Since it has been a year since I reviewed the criteria used to select solutions for publication, let me do so now.

As responses to problems arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred. I favor contributions from correspondents whose solutions have not previously appeared, as well as solutions that are neatly written or typed, since these produce fewer typesetting errors.

Electronic mail can be sent to me using the internet address gottlieb @ nyu.edu, and I can format responses with the standard unix tools (troff, eqn, tbl, pic, −me −ms macros).

Problems

JUL 1. We begin with a computer-oriented problem from Matthew Fountain, inspired by Fermat:

Write an efficient computer program for searching for positive integer solutions to the equation

$x^3 + y^3 + c = z^3$

and list all values of c between 0 and 100 for which you find solutions. You will need to bound x and y (thereby bounding z). Efficiently searching all values less than a few thousand is a modest (but not trivial) computation.

JUL 2. The following naymandidge (defined below) is from Neil Macdonald's puzzle column in the September-October issue of *Computers and People*:

In a naymandidge, an array of random or psuedo-random digits ("produced by *nature*"), such as that at the bottom of this column, has been subjected to a "definite systematic operation" ("chosen by *nature*"). The problem ("which *man* is faced with") is to figure out what was *nature's* operation. A "definite systematic operation" meets the following requirements: the operation must be performed on all the digits of a definite class which can be designated; the result must display some kind of evident, systematic, rational order and completely remove some kind of randomness; and the operation must be expressible in not more than four English words. (But *man* can use more words to express the solution and still win.)

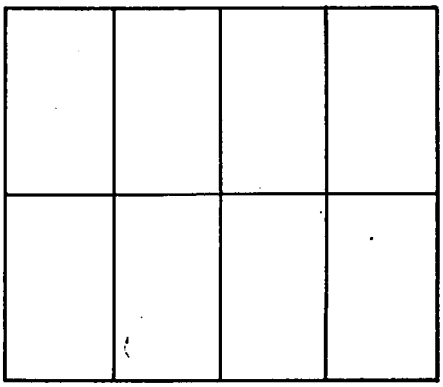
4 8 0 1 8 9 1 7 5 1 0 4 4 6 2 1 7 4 7 7
5 1 8 6 4 8 9 1 4 4 3 0 0 4 2 1 0 9 8 7
9 3 2 9 9 6 5 2 3 5 6 0 5 8 2 2 9 7 5 0
4 9 6 2 4 8 7 7 5 3 6 4 5 0 7 1 9 7 9 3
8 0 1 5 0 2 9 4 6 0 9 9 4 3 3 1 1 6 7 0
8 5 7 8 6 6 2 2 4 6 9 8 7 5 8 1 7 9 2 1
4 7 4 3 8 7 4 7 6 1 1 5 6 1 4 2 8 4 0 4
0 5 4 1 7 4 1 5 2 1 1 2 7 0 3 2 2 7 9 3
6 8 9 9 2 2 9 4 6 7 7 2 4 1 0 1 6 8 6 8
2 6 6 7 8 0 9 7 6 5 1 7 8 4 3 2 0 1 0 0

JUL 3. Dave Mohr has a pure sinusoidal tone to which he proposes to add a second pure sinusoidal tone of identical amplitude and frequency but with a random phase shift. What is the probability that the resulting sound is lower in volume than the original tone?

JUL 4. Frank Rubin is interested in doubly palindromic numbers, i.e. a number N that is palindromic when expressed in each of two bases p and q, where gcd(p,q) = 1 and N exceeds the product pq. Are there any such N? If so, is there a largest?

JUL 5. Robert Johnson needs help in folding his maps:

Take an 8 1/2 × 11 inch piece of paper (preferably with writing on both sides) and fold it in half three times, as if you were about to store it into your shirt pocket. Now unfold it. If you are like most of us, the fold lines will separate the paper into two rows of four rectangles all of which are identical:



In how many unique ways can you completely fold up the paper, assuming that you only make flat folds along the fold lines? Now suppose you had a strip of paper containing one row of six rectangles. How many ways can you fold that? What are the odds that the large map inside your automobile glove compartment is correctly folded?

Speed Department

SD 1. Jim Landau recalls that Robert Moeser's solution of 1986 JAN 3, while discussing reciprocals of prime numbers, comes to a conclusion and then says "viola!" Mr. Landau now asks, What relation do musical instruments have to reciprocals of primes?

SD 2. On a one-year certificate of deposit, Stephen McAdam's bank pays 8% compounded quarterly, with a 2% bonus during the first quarter (so the rate that quarter is 10%). Another bank also pays 8% compounded quarterly, but they pay the 2% bonus during the last quarter. Which offers the better deal?

Solutions

F/M 1. Define an ordering of n! permutations of n objects by considering the objects as the first n letters of the alphabet and arranging the n! permutations in alphabetical order. For n = 3 this gives

1. A B C	4. B C A
2. A C B	5. C A B
3. B A C	6. C B A

The mth permutation is now defined as the permutation that is number m in alphabetical order. Devise an nonrecursive algorithm and/or write a program which will find the mth permutation of n objects. By nonrecursive is meant that the algorithm

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

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with TEXT_IO, INTEGER_TEXT_IO, FACTORIAL;

procedure PERMUTE is
  SET : STRING (1..26) := "ABCDEFGHIJKLMNOPQRSTUVWXYZ";
  M, N, J : INTEGER;
begin
  TEXT_IO.PUT ("Enter the number of elements: ");
  INTEGER_TEXT_IO.GET (N);
  TEXT_IO.PUT ("Enter the permutation number: ");
  INTEGER_TEXT_IO.GET (M);

  for I in 1..N - 1 loop
    J := (M * FACTORIAL (N - I) - 1) / FACTORIAL (N - I);
    SET (I..I + J - 1) := SET (I + J - 1) & SET (I..I + J - 2);
    M := M - ((J - 1) * FACTORIAL (N - I));
  end loop;

  TEXT_IO.PUT_LINE (SET (1..N));
end PERMUTE;
```

can find the mth permutation without knowing (or computing) the (M − 1)th or any other permutation.

Stephen Merola writes:

Here is a non-recursive solution. A program, written in ADA under VAX/VMS, is shown at the top of this column. The program iterates from 1 to n calculating which element of the set belongs in that position. Assuming that we're using the letters of the alphabet, the program starts with a sorted array, e.g. for n = 4 we have ABCD. The ith element of the final array is the jth element of the remaining elements where j is given by:

$j = [m/(n-1)!]$

Here's some pseudo code:

for I = 1 to N = 1 do
J = ceiling ($M/(N-I)!$)
SET (I..I+J−1) = SET (I+J−1) & SET (I..I+J−2)
M = M − (J−1)*(N−I)!
end do

For initial conditions, N = 4, SET = ABCD, M = 6:

After iteration 1, J = 1, N = 3, SET = ABCD, M = 6

After iteration 2, J = 3, N = 2, SET = ADBC, M = 2

After iteration 3, J = 2, N = 1, SET = ADCB, M = 1

At the end of this loop, the array of characters, SET, initially sorted, will now be in the order of the Mth permutation.

Also solved by Matthew Fountain, Robert Bart, Roger Spellman, Lorenzo Sadun, Richard King, Richard Hess, Louis Howell, John Chandler, and Steven Feldman.

F/M 2. Given positive integers x and y, find positive integers a, b, c, and z satisfying $a^x + b^y = c^z$.

If z = 1, the problem is trivial: we can choose a and b arbitrarily and let
$c = a^x + b^y$.

Lorenzo Sadun suggests
$a = 2^y$, $b = 2^x$, $c = 2$, $z = xy + 1$.

Also solved by Matthew Fountain, Ken Rosato, John Chandler, Louis Howell, Richard Hess, Richard King, Robert Bart, and the proposer, Frank Rubin.

F/M 3. Find the smallest integer A such that the first ten digits of the square root of A are distinct.

The following solution is from Louis H. Howell: I didn't see a way to do this analytically, so I made a short program to search for solutions. In less than a second on a Sun3 workstation it found that the number 1362 has 36.9052841744919... as its square root. There are five other solutions less than 10000: 1843, 2540, 4280, 5507, and 6896. I saw no particular pattern to later solutions, except that the solutions tend to be rather bunched up in some intervals, while other large intervals have no solutions at all.

Also solved by Matthew Fountain, Richard Hess, Steven Feldman, John Prussing, Roger Spellman, Robert Bart, and the proposer, Nob Yoshighara.

F/M 4. Form palindromes of the type "—— was I ere I saw ——," where the *last* word of the sentence is a place name, and where the cleverness of the composition depends on to whom you attribute it. The classic is "able was I ere I saw Elba," attributed to Napoleon.

I am printing all the solutions received for this one, not claiming that I fully understand each one:

Frederick Furland sent two:

Sage v. salt at Las Vegas—Food controversy headline

A laconic in Ocala—Calvin Coolidge visits Florida headline

Robert Bart also sent two:

Not so, Ben, was . . . saw N.E. Boston—Washington to Franklin

No Geronimo or on was . . . saw no room in Oregon—Crazy Horse to Egyptian archeologist

John Chandler adds:

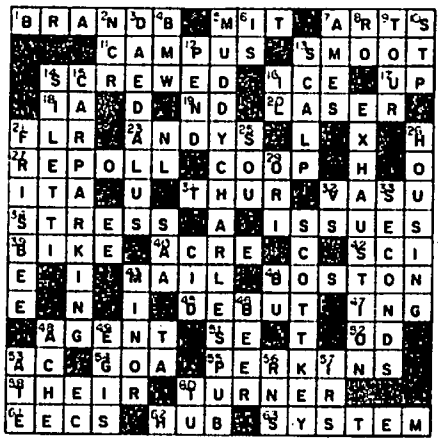
Nowhere was I ere I saw Erehwon—James Hilbert

Finally, five from the proposer, Solomon Golomb:

A rat was I ere I saw Tara—Rhett Butler
Zeus was I ere I saw Suez—Nasser
Naive was I ere I saw Evian—M. Perrier
An ole crab was I ere I saw Barcelona—Franco
A slut was I ere I saw Tulsa—name withheld

FM 5. Much to my surprise, John Chandler did not have too much trouble with this crossword. He writes as follows:

This isn't really a proper crossword puzzle, since there are 27 squares with no "cross" word, and the fact that clues for 13A, 37A, 21D, 35D, 36D, 38D, and 41D are missing doesn't help at all. Indeed, the labels and clues for 21 and 22 are scrambled, there are typos in the clues for 60D (should be "Et—, Brute") and 18A (a suffix can't possibly be medial: I assume it should be medical). Still, I can't resist a crossword . . .



I must confess that I'm a little unsure of 14D (and consequently 18A) and 32A (which crosses an undefined word), but otherwise, it seems pretty clear-cut.

Also solved by Thomas Goldfrank and the original *Graduate Student News* proposer, David Wagger. Mr. Wagger notes that *GSN* did add a few typos.

Better Late Than Never

1987 OCT 4. John Silvasy believes the width is 26.458 and the length is 13.333.

1987 OCT 5. Thomas Murley reports that he wrote a paper on this subject at M.I.T. in 1964.

JAN 1. Matthew Fountain has responded.

JAN 2. Matthew Fountain and Edward Dawson have responded.

JAN 3. Charles Markham, Matthew Fountain, and Edward Dawson have responded.

JAN 2. Doug Tritsch, Matthew Fountain, and Edward Dawson have responded.

Proposers' Solutions to Speed Problems

SD 1. By a well known theorem of Euler,

$$\prod_p (1 - 1/p)^{-1} = \sum_{n=1}^{\infty} 1/n$$

But the right hand side is the famous "Harmonic series."

SD 2. Neither; they are the same.